GEST 011, Newton's Clock & Heisenberg's Dice, Fall 2013

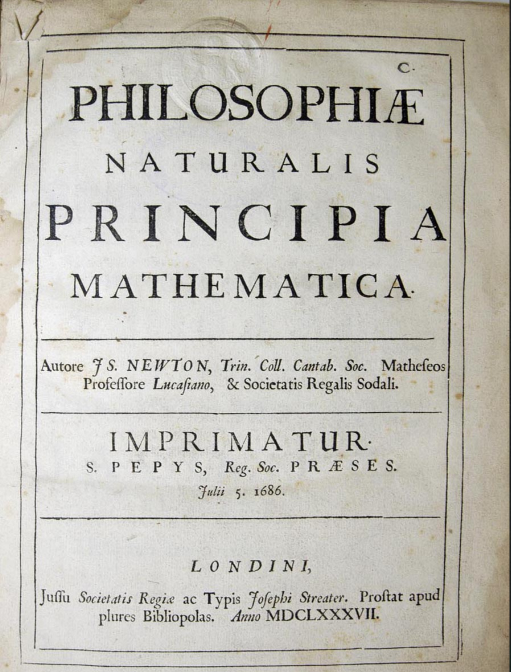
The Principia

(Philosophiae Naturalis Principia Mathematica)

Mahn-Soo Choi (Korea University)

October 7, 2013 (v5.10)

Image from http://katolisitas.org/ (freely available for nonprofit usage)



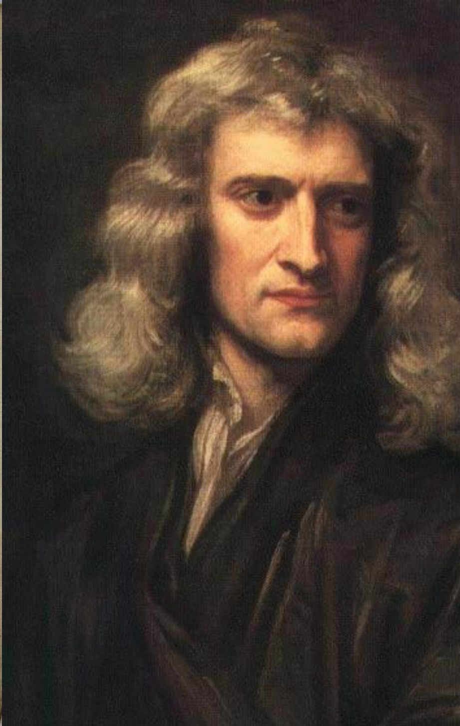
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

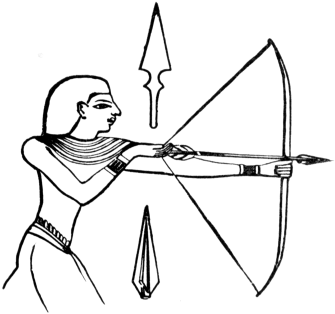
Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.



Left image courtesy of Manchester Libraries
Right image from Wikipedia

What Is Motion?

Zeno's Arrow Paradox

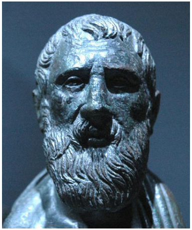


http://etc.usf.edu/

Zeno of Elea, "The object at each moment is at a fixed position."

How can it then move?

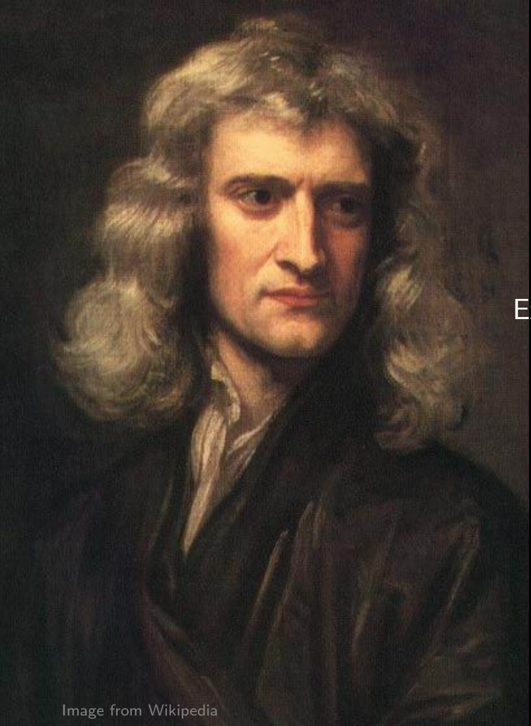
http://people.rit.edu/



http://www.daviddarling.info/

What is "motion"?

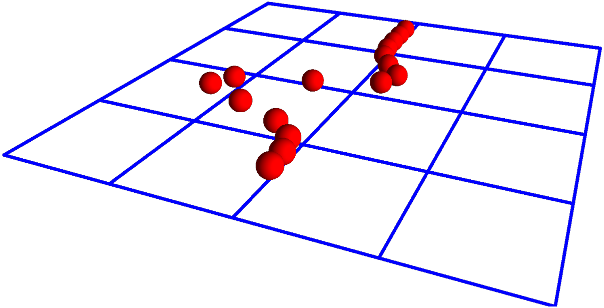
How to "describe" it?



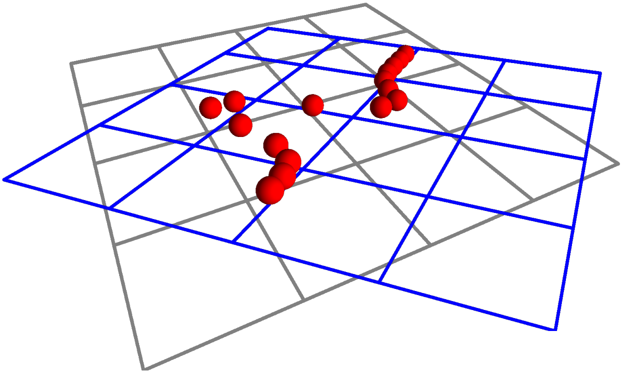
Sir Isaac Newton (1643–1727)
Explains how to describe motion

Minimum Requirements

Reference Frame



Reference Frame



Time

Image from http://www.pjcj.net/

Time

What is time?

Image from `http://www.pjcj.net/`

Two Methods

To Describe Motion

Recording the Position

(as a function of time)

Image from http://people.rit.edu

Using "Rate"

(amount of change per unit time)

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

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$$a(t_1) = \lim_{t_2 \to t_1} \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{dv}{dt}$$

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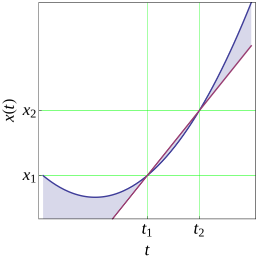


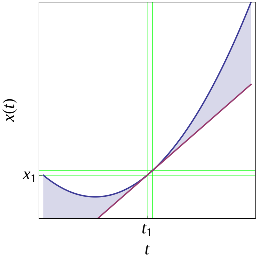
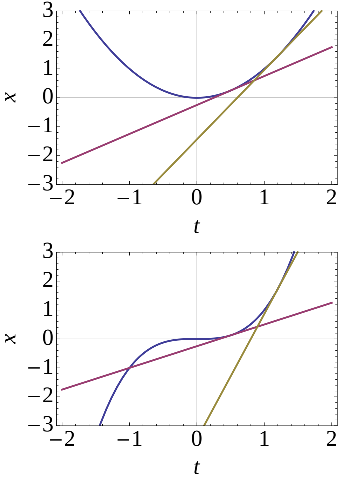


Table of Derivatives

$x(t)$	$\Rightarrow$	$\dfrac{dx}{dt}$
1	$\Rightarrow$	0
t	$\Rightarrow$	1
t^2	$\Rightarrow$	$2t$
t^n	$\Rightarrow$	$nt^{n-1} \quad (n \in \mathbb{Z})$
$\sin(t)$	$\Rightarrow$	$+\cos(t)$
$\cos(t)$	$\Rightarrow$	$-\sin(t)$
$\exp(t)$	$\Rightarrow$	$\exp(t)$



How to “Predict” the Motion

Equation of Motion

(Newton's 2nd Law of Motion)

$$(\text{force}) = (\text{mass}) \times (\text{acceleration})$$

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$$F = m\frac{dv}{dt}$$

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$$(\text{force}) = (\text{mass}) \times (\text{acceleration})$$

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$$F = m\frac{dv}{dt} \qquad F = m\frac{d}{dt}\frac{dx}{dt} \qquad F = m\frac{d^2x}{dt^2}$$

$$x(t) = ?$$

How to solve the equation?

How to Solve an Equation

(algebraic equation)

$$x + 3 = 10 \quad \Rightarrow \quad x = ?$$

$$5x = 30 \quad \Rightarrow \quad x = ?$$

$$x^2 = 144 \quad \Rightarrow \quad x = ?$$

$$10^x = 1000 \quad \Rightarrow \quad x = ?$$

$$e^x = 1 \quad \Rightarrow \quad x = ?$$

How to Solve an Equation

(algebraic equation)

$$
\begin{aligned}
x + 3 = 10 \quad &\Rightarrow \quad x = 10 - 3 \\
5x = 30 \quad &\Rightarrow \quad x = \frac{30}{5} \\
x^2 = 144 \quad &\Rightarrow \quad x = \sqrt{144} \\
10^x = 1000 \quad &\Rightarrow \quad x = \log_{10} 1000 \\
e^x = 1 \quad &\Rightarrow \quad x = \log_e 1 = \log 1
\end{aligned}
$$

Table of Derivatives

$x(t)$	$\Rightarrow$	$\dfrac{dx}{dt}$
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t^2	$\Rightarrow$	$2t$
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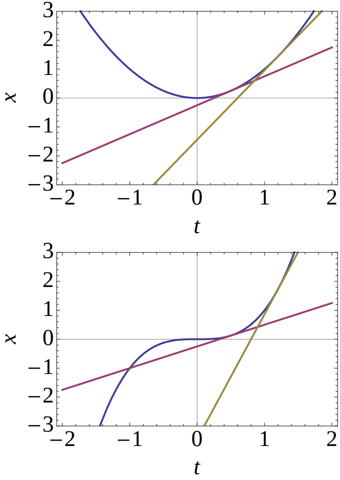

How to Solve an Equation

(differential equation)

$$\frac{dx}{dt} = 0 \quad \Rightarrow \quad x(t) = ?$$

$$\frac{dx}{dt} = 1 \quad \Rightarrow \quad x(t) = ?$$

$$\frac{dx}{dt} = 2 \quad \Rightarrow \quad x(t) = ?$$

$$\frac{dx}{dt} = t \quad \Rightarrow \quad x(t) = ?$$

$$\frac{dx}{dt} = 3t \quad \Rightarrow \quad x(t) = ?$$

$$\frac{dx}{dt} = x \quad \Rightarrow \quad x(t) = ?$$

How to Solve an Equation

(differential equation)

$$\frac{dx}{dt} = 0 \quad \Rightarrow \quad x(t) = C$$

$$\frac{dx}{dt} = 1 \quad \Rightarrow \quad x(t) = t + C$$

$$\frac{dx}{dt} = 2 \quad \Rightarrow \quad x(t) = 2t + C$$

$$\frac{dx}{dt} = t \quad \Rightarrow \quad x(t) = \frac{1}{2}t^2 + C$$

$$\frac{dx}{dt} = 3t \quad \Rightarrow \quad x(t) = \frac{3}{2}t^2 + C$$

$$\frac{dx}{dt} = x \quad \Rightarrow \quad x(t) = C\exp(t)$$

How to Solve an Equation

(differential equation)

$$\boxed{m\frac{dv}{dt} = mg},$$

How to Solve an Equation

(differential equation)

$$\boxed{m\frac{dv}{dt} = mg}\,, \qquad \frac{dv}{dt} = g\,,$$

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How to Solve an Equation

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$$\boxed{m\frac{dv}{dt} = mg}\,, \qquad \frac{dv}{dt} = g\,, \qquad v(t) = gt + v_0$$

$$\boxed{\frac{dx}{dt} = gt + v_0}\,,$$

How to Solve an Equation

(differential equation)

$$\boxed{m\frac{dv}{dt} = mg}, \quad \frac{dv}{dt} = g\,, \quad v(t) = gt + v_0$$

$$\boxed{\frac{dx}{dt} = gt + v_0}, \quad x(t) = \frac{1}{2}gt^2 + v_0 t + x_0$$

How to Solve an Equation

(differential equation)

$$\boxed{m\frac{dv}{dt} = mg}\,, \quad \frac{dv}{dt} = g\,, \quad v(t) = gt + v_0$$

$$\boxed{\frac{dx}{dt} = gt + v_0}\,, \quad x(t) = \frac{1}{2}gt^2 + v_0 t + x_0$$

We need to measure two quantities x_0 and v_0 to predict the future of the motion.

State

“State”
(in dictionaries)

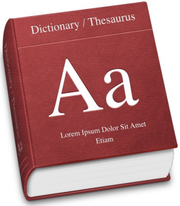


Image from

http://apple.com/

the particular condition that someone or something is in at a specific time

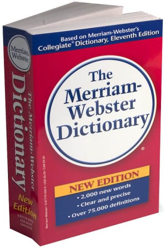
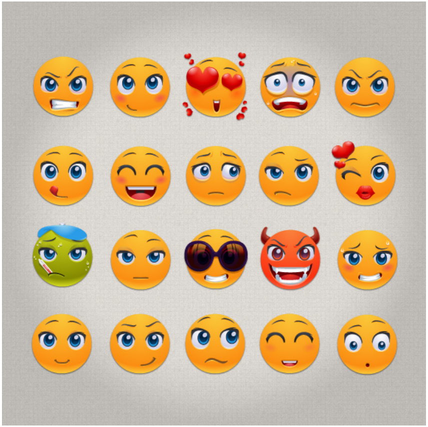


Image from

www.merriam-webster.com

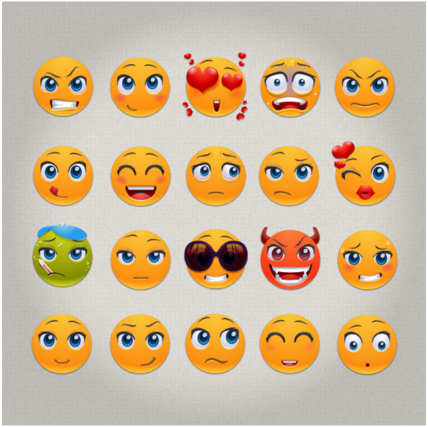
mode or condition of being

State of Mind



http://petshopbox.com/

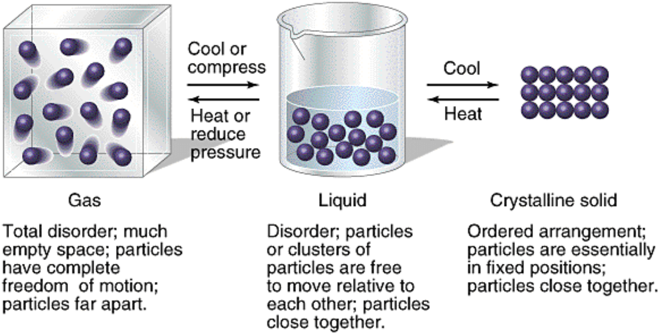
State of Mind



http://petshopbox.com/

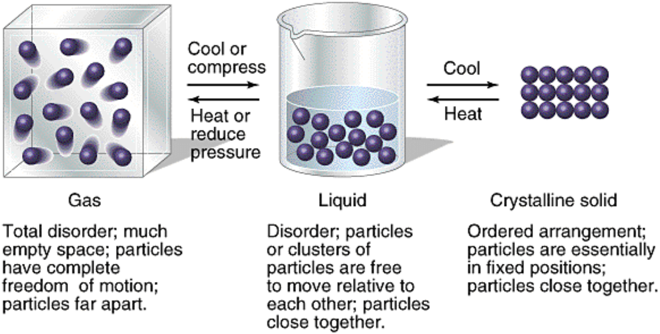
Happiness	10 %
Sadness	25 %
Annoyance	5 %
Excitement	15 %
⋮	⋮

State of Matter



http://www.chem.ufl.edu/~itl/

State of Matter



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Pressure, Volume, Density, Temperature, . . .

State of Dice and Pennies

http://stores.auction.co.kr/

http://www.canstockphoto.com/

State of Dice and Pennies

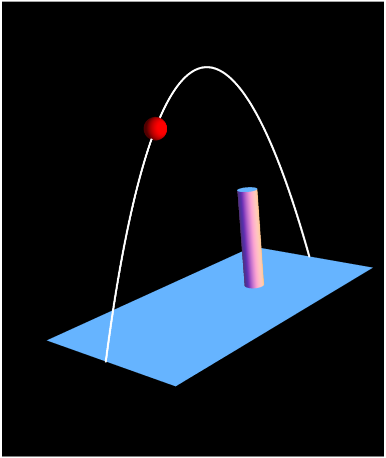
http://stores.auction.co.kr/

Face	head, tail

http://www.canstockphoto.com/

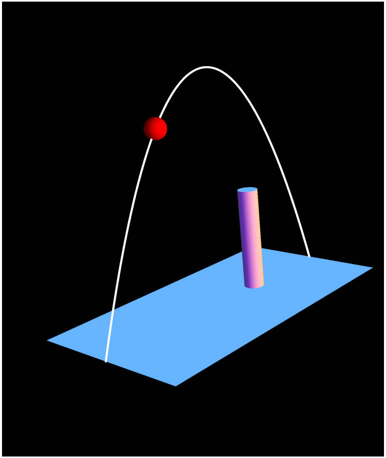
Pips	1,2,3,4,5,6

State of Flying Particle



http://demonstrations.wolfram.com/

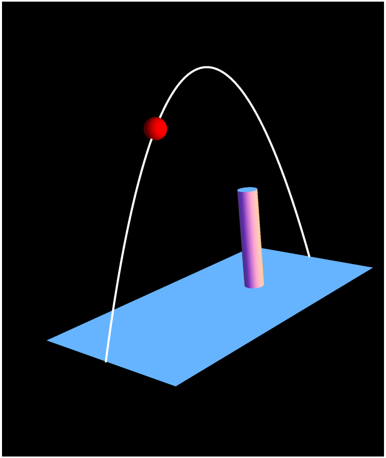
State of Flying Particle



http://demonstrations.wolfram.com/

Mass	3	kg
Charge	0	C
Energy	5	J
Horizontal position	5	m
Vertical position	7	m
Horizontal velocity	5	m/s
Vertical velocity	8	m/s
Horizontal acceleration	5	m
Vertical acceleration	7	m
⋮	⋮	

State of Flying Particle



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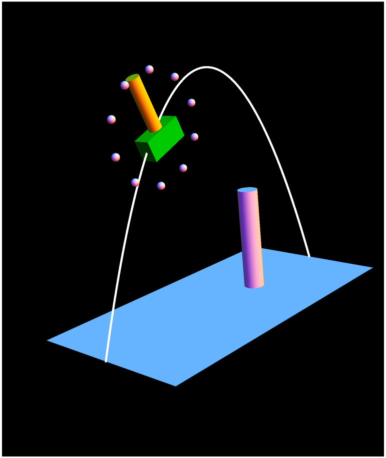
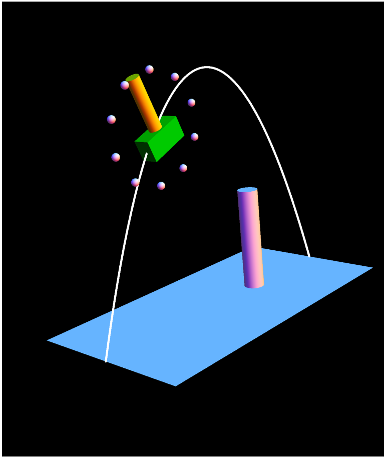
State of Flying Hammer



Graphic by Sndor Kabai /

`http://demonstrations.wolfram.com/`

State of Flying Hammer

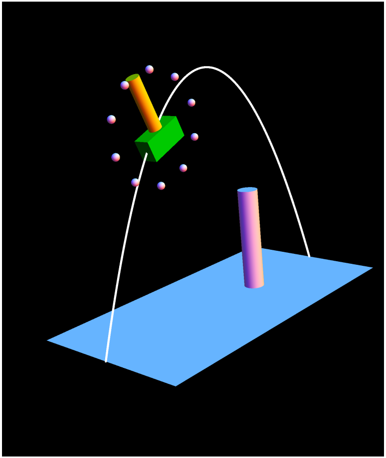


Graphic by Sndor Kabai /

http://demonstrations.wolfram.com/

Mass
Charge
Energy
Horizontal position
Vertical position
Horizontal velocity
Vertical velocity
Angle
Anglular velocity
$\vdots$

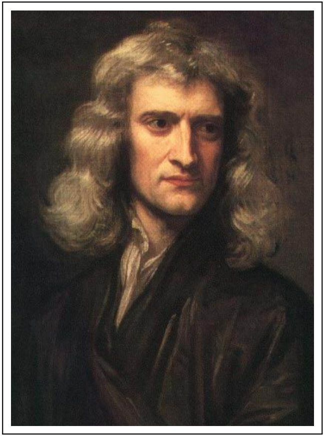
State of Flying Hammer



Graphic by Sndor Kabai / http://demonstrations.wolfram.com/

Mass
Charge
Energy
Horizontal position
Vertical position
Horizontal velocity
Vertical velocity
Angle
Anglular velocity
$\vdots$

State of a Particle in Motion
(Classical Mechanics)



Sir Isaac Newton (1643–1727)

- Position $\boldsymbol{r}$ & Velocity $\boldsymbol{v}$
- Newton's equation of motion

  (force) = (mass) × (acceleration)

$$\boxed{\boldsymbol{F} = m\frac{d\boldsymbol{v}}{dt} = m\frac{d^2\boldsymbol{r}}{dt^2}}$$

The Prinicipia Mathematica

Principia probant, non probantur.
(Principles prove; they are not proved.)

Fundamental principles require no proof;
they are assumed *a priori*.

The Prinicipia Mathematica

(Newton laws of motion)

Newton's 1st Law

If no *net* force acts on a body, the body's velocity cannot change.

The Prinicipia Mathematica

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Newton's 2nd Law

The net force on a body is equal to the product of the body's mass and its acceleration:

$$(\text{force}) = (\text{mass}) \times (\text{acceleration})$$

The Prinicipia Mathematica

(Newton laws of motion)

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If no *net* force acts on a body, the body's velocity cannot change.

Newton's 2nd Law

The net force on a body is equal to the product of the body's mass and its acceleration:

$$\text{(force)} = \text{(mass)} \times \text{(acceleration)}$$

Newton's 3rd Law

When two bodies interact, the forces on the bodies from each other are always in magnitude and opposite in direction.

The Structure of Theory

Theory in Math

- Axioms
- Definitions
- Theorems

For logical arguments and efficient organization of thoughts.

The Structure of Theory

Theory in Physics

- Axioms ("Laws", "Principles")
- Definitions ("Interpretations")
- Theorems ("Laws", "Principles")
- Interpretations
- Experimental tests

* Analogies

For fundamental understanding of nature and natural phenomena.

One of the Most Beautiful Theories!

Self-containing
Self-consistent
Beautiful structure

References