Mobile Communications (KECE425)

Lecture Note 3

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Summary

- Path loss in free space
- Shadowing
- Link budget

Power Attenuation in Free Space

- Received signal power at d denoted as $\Omega_p(d)$ in free space

$$\Omega_p(d) = \Omega_t k \left(\frac{\lambda_c}{4\pi d} \right)^2$$

where

- Ω_t: transmit power
- λ_c: wavelength where $\lambda_c = \frac{c}{f_c}$.
- k: a constant of proportionality
- d: distance between the transmitter and the receiver

- In decibel domain, we have

$$\Omega_{p(\mathrm{dBm})}(d) = 10\log_{10}(\Omega_p(d) \times 10^3)$$

$$= 10\log_{10}\left[\Omega_t k \left(\frac{\lambda_c}{4\pi d}\right)^2 \times 10^3\right]$$

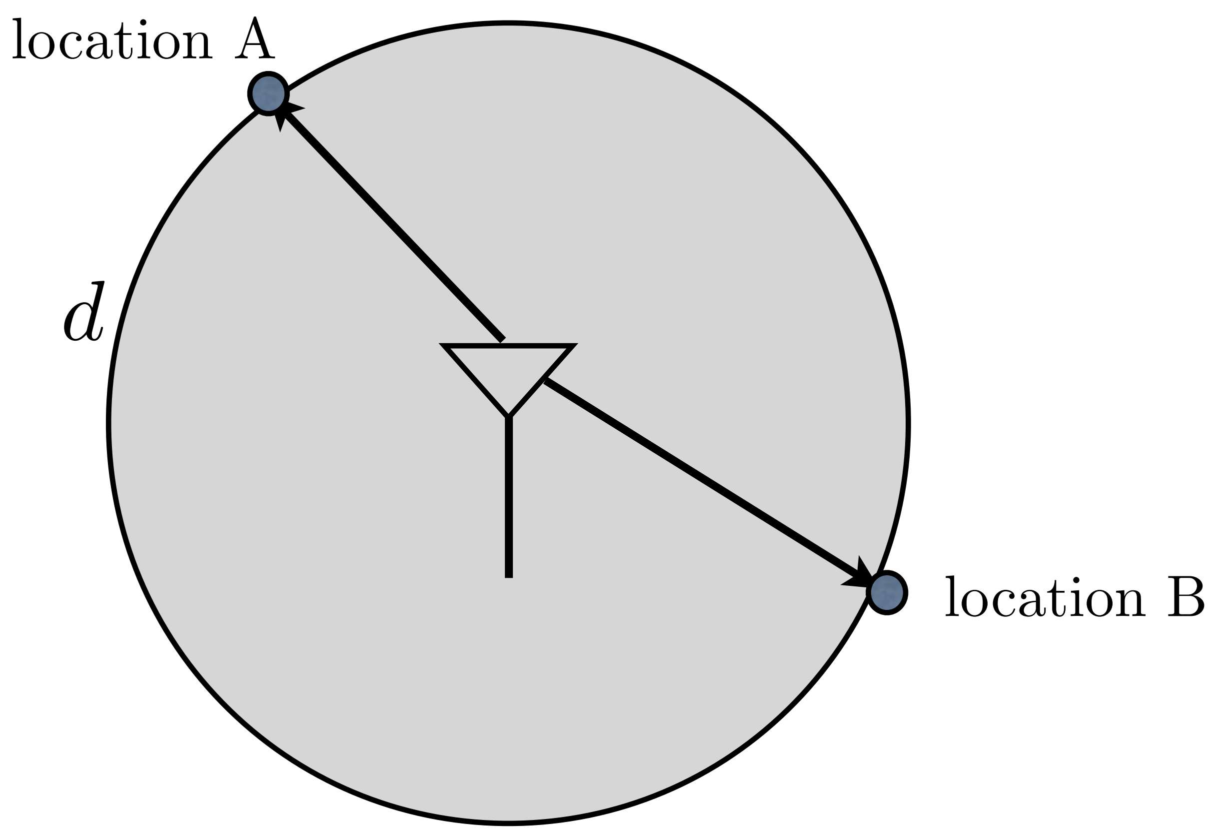
$$= \underbrace{10\log_{10}\left(\Omega_t \times 10^3\right)}_{=\Omega_{t(\mathrm{dBm})}} + 10 \times 2\log_{10}\left(\frac{\lambda_c}{4\pi d}\right) + 10\log_{10} k$$

$$= \Omega_{t(\mathrm{dBm})} - 20\log_{10}(d) + 10\log_{10}(k')$$

$$\text{where} \quad k' = \frac{\lambda_c^2}{16\pi^2}$$

- In free space, the received signals of the locations at the same distance are all the same as

$$\Omega_{p(\mathrm{dBm})}(d) = \Omega_{t(\mathrm{dBm})} - 20\log_{10}(d) + 10\log_{10}(k')$$



Power at A = Power at B

- Let us assume that the power at the distance d_0 is $\Omega_{p(\mathrm{dBm})}(d_0)$:

$$\Omega_{p(\mathrm{dBm})}(d_0) = \Omega_{t(\mathrm{dBm})} - 20\log_{10}(d_0) + 10\log_{10}(k')$$

or equivalently we can write

$$\Omega_{p(\mathrm{dBm})}(d_0) + 20\log_{10}(d_0) = \Omega_{t(\mathrm{dBm})} + 10\log_{10}(k')$$

- Then we can express the power at a certain distance d as $\Omega_{p(\mathrm{dBm})}(d_0)$ as

$$\begin{aligned}\Omega_{p(\mathrm{dBm})}(d) &= \boxed{\Omega_{t(\mathrm{dBm})}} - 20\log_{10}(d) + \boxed{10\log_{10}(k')} \\ &= \boxed{\Omega_{p(\mathrm{dBm})}(d_0) + 20\log_{10}(d_0)} - 20\log_{10}(d) \\ &= \Omega_{p(\mathrm{dBm})}(d_0) - 20\log_{10}(d/d_0)\end{aligned}$$

Example

- Assume that the measured power in free space at distance 500 m is 16 dBm:

$$\Omega_{p(\mathrm{dBm})}(0.5\mathrm{km}) = 16\,\mathrm{dBm}$$

- Calculate the power at distance $d = 1$ km, $d = 2$ km, and $d = 3$ km.

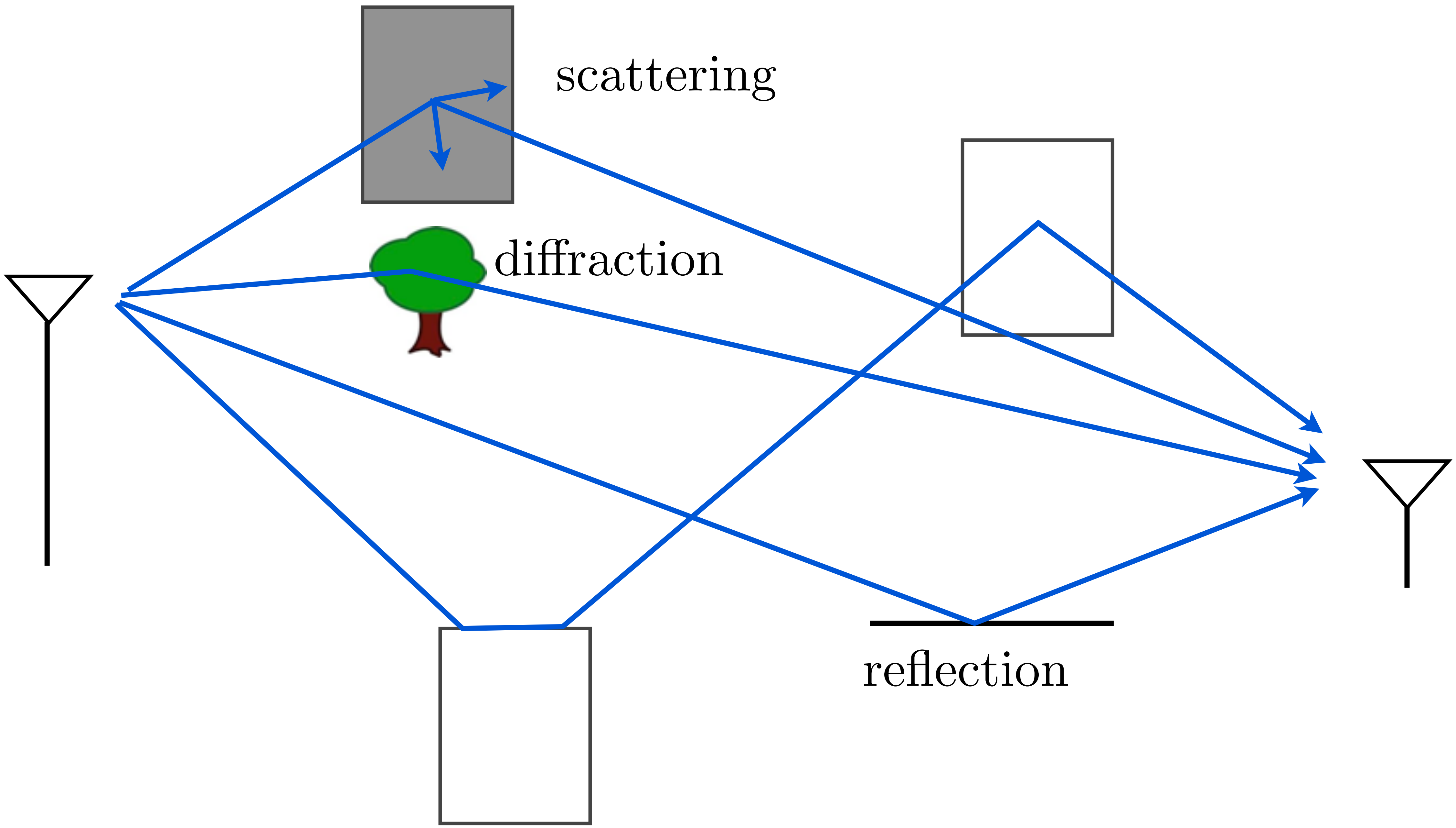
$$\Omega_{p(\mathrm{dBm})}(1\mathrm{km}) = 16 - 20\log_{10}(1/0.5) = 9.9794\,\mathrm{dBm}$$

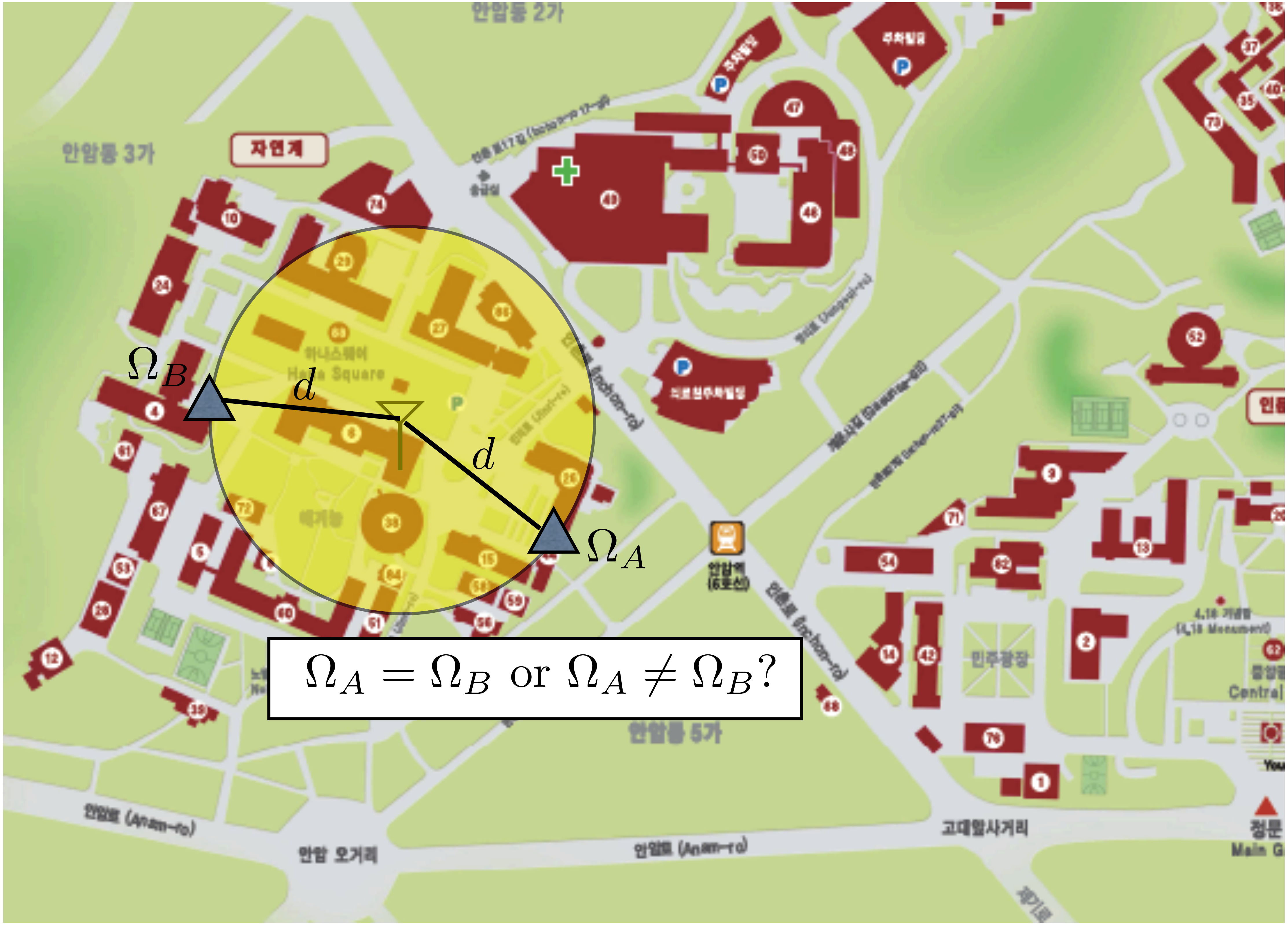
$$\Omega_{p(\mathrm{dBm})}(2\mathrm{km}) = 16 - 20\log_{10}(2/0.5) = 3.9588\,\mathrm{dBm}$$

$$\Omega_{p(\mathrm{dBm})}(3\mathrm{km}) = 16 - 20\log_{10}(3/0.5) = 0.4370\,\mathrm{dBm}$$

Wireless Radio Propagation

- Radio signals generally propagate according to three mechanisms:





$$\Omega_A = \Omega_B \text{ or } \Omega_A \neq \Omega_B?$$

Shadowing

The received signal power is random variable which is called shadowing.

Path Loss in Wireless Environment

- From the experimental measurements, the received power is the form of:

$$\Omega_p(d) = \Omega_t k \left(\frac{\lambda_c}{4\pi d}\right)^\beta \cdot \epsilon$$

random variable

where β is called "path loss exponent" ranged from 2 to 8.

- Received (average) power in decibel domain:

$$\Omega_{p(\mathrm{dBm})}(d) = 10\log_{10}(\Omega_t \times 1000) - 10\beta\log_{10}(d) + 10\log_{10}(k') + \epsilon_{(\mathrm{dB})}$$

$$= \Omega_{t(\mathrm{dBm})} + 10\log_{10}(k') - 10\beta\log_{10}(d) + \epsilon_{(\mathrm{dB})}$$

where $\epsilon_{(\mathrm{dB})} = 10\log_{10}(\epsilon)$ is random variable.

Area Mean and Local Mean

$$\Omega_{p(\mathrm{dBm})}(d) = \Omega_{t(\mathrm{dBm})} + 10\log_{10}(k') - 10\beta\log_{10}(d) + \epsilon_{(\mathrm{dB})}$$

- It has been known that $\epsilon_{(\mathrm{dB})}$ is zero-mean Gaussian random variable with a certain variance σ_ϵ^2:

$$\epsilon_{(\mathrm{dB})} \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

- Let us define

$$\mu_{\Omega_{p(\mathrm{dBm})}}(d) = \Omega_{t(\mathrm{dBm})} + 10\log_{10}(k') - 10\beta\log_{10}(d)$$

which is called "area mean".

- Local mean $\Omega_{p(\mathrm{dBm})}$ can be written as

$$\Omega_{p(\mathrm{dBm})}(d) = \mu_{\Omega_{p(\mathrm{dBm})}}(d) + \epsilon_{(\mathrm{dB})}$$

- Note the following relation:

$$\mu_{\Omega_{p(\mathrm{dBm})}}(d_0) = \boxed{\Omega_{t(\mathrm{dBm})} + 10\log_{10}(k')} - 10\beta\log_{10}(d_0)$$

$$\begin{aligned}\mu_{\Omega_{p(\mathrm{dBm})}}(d) &= \boxed{\Omega_{t(\mathrm{dBm})} + 10\log_{10}(k')} - 10\beta\log_{10}(d) \\ &= \boxed{\mu_{\Omega_{p(\mathrm{dBm})}}(d_0) + 10\beta\log_{10}(d_0)} - 10\beta\log_{10}(d) \\ &= \mu_{\Omega_{p(\mathrm{dBm})}}(d_0) - 10\beta\log_{10}(d/d_0)\end{aligned}$$

- Then the local mean at d can be written as

$$\Omega_{p(\mathrm{dBm})}(d) = \underbrace{\boxed{\mu_{\Omega_{p(\mathrm{dBm})}}(d_0) - 10\beta\log_{10}(d/d_0)}}_{= \mu_{\Omega_{p(\mathrm{dBm})}}(d)} + \epsilon_{(\mathrm{dB})}$$

- Local mean is also Gaussian RV:

$$\Omega_{p(\mathrm{dBm})}(d) \sim \mathcal{N}\left(\mu_{\Omega_{p(\mathrm{dBm})}}(d), \sigma_\epsilon^2\right)$$

PDF of Local Mean in dBm

- PDF of local mean in dBm

$$p_{\Omega_{p(\mathrm{dBm})}(d)}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left[-\frac{(x-\mu_{\Omega_{p(\mathrm{dBm})}}(d))^2}{2\sigma_\Omega^2}\right]$$

where

$$\mu_{\Omega_{p(\mathrm{dBm})}}(d) = \mu_{\Omega_{p(\mathrm{dBm})}}(d_0) - 10\beta\log_{10}(d/d_0) \quad (\mathrm{dBm})$$

σ_Ω : shadow standard deviation ranged from 5 to 12 dB

Hence, we say the local mean follows the *log-normal* distribution.

PDF of Local Mean in Linear Scale

- Note that

$$\Omega_p(d) = 10^{\Omega_{p(\mathrm{dB})}(d)/10} \quad \text{where } \Omega_{p(\mathrm{dB})}(d) = \Omega_{p(\mathrm{dBm})}(d) - 30.$$

- Let $Y = \Omega_p(d)$ and $X = \Omega_{p(\mathrm{dB})}(d)$. Then,

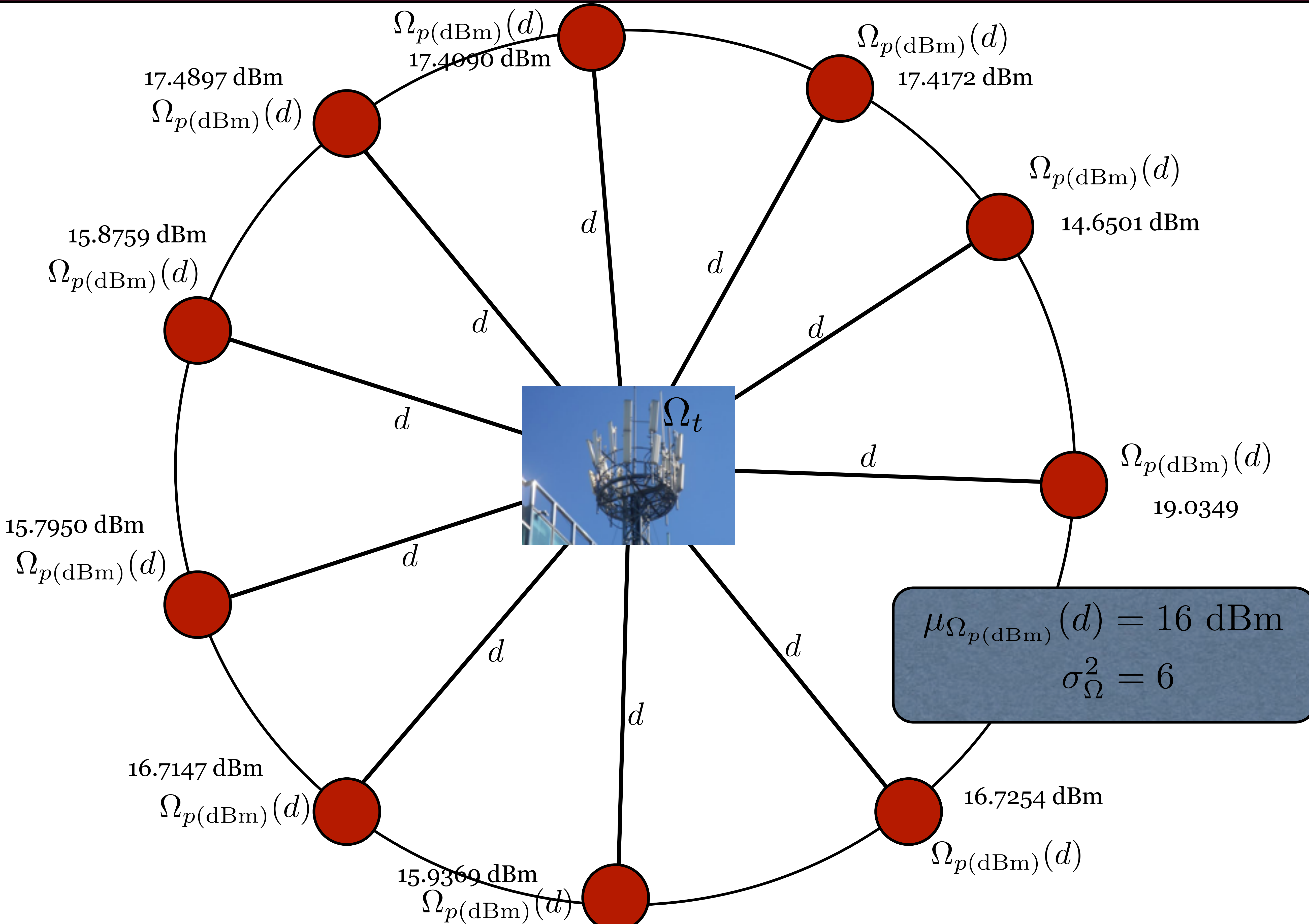
$$\frac{dY}{dX} = 10^{X/10} \ln 10 = Y \ln 10$$

- Transformation of random variable

$$p_{\Omega_p(d)}(y) = p_{\Omega_{p(\mathrm{dB})(d)}}(x) \frac{dX}{dY}\bigg|_{Y=10\log_{10}(X)}$$

- PDF of local mean in linear scale:

$$p_{\Omega_{p(\mathrm{dB})}}(y) = \frac{\zeta}{y\sqrt{2\pi}\sigma_\Omega} \exp\left[-\frac{(10\log_{10}(y) - \mu_{\Omega_{p(\mathrm{dB})}}(d))^2}{2\sigma_\Omega^2}\right]$$


$\Omega_{p(\mathrm{dBm})}(d)$
17.4090 dBm
$\Omega_{p(\mathrm{dBm})}(d)$
17.4172 dBm
17.4897 dBm
$\Omega_{p(\mathrm{dBm})}(d)$
$\Omega_{p(\mathrm{dBm})}(d)$
14.6501 dBm
15.8759 dBm
$\Omega_{p(\mathrm{dBm})}(d)$
Ω_t
$\Omega_{p(\mathrm{dBm})}(d)$
19.0349
15.7950 dBm
$\Omega_{p(\mathrm{dBm})}(d)$
$\mu_{\Omega_{p(\mathrm{dBm})}}(d) = 16$ dBm
$\sigma_{\Omega}^2 = 6$
16.7147 dBm
$\Omega_{p(\mathrm{dBm})}(d)$
16.7254 dBm
$\Omega_{p(\mathrm{dBm})}(d)$
15.9369 dBm
$\Omega_{p(\mathrm{dBm})}(d)$
d

Remarks

- Shadow standard deviation
  - In macro-cellular, $\sigma_\Omega = 8\,\mathrm{dB}$ is a typical value.
  - Nearly independent of the radio path length d
- Area mean

$$\mu_{\Omega_{p(\mathrm{dBm})}}(d) = \mu_{\Omega_{p(\mathrm{dBm})}}(d_0) - 10\beta \log_{10}(d/d_0) \quad (\mathrm{dBm})$$

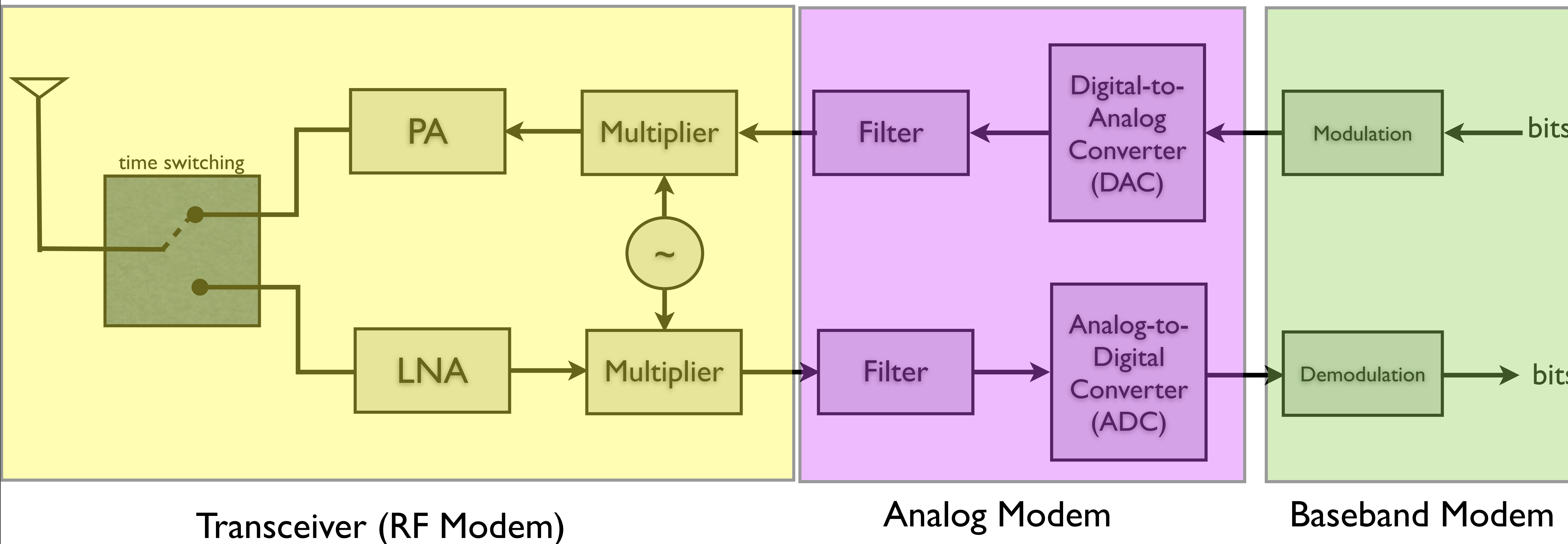
- Local mean

Gaussian RV with zero mean and variance σ_Ω

$$\Omega_{p(\mathrm{dBm})}(d) = \mu_{\Omega_{p(\mathrm{dBm})}}(d) + \epsilon_{(\mathrm{dB})}$$

  - Local mean is the received power with shadowing.

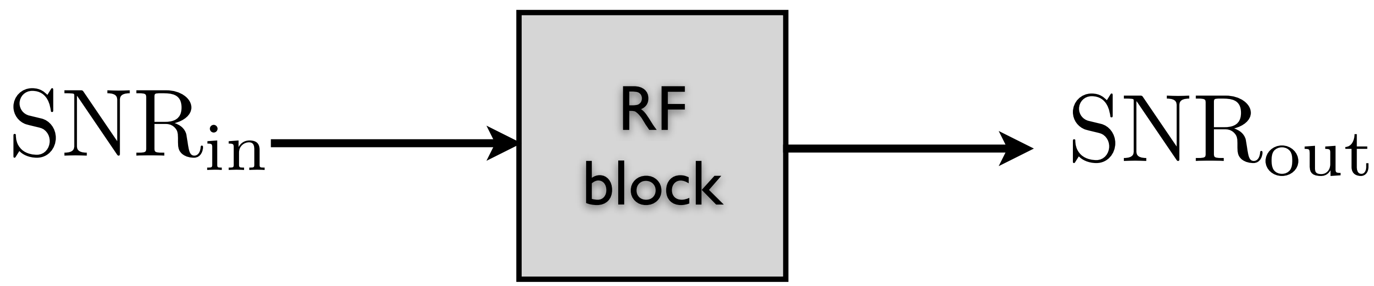
MODEM Architecture



- RF block is characterized by "noise figure" and "gain"

Noise Figure

- Noise factor F is defined as



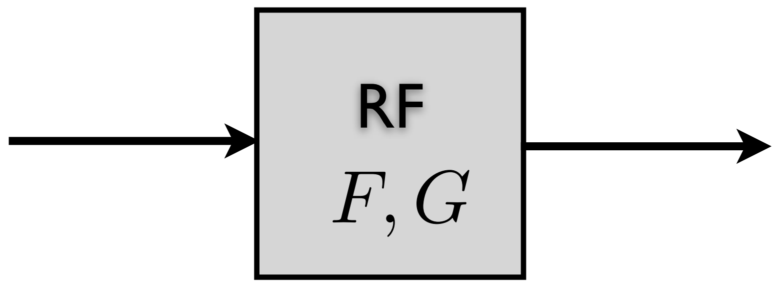
$$F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$$

- Noise figure NF is defined as

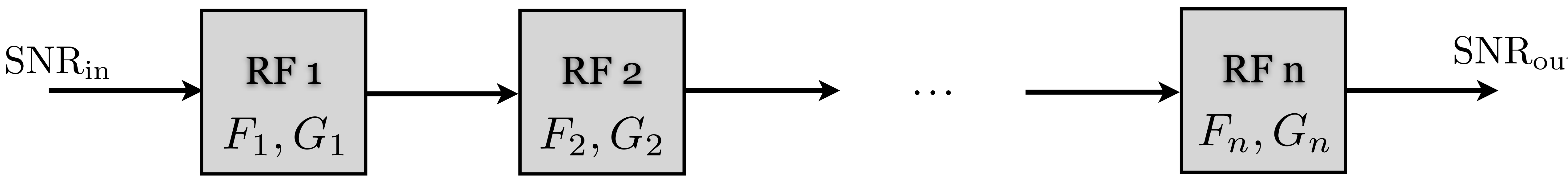
$$NF = 10\log_{10}(F) = 10\log_{10}\left(\frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}\right)$$

$$= \mathrm{SNR_{in,dB}} - \mathrm{SNR_{out,dB}}$$

- RF block is characterized by "noise figure" and "gain"



- Friis' formula



$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots + \frac{F_n - 1}{G_1 G_2 G_3 \cdots G_{n-1}}$$

$$F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$$

Noise and Received Power

- Total input noise power to the receiver

$$N = kT_0 B_w F$$

- Effective received carrier power

$$\Omega_p = \frac{\Omega_t G_T G_R}{L_{R_X} L_P}$$

- Received carrier-to-noise ratio

$$\Gamma = \frac{\Omega_p}{N} = \frac{\Omega_t G_T G_R}{kT_0 B_w F L_{R_X} L_p}$$

$$\begin{aligned}
\Omega_t &= \text{transmitted carrier power} \\
G_T &= \text{transmitter antenna gain} \\
L_p &= \text{path loss} \\
G_R &= \text{receiver antenna gain} \\
\Omega_p &= \text{received signal power} \\
T_0 &= \text{receiving system noise temperature in degrees Kelvin} \\
B_w &= \text{receiver noise bandwidth} \\
N_0 &= \text{white noise power spectral density} \\
R_c &= \text{modulated symbol rate} \\
k &= 1.38 \times 10^{-23} Ws/K \text{ Boltzmann's constant} \\
F &= \text{Noise figure, typically to 5 to 6dB} \\
L_{R_X} &= \text{receiver implementation loss}
\end{aligned}$$

Link Budget

- Modulated symbol energy-to-noise ratio

$$\frac{E_c}{N_0} = \Gamma \times \frac{B_w}{R_c}$$

- Link budget is defined as the symbol energy-to-noise ratio such as

$$\frac{E_c}{N_0} = \frac{\Omega_t G_T G_R}{k T_0 R_c F L_{R_x} L_p}$$

or in decibel unit as

$$\begin{aligned} (E_c/N_0)_{(\mathrm{dB})} &= \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} \\ &- kT_{0(\mathrm{dBm})/\mathrm{Hz}} - R_{c(\mathrm{dBHz})} - F_{(\mathrm{dB})} - L_{R_x(\mathrm{dB})} - L_{p(\mathrm{dB})} \end{aligned}$$