Communication Systems II

[KECE322_01]

<2012-2nd Semester>

Lecture #18

2012. 11. 7

School of Electrical Engineering

Korea University

Prof. Young-Chai Ko

Outline

- Signal design for bandlimited channels
    - The Nyquist criterion

PAM Transmission through Bandlimited Baseband Channels

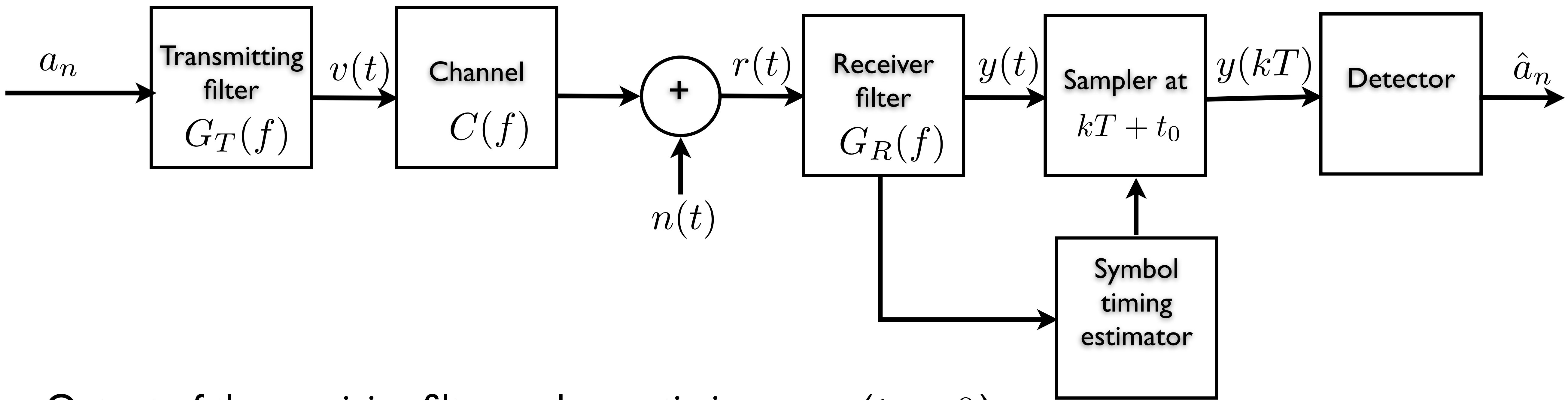
- PAM transmit signals

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g_T(t - nT),$$

where $T = \dfrac{k}{R_b}$ is the symbol interval, R_b is the bit rate and $\{a_n\}$ is a sequence of the amplitude levels corresponding to the sequence of k-bit blocks of information bits.

- Received signals

$$r(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) + n(t)$$



Output of the receiving filter under no timing error ($t_0 = 0$):

$$y(t) = \sum_{n=-\infty}^{\infty} a_n x(t - nT) + w(t)$$

where $\quad x(t) = h(t) * g_R(t) = g_T(t) * c(t) * g_R(t)$

Sampled signal

$$y(mT) = \sum_{n=-\infty}^{\infty} a_n x(mT - nT) + w(mT)$$

or equivalently,

$$y_m \quad = \quad \sum_{n=-\infty}^{\infty} a_n x_{m-n} + w_m = x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + w_m$$

$$y_m = x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + w_m$$

desired symbol

Inter-Symbol interference (ISI)

Note that

$$x(t) = h(t) * g_R(t)$$

if the receiving filter $g_R(t)$ is matched to $h(t)$, then

$$\begin{aligned} x(0) \triangleq x_0 &= \int_{-\infty}^{\infty} h(\lambda)h(\lambda)\, d\lambda \\ &= \int_{-\infty}^{\infty} h^2(t)\, dt \\ &= \int_{-\infty}^{\infty} |H(f)|^2\, df = \int_{-W}^{W} |G_T(f)|^2 |C(f)|^2\, df = \mathcal{E}_h \end{aligned}$$

Signal Design for Bandlimited Channels

- ISI signal

$$y_m = x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + w_m$$

- Bandlimited channel model

$$C(f) = \begin{cases} C_0 e^{-j2\pi f t_0}, & |f| \leq W \\ 0, & |f| > W \end{cases}$$

- Output of the receiving filter

$$X(f) = G_T(f)C(f)G_R(f) = G_T(f)C(f)C_0 e^{-j2\pi f t_0}$$

  - Assuming $C_0 = 1$ and $t_0 = 0$

$$X(f) = G_T(f)C(f), \qquad |f| \leq W$$

- Zero ISI condition

$$x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

  - which is called *Nyquist condition* for zero ISI.

- Nyquist condition for zero ISI

  - A necessary and sufficient condition for $x(t)$ to satisfy

$$x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

    is that its Fourier transform $X(f)$ must satisfy

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T.$$

- Proof

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t}\, df.$$

At the sampling instants $t = nT,$ it becomes

$$\begin{aligned}
x(nT) &= \int_{-\infty}^{\infty} X(f) e^{j2\pi f nT}\, df \\
&= \sum_{m=-\infty}^{\infty} \int_{-(2m-1)/2T}^{(2m+1)/2T} X(f) e^{j2\pi f nT}\, df \\
&= \sum_{m=-\infty}^{\infty} \int_{-1/2T}^{1/2T} X\left(f + \frac{m}{T}\right) e^{j2\pi f nT}\, df \\
&= \int_{-1/2T}^{1/2T} \left[\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) \right] e^{j2\pi f nT}\, df \\
&= \int_{-1/2T}^{1/2T} Z(f) e^{j2\pi f nT}\, df,
\end{aligned}$$

where $Z(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)$

$$Z(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) \qquad\qquad x(nT) = \int_{-1/2T}^{1/2T} Z(f) e^{j2\pi f nT}\, df,$$

- $Z(f)$ is periodic function with period $\frac{1}{T}$; therefore it can be expanded in terms of its Fourier series coefficients $\{z_n\}$ as

$$Z(f) = \sum_{n=-\infty}^{\infty} z_n e^{j2\pi n f T}$$

where

$$z_n = T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} Z(f) e^{-j2\pi n f T}\, df.$$

Compare the following two:

$$z_n = T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} Z(f) e^{-j2\pi n f T}\, df, \quad \text{and} \quad x(nT) = \int_{-1/2T}^{1/2T} Z(f) e^{j2\pi f nT}\, df,$$

Then we have: $z_n = T x(-nT)$

✦ Zero ISI condition

$$z_n = \begin{cases} T, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

since $z_n = Tx(-nT)$ and zero ISI condition is $x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}.$

- which yields

symbol duration

$$Z(f) = T,$$

or equivalently

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T.$$

- Let us define

$$B(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)$$

  - Then the Nyquist criterion is

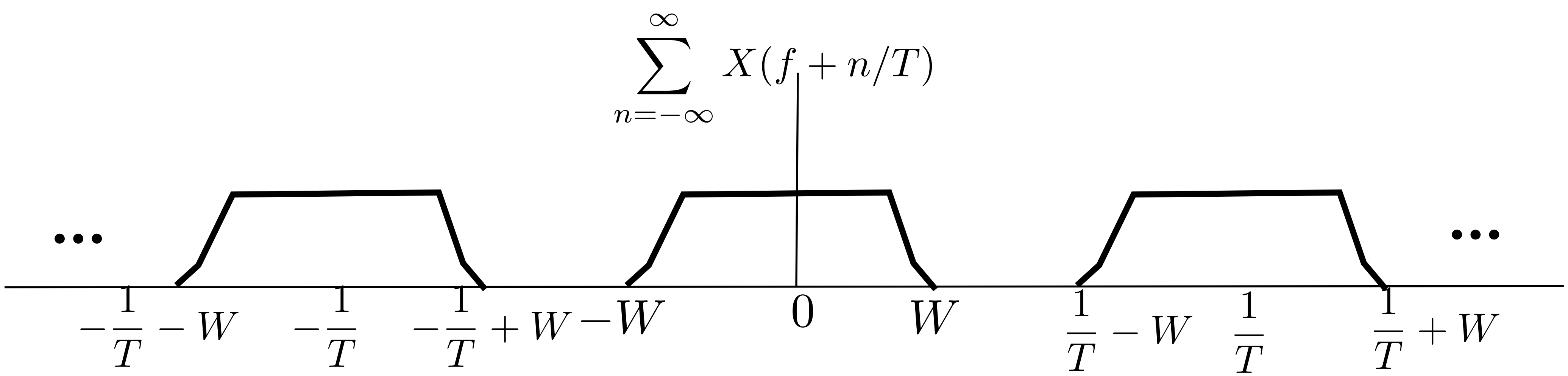
$$B(f) = T.$$

- Suppose that the channel has a bandwidth of W.
  - Then $C(f) = 0$ for $|f| > W$; consequently, $X(f) = 0$ for $|f| > W$.
  - Note that the Nyquist criterion for zero ISI is $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T$.
- We distinguish three cases.

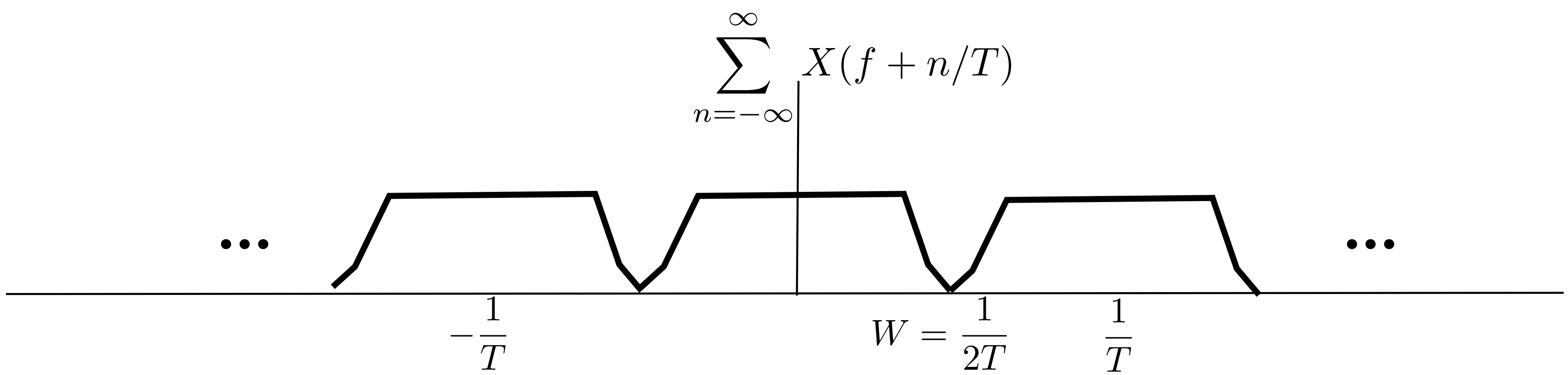
1. When $T < \frac{1}{2W}$, or equivalently, $\frac{1}{T} > 2W$
2. When $T = \frac{1}{2W}$
3. When $T > \frac{1}{2W}$

- When $T < \frac{1}{2W}$, or equivalently, $\frac{1}{T} > 2W$, $B(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)$ consists of non-overlapping replicas of $X(f)$, separated by $1/T$.

  - There is no choice for $X(f)$ to ensure $B(f) \equiv T$ in this case and there is no way that we can design a system with no ISI.



When $T = 1/2W$, or equivalently, $1/T = 2W$, the replication of $X(f)$ separated by $1/T$ has the form as below



- In this case there exists only one $X(f)$ that results $B(f) = T$, namely,

$$X(f) = \begin{cases} T, & (|f| < W) \\ 0, & (\text{otherwise}) \end{cases}$$

- which corresponds to

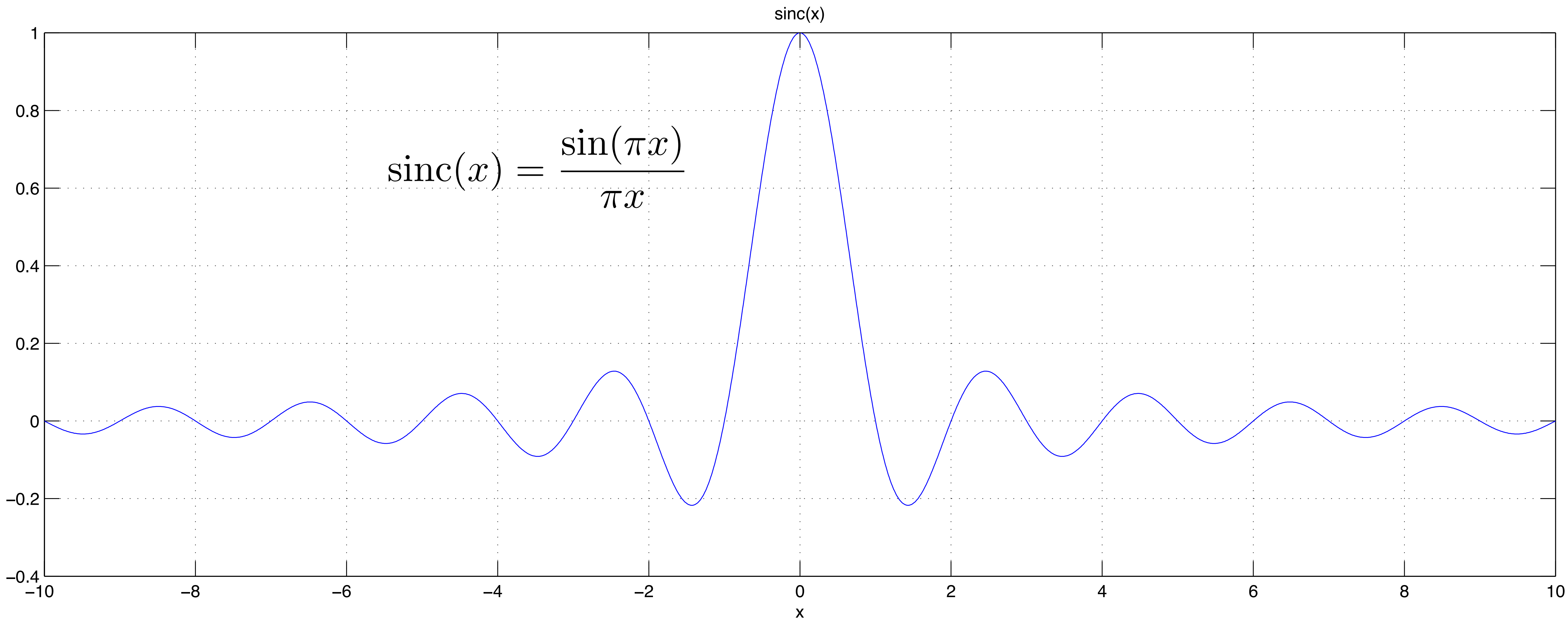
$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T} \equiv \mathrm{sinc}\left(\frac{\pi t}{T}\right)$$

- This means that the smallest value of T for which transmission with zero ISI is possible is $T = 1/2W,$ and for this value, $x(t)$ has to be a sinc function.

- The **difficulty** with this choice of $x(t)$ is that it is non-causal and therefore non-realizable.

- To make it realizable, usually a delayed version of it, i.e., $\mathrm{sinc}[\pi(t-t_0)/T]$ is used and t_0 is chosen such that for $t<0$, we have $\mathrm{sinc}[\pi(t-t_0)/T] \approx 0$.

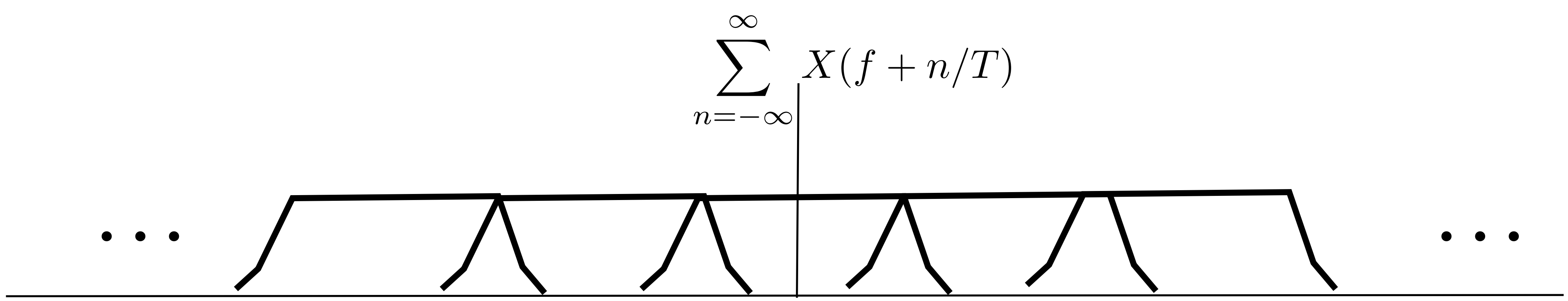
- Of course, with this choice of $x(t)$, the sampling time must be shifted to $mT + t_0$.



- A second **difficulty** with this pulse shape is that its rate of convergence to zero is low. The tails of $x(t)$ decays as $1/t$, consequently, a small mistiming error in sampling the output of the matched filter at the demodulator results in an infinite series of ISI components.

- When $T > 1/2W$, $B(f)$ consists of overlapping replications of $X(f)$ separated by $1/T$.

  - In this case, there exists numerous choice for $X(f)$ such that $B(f) \equiv T$.



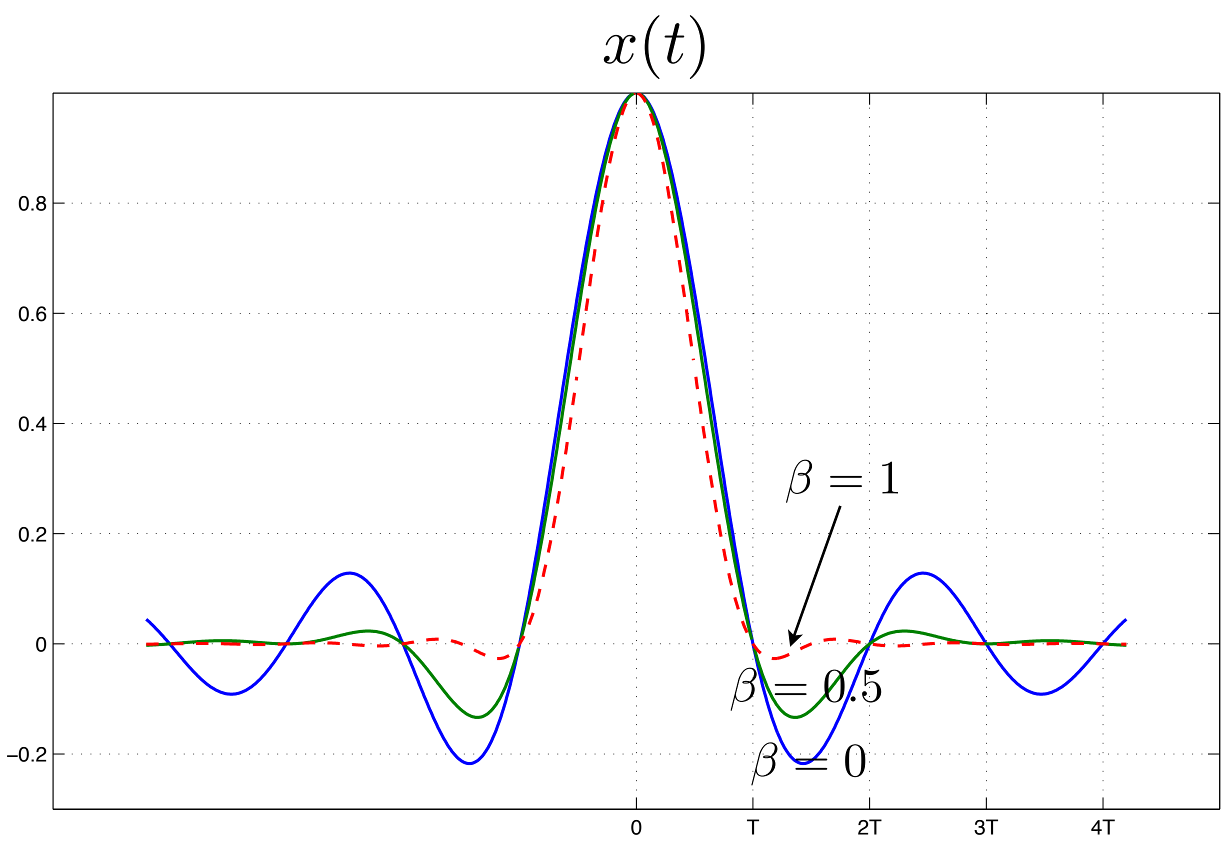
- A particular pulse spectrum for the $T > 1/2W$ case, that has desirable spectral properties and has been widely used in practice is the *raised cosine spectrum*.

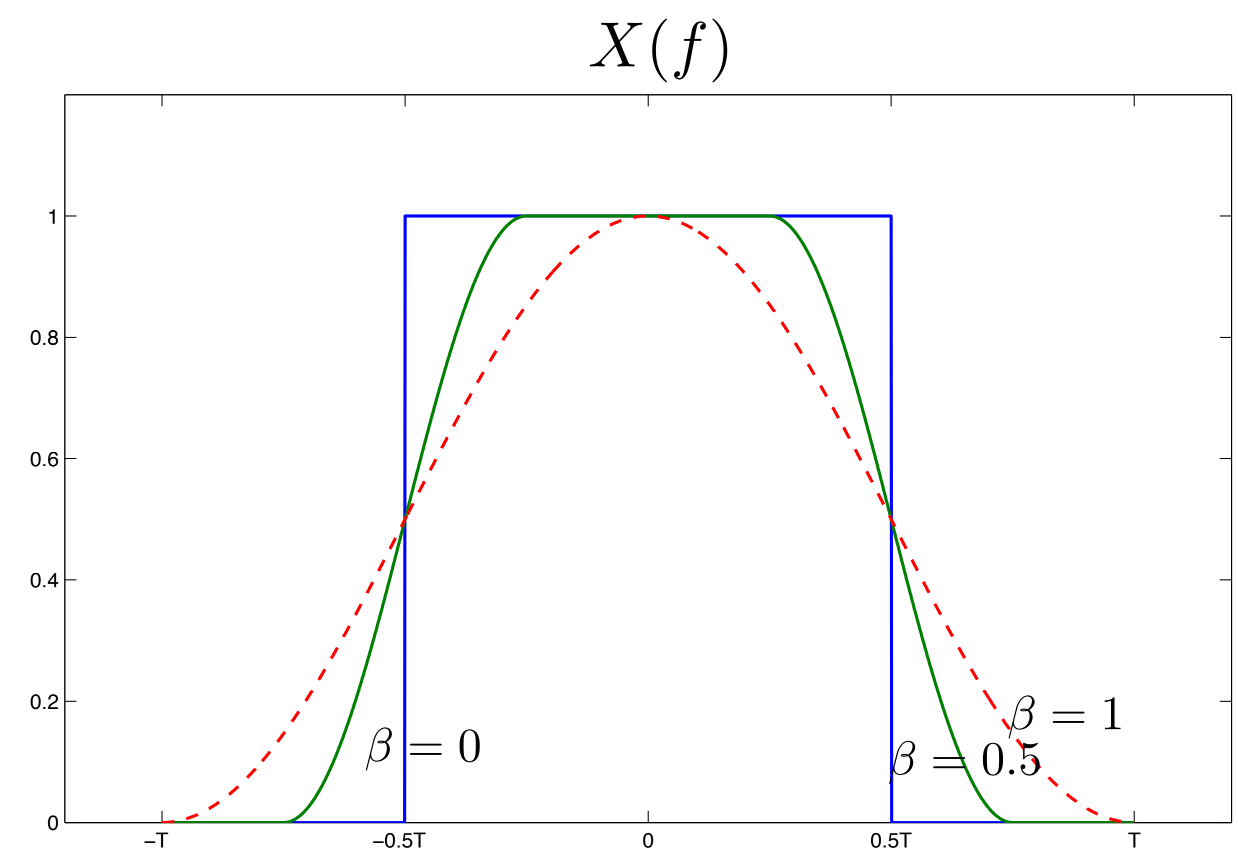
$$X_{rc}(f) = \begin{cases} T & \left(0 \le |f| \le \frac{1-\beta}{2T}\right) \\ \frac{T}{2}\left\{1+\cos\left[\frac{\pi t}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right]\right\} & \left(\frac{1-\beta}{2T} \le |f| \le \frac{1+\beta}{2T}\right) \\ 0 & \left(|f| > \frac{1+\beta}{2T}\right) \end{cases}$$

where $0 \le \beta \le 1$ is called roll-off factor.

Raised cosine pulse

$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\pi \beta t/T)}{1 - 4\beta^2 t^2/T^2} = \text{sinc}\left(\frac{t}{T}\right) \frac{\cos(\pi \beta t/T)}{1 - 4\beta^2 t^2/T^2}$$





- Note that $x(t)$ is normalized so that $x(0) = 1$.

12년 11월 7일 수요일

Raised cosine pulse

$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\pi\beta t/T)}{1 - 4\beta^2 t^2/T^2} = \mathrm{sinc}\left(\frac{t}{T}\right) \frac{\cos(\pi\beta t/T)}{1 - 4\beta^2 t^2/T^2}$$

- For $\beta = 0$, the pulse reduces to $x(t) = \mathrm{sinc}\left(\frac{t}{T}\right)$ and the symbol rate is $\frac{1}{T} = 2W.$
- For $\beta = 1,$ the symbol rate is $\frac{1}{T} = W.$
- In general, the tails of $x(t)$ decays as $1/t^3$ for $\beta > 0.$ Consequently, a mistiming error in sampling leads to a series of ISI components that converges to a finite value.