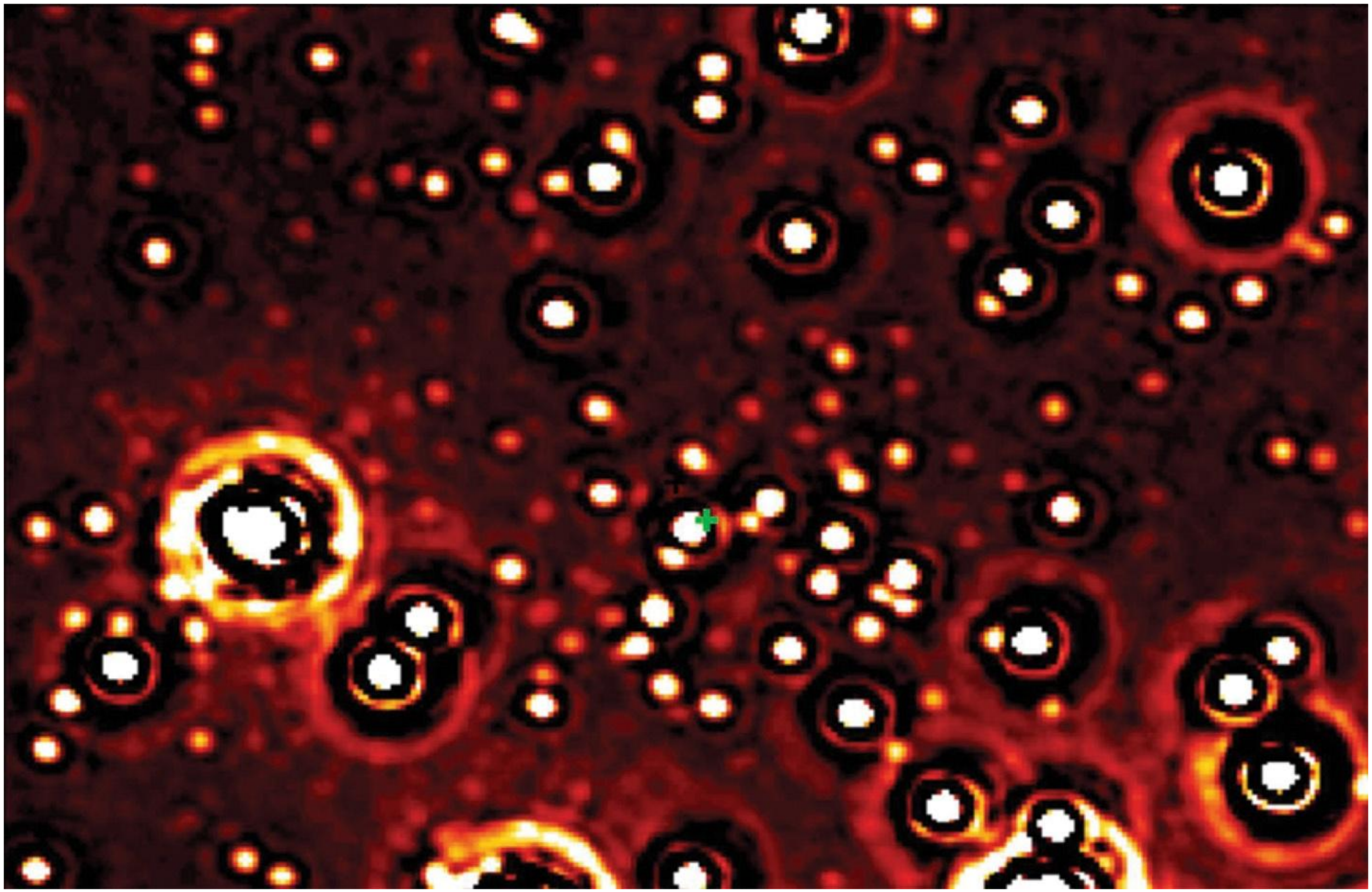
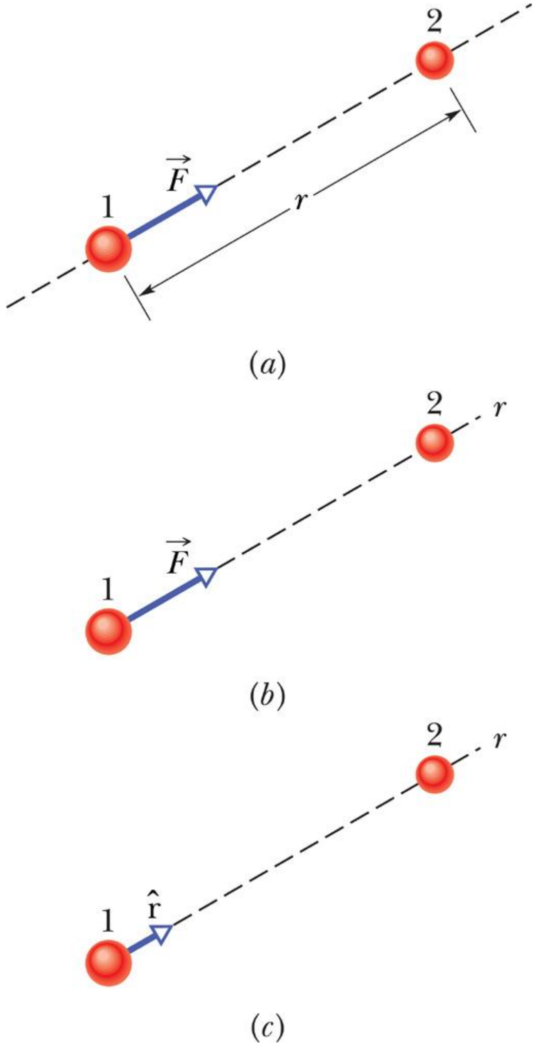
Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

Chap. 12 Gravitation



Newton's law of gravitation



$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r}$$

Gravitation constant G

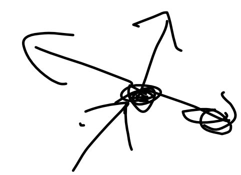
$$\begin{aligned} G &= 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2} \\ &= 6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2} \end{aligned}$$

Newton's shell theorem

Shell theorem

(1) 공 모양의 균일한 껍질은 마치 모든 질량이 중심에 모여있는 것처럼 외부의 입자를 잡아당긴다.

(2) 공 모양의 균일한 껍질 내부에 있는 입자는 이 껍질에 의한 중력이 상쇄되어 없어진다.



중력과 중첩원리

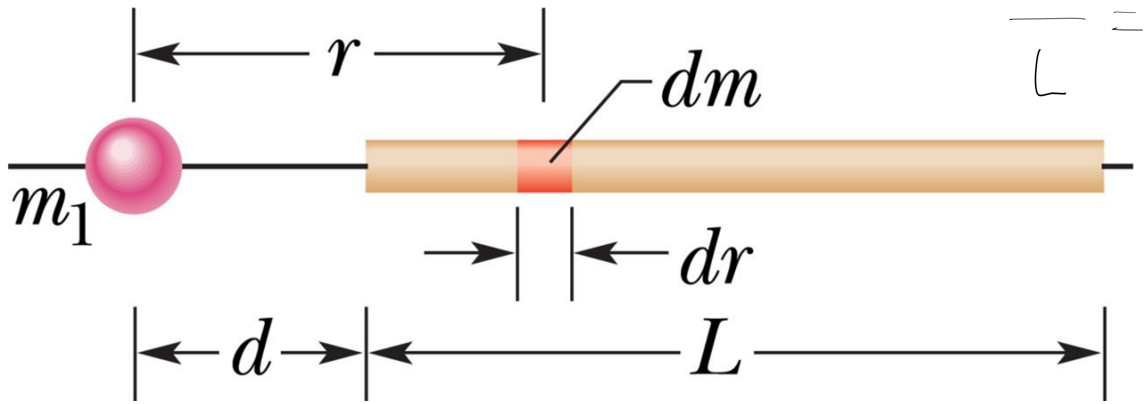
$$\vec{F}_{1,\mathrm{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}$$

$$= \sum_{i=2}^{n} \vec{F}_{1i}$$

연속적인 물체의 경우

$$\vec{F}_1 = \int d\vec{F}$$

Sample prob.



$L = 3.0\ \text{cm},\ M = 5.0\ \text{kg},\ d = 23\ \text{cm},\ m_1 = 0.67\ \text{kg}$

$$dF = \frac{Gm_1}{r^2}\,dm \qquad F = \int dF = \frac{Gm_1 M}{L}\int_d^{L+d}\frac{dr}{r^2}$$

$$dm = \frac{M}{L}\,dr \qquad = \frac{Gm_1 M}{d(L+d)} = 3.0\times 10^{-10}\ \text{N}$$

지표면 근처의 중력

$$F = G\frac{Mm}{r^2} = ma_g \longrightarrow a_g = \frac{GM}{r^2}$$

Variation of a_g with Altitude

Altitude (km)	a_g (m/s²)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

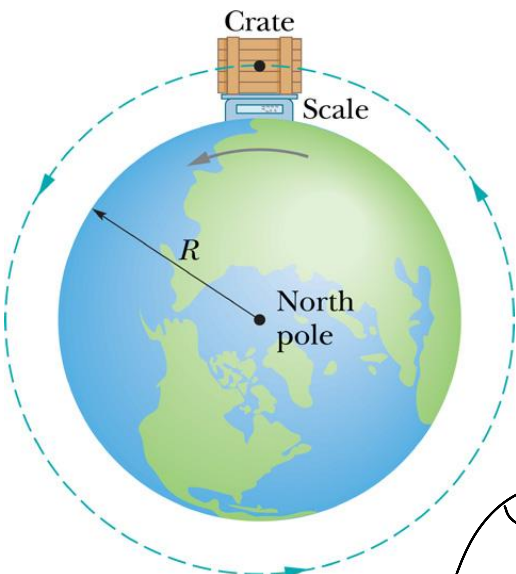
이유

(1) 지구는 균일하지 않다.

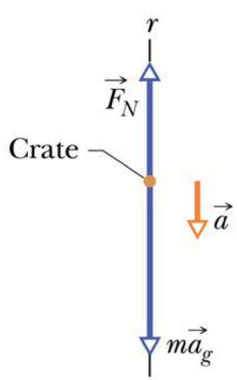
(2) 지구는 타원체이다.

(3) 지구는 자전하고 있다.

지구가 자전하면?



(a)

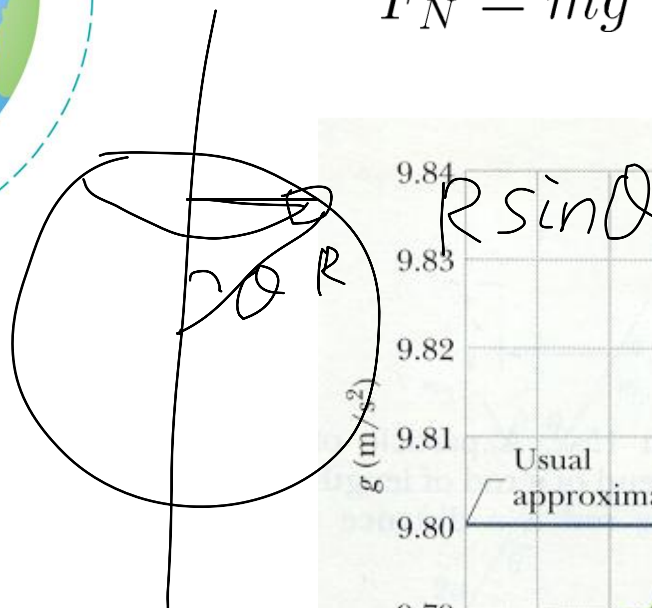


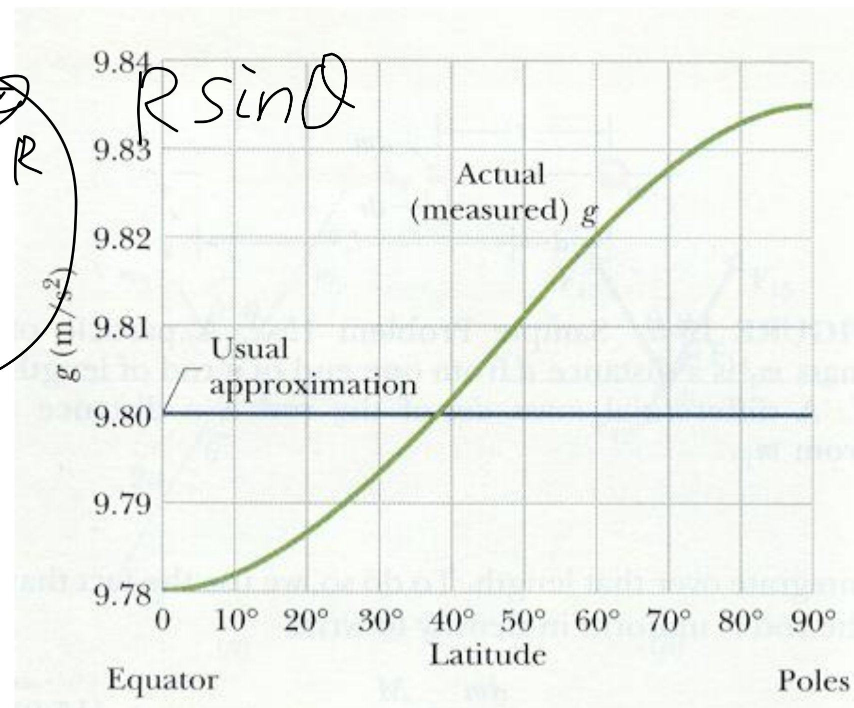
(b)

$$ma_g - F_N = m\omega^2 R$$

$$F_N = mg$$

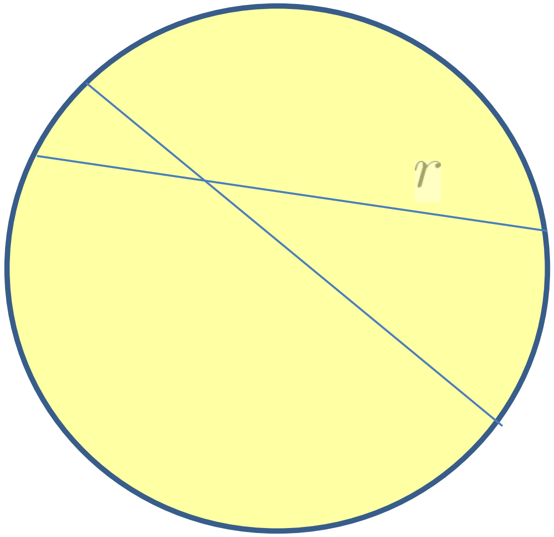
$$g = a_g - \omega^2 R$$





지구 내부의 중력

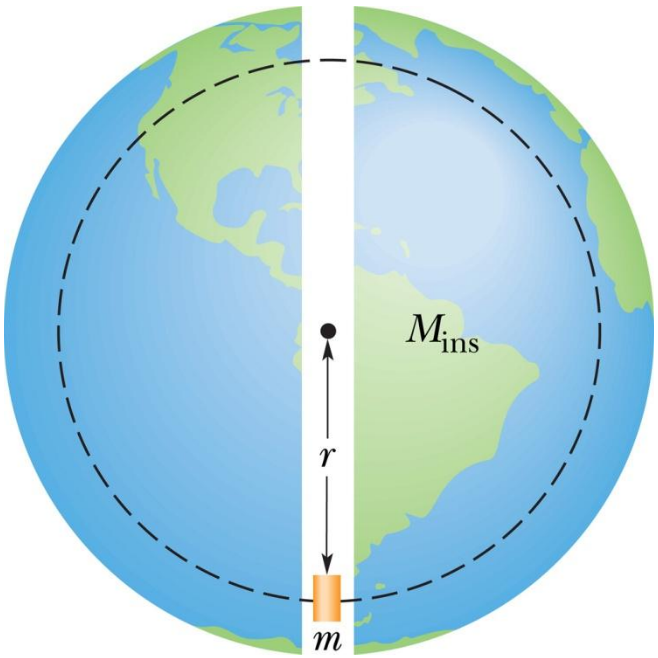
껍질 정리에 의하면 중력은 작용하지 않는다.



$dM \propto r^2$

$$dF \propto \frac{dM}{r^2} \propto \text{constant}$$

Sample prob.



$$F = \frac{GmM_{\text{ins}}}{r^2}$$

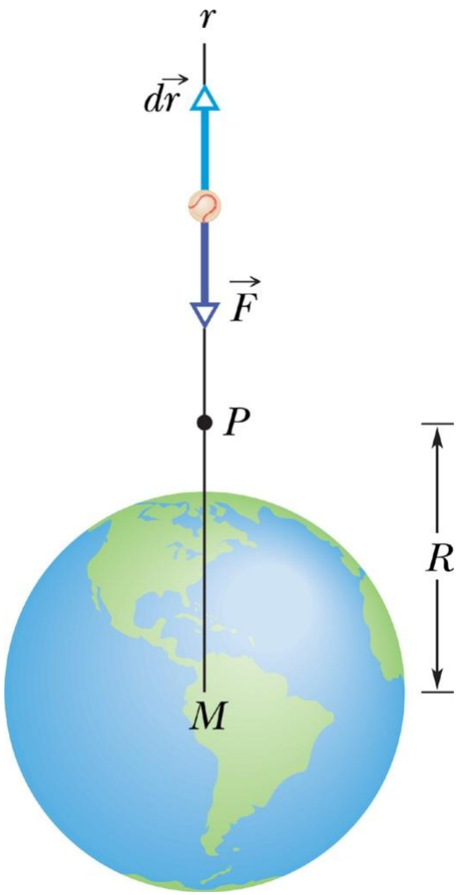
$$M_{\text{ins}} = \rho V_{\text{ins}} = \rho\frac{4\pi r^3}{3}$$

$$F = -\frac{4\pi}{3}Gm\rho r = -Kr$$

Gravitational potential energy

$$U = -\frac{GMm}{r}$$

무한대에서의 값을 0으로 정할 때



$$\vec{F}(r) \cdot d\vec{r} = -\frac{GMm}{r^2} dr$$

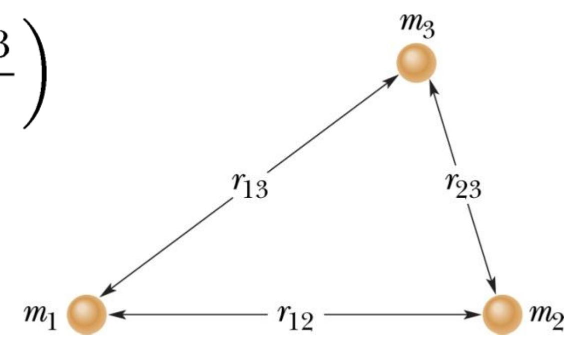
$$\begin{aligned} W &= \int_R^\infty \vec{F}(r) \cdot d\vec{r} \\ &= -GMm \int_R^\infty \frac{dr}{r^2} = \left[\frac{GMm}{r}\right]_R^\infty = -\frac{GMm}{R} \end{aligned}$$

$$\Delta U = -W \longrightarrow U_\infty - U = -W$$

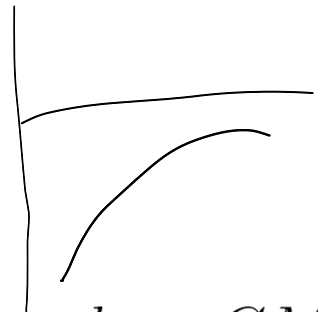
$$U = W = -\frac{GMm}{R}$$

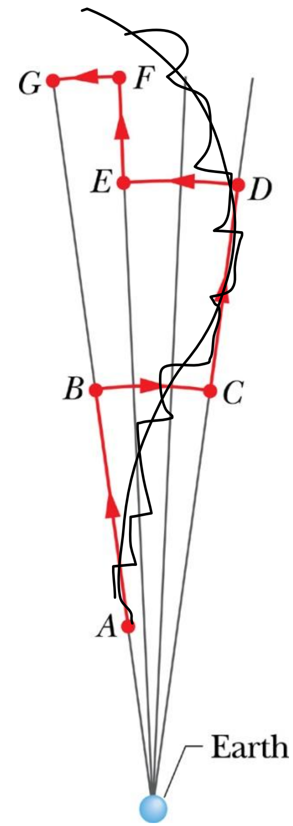
특징

(1) 중첩 $U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{GM_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right)$



(2) 경로에 무관

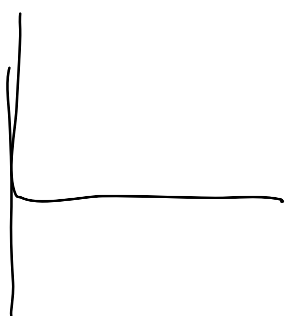




(3) Potential energy와 힘

$$F = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{GMm}{r}\right)$$
$$= -\frac{GMm}{r^2}$$

(4) Escape velocity

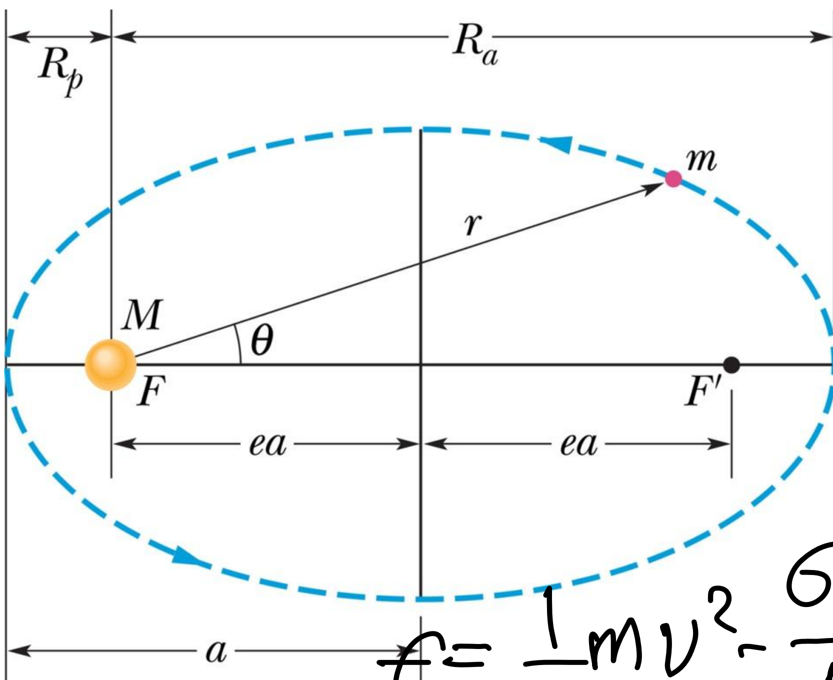


$$K + U = \frac{1}{2}mv^2 - \frac{GMm}{R} = 0$$

$$v = \sqrt{\frac{2GM}{R}}$$

Kepler의 법칙

1. 궤도법칙: 행성은 태양을 초점으로 하는 타원궤도를 돈다.



a 장축 (major axis)
e 이심률 (eccentricity)
(지구: 0.0167)

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

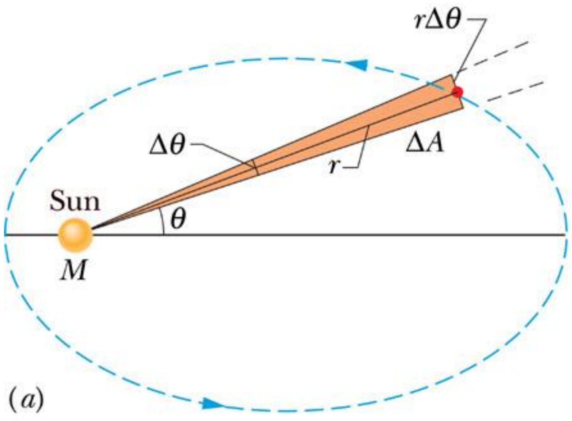
$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

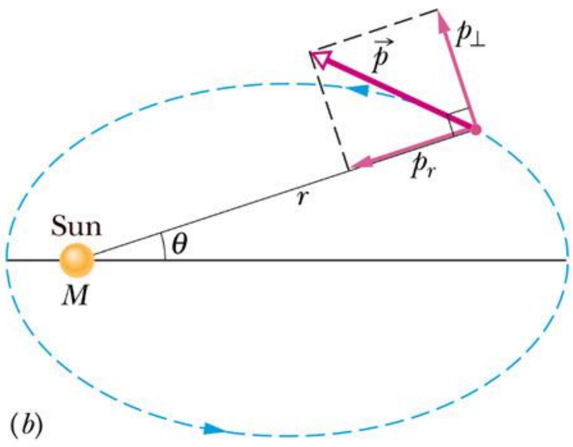
$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$E = K + U = -\frac{GMm}{2a}$$

e=0.74

2. 면적법칙: 면적속도는 항상 일정하다.





$$\Delta A \approx \frac{1}{2} r^2 \Delta\theta$$

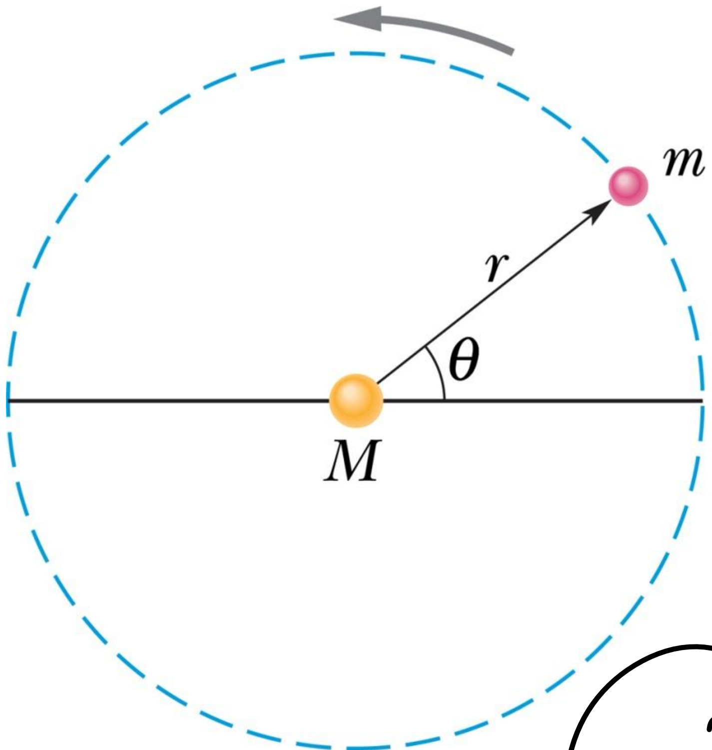
$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega$$

$$\frac{dA}{dt} = \frac{L}{2m}$$

$$L = r p_{\perp} = r m v_{\perp} = r(m\omega r) = m r^2 \omega$$

각운동량 보존과 동등

3. 주기법칙: $T^2 \propto a^3$



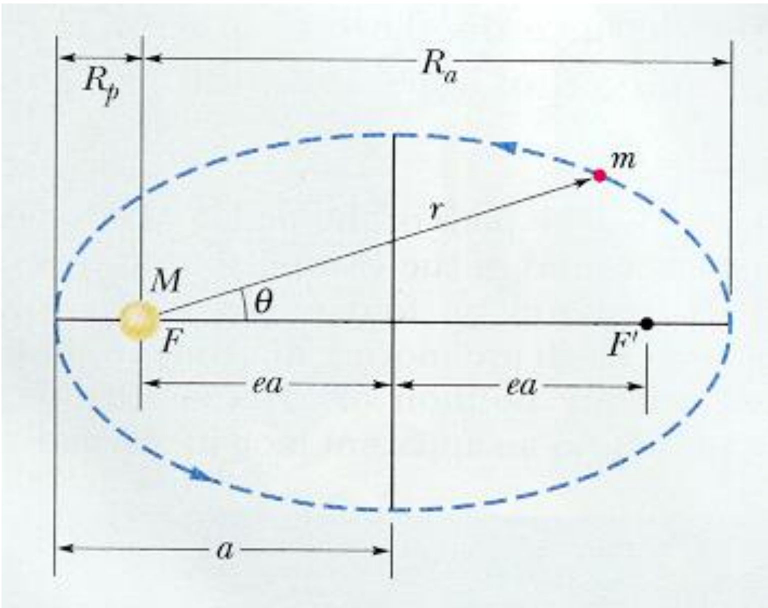
$$\frac{GMm}{r^2} = mr\omega^2 = mr\frac{4\pi^2}{T^2}$$

$$T\omega = 2\pi \longrightarrow \omega = \frac{2\pi}{T}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Sample prob.



Halley 혜성

$R_p = 8.9 \times 10^{10}$ m, $T = 76$ y

(a) 원일점(aphelion) 거리 R_a

$$R_a + R_p = 2a$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3$$

$$a = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = 2.7 \times 10^{12}\ \text{m}$$

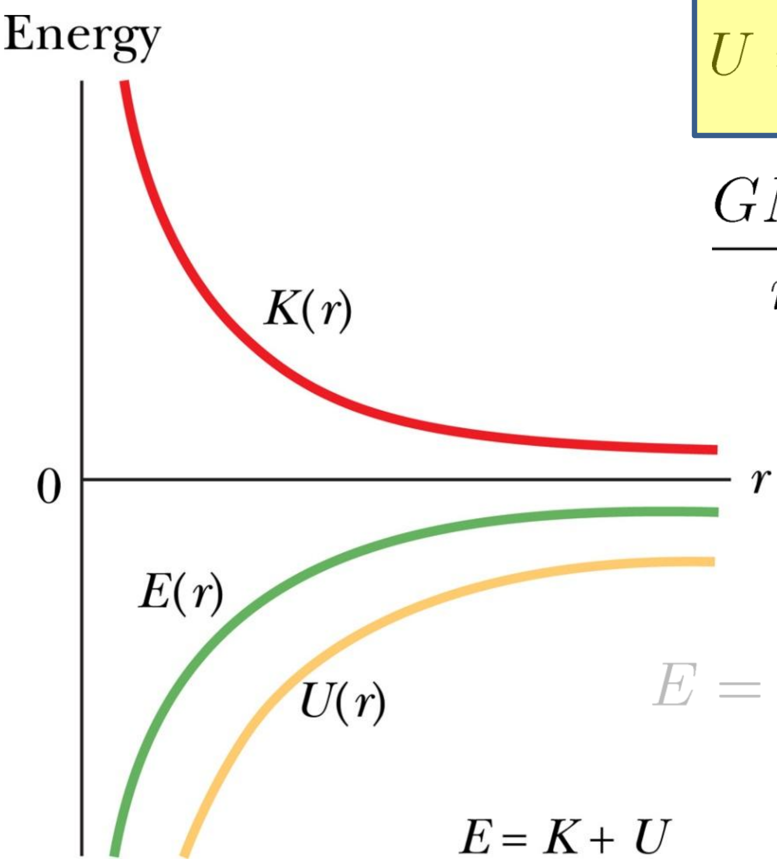
$$R_a = 2a - R_p = 5.3 \times 10^{12}\ \text{m}$$

(b) 궤도의 이심률 e

$$ea = a - R_p$$

$$e = \frac{a - R_p}{a} = 1 - \frac{R_p}{a} = 0.97$$

위성: 궤도와 에너지



$$U = -\frac{GMm}{r}$$

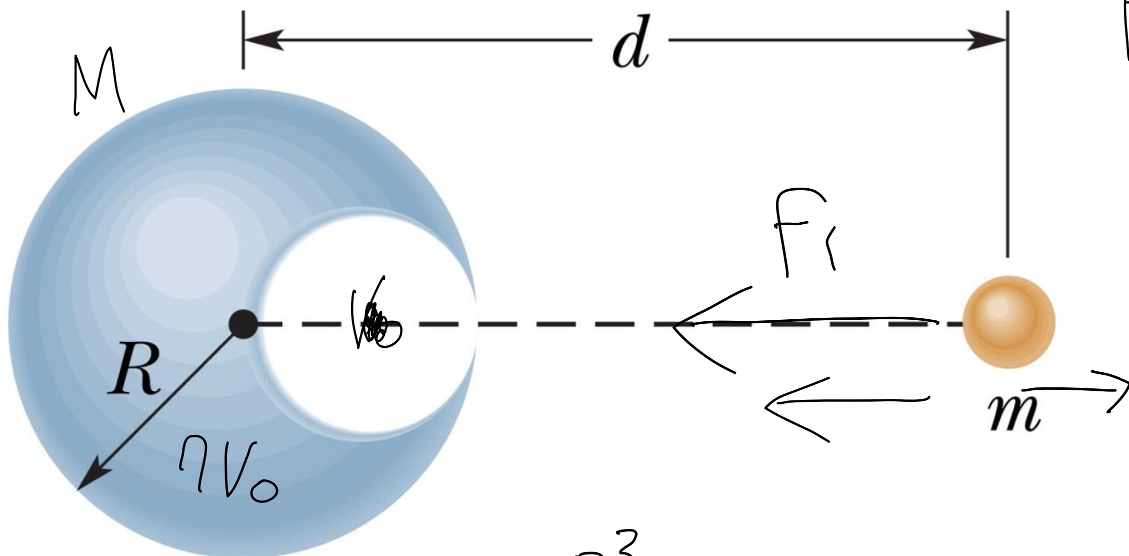
$$\frac{GMm}{r^2} = \frac{mv^2}{r} \longrightarrow K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$K = -\frac{U}{2} \text{(circular orbit)}$$

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

$$E = -K = -\frac{GMm}{2a} \text{(elliptic orbit)}$$

Problem 1



$$F_1 = \frac{G m \frac{8}{7} M}{d^2}$$

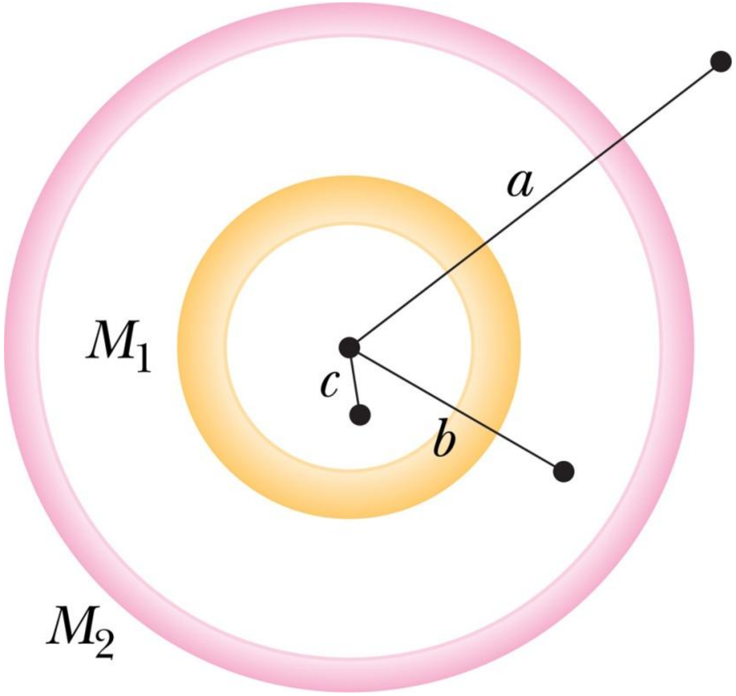
$$F_2 = \frac{G m \frac{M}{7}}{\left(d - \frac{R}{2}\right)^2}$$

$$\frac{4}{3}\pi R^3 \qquad \frac{4}{3}\pi \frac{R^3}{8}$$

$$F = F_1 - F_2 = GmM\left(\frac{8}{7d^2} - \frac{1}{7}\frac{1}{\left(d-\frac{R}{2}\right)^2}\right)$$

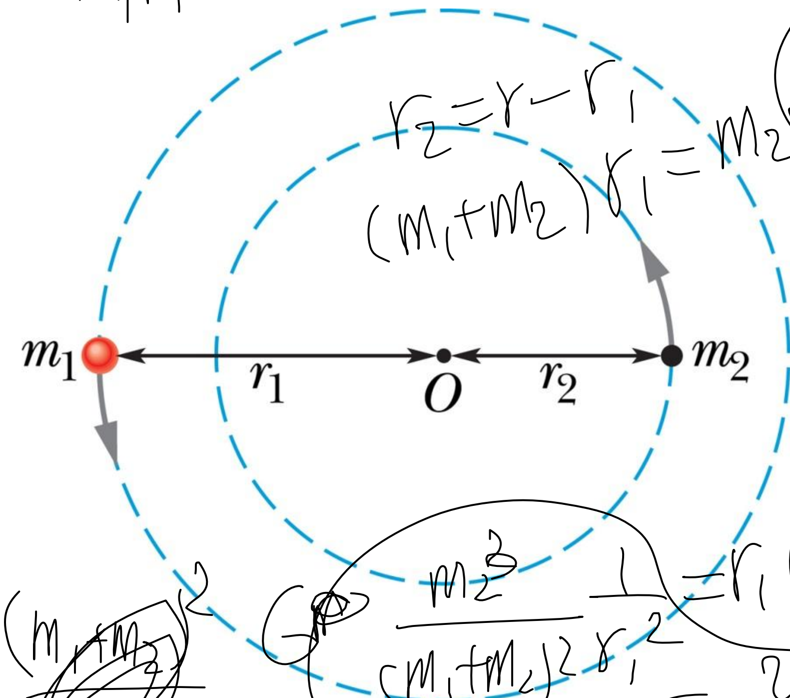
Problem 2

a, b, c에서 질량 m에 대한 중력 구하기



Problem 3: 블랙홀 찾기

$$\frac{Gm_1m_2}{r^2} = m_1 a = m_1 r_1 \omega^2 = m_1 r_1 \left(\frac{2\pi}{T}\right)^2$$



$$r_1 = \frac{m_2 r}{m_1 + m_2} \qquad T = \frac{2\pi r_1}{v}$$

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{v^3 T}{2\pi G} = 3.47 M_s$$

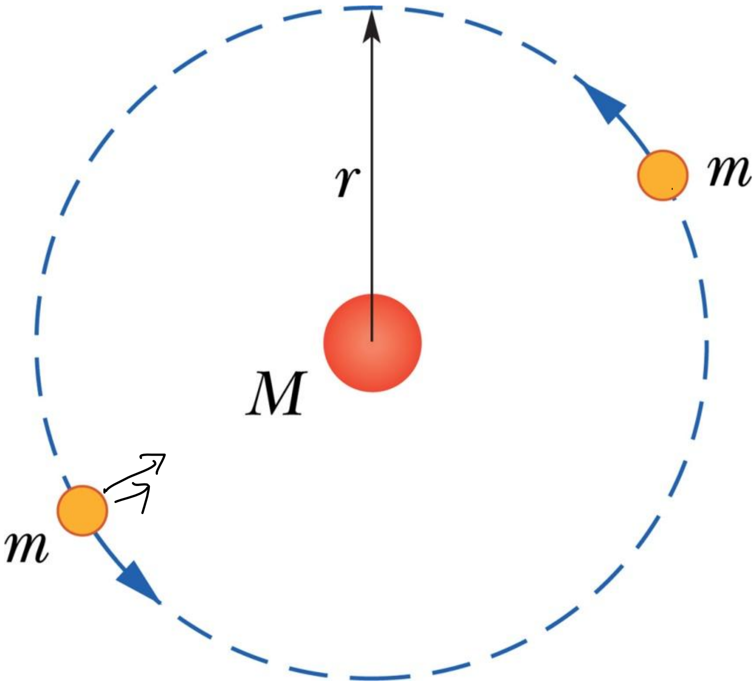
$m_1 \approx 6M_s,$

$v = 270$ km/s,

$T = 1.70$ day

$$m_2 \approx 9M_s \rightarrow black \;\; hole(?)$$

Problem 4



$$\frac{GMm}{r^2} + \frac{Gm^2}{4r^2} = \cancel{m}\cancel{r}\omega^2$$

$$\frac{G}{r^3}\left(M + \frac{m}{4}\right) = \left(\frac{2\pi}{T}\right)^2$$