KECE321 Communication Systems I

(Haykin Sec. 4.8)

Lecture #18, May 14, 2012

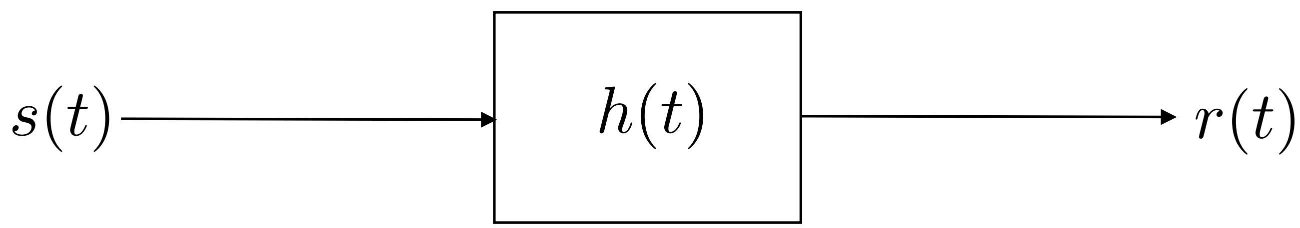
Prof. Young-Chai Ko

Summary

- Demodulation of FM signals
  - Frequency discriminator
  - Phase locked loop (PLL)

Review

- Low-pass equivalent representation of pass-band (narrow-band) signal



  - $s(t),\ \ r(t)$: pass-band signal
  - $h(t)$: pass-band system
  - Then,

equivalent low-pass signal

$$\begin{aligned} s(t) &= \Re[\tilde{s}(t)e^{j2\pi f_c t}] \\ r(t) &= \Re[\tilde{r}(t)e^{j2\pi f_c t}] \end{aligned}$$

$$\begin{aligned} S(f) &= \frac{1}{2}[\tilde{S}(f-f_c)+\tilde{S}^*(-f-f_c)] \\ R(f) &= \frac{1}{2}[\tilde{R}(f-f_c)+\tilde{R}^*(-f-f_c)] \end{aligned}$$

- Let us define the low-pass system response

$$\tilde{H}(f - f_c) = \begin{cases} H(f) & (f > 0) \\ 0 & (f < 0) \end{cases}$$

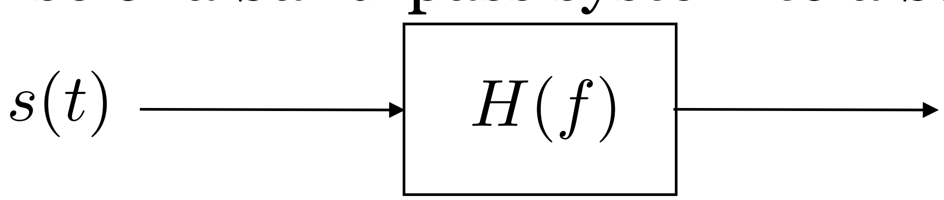
$$\tilde{H}^*(-f - f_c) = \begin{cases} 0 & (f > 0) \\ H^*(-f) & (f < 0) \end{cases}$$

- Then, we have

$$H(f) = \tilde{H}(f - f_c) + \tilde{H}^*(-f - f_c)$$

$$h(t) = 2\Re[\tilde{h}(t)e^{j2\pi f_c t}]$$

- Response of a band-pass system to a band-pass signal



$$r(t) = \int_{-\infty}^{\infty} s(\tau)h(t - \tau)\, d\tau$$

$$R(f) = S(f)H(F)$$

- Then, we can write

$$
\begin{aligned}
R(f) &= S(f)H(F) \\
&= \frac{1}{2}[\tilde{S}(f-f_c)+\tilde{S}^*(-f-f_c)][\tilde{H}(f-f_c)+\tilde{H}^*(-f-f_c)] \\
&= \frac{1}{2}[\tilde{S}(f-f_c)\tilde{H}(f-f_c)+\tilde{S}^*(-f-f_c)\tilde{H}^*(-f-f_c)] \\
&= \frac{1}{2}[\tilde{R}(f-f_c)+\tilde{R}^*(-f-f_c)]
\end{aligned}
$$

where $\tilde{R}(f)=\tilde{S}(f)\tilde{H}(f)$

$$\tilde{r}(t)=\int_{-\infty}^{\infty}\tilde{s}(\tau)\tilde{h}(t-\tau)\,d\tau$$

- Now let us go back to the filter of the differentiator to be used for the demodulation of the FM signal.

  - The transfer function of the filter is

$$H_1(f) = \begin{cases} j2\pi[f - (f_c - B_T/2)], & f_c - (B_T/2) \le |f| \le f_c + B_T/2 \\ 0, & \text{otherwise} \end{cases}$$

  - Then its equivalent low pass form is

$$\tilde{H}_1(f) = \begin{cases} j2\pi[f + (B_T/2)], & -B_T/2 \le f \le B_T/2 \\ 0, & \text{otherwise} \end{cases}$$

  - Then,

$$\begin{aligned} \tilde{S}_1(f) &= \tilde{H}_1(f)\tilde{S}(f) \\ &= \begin{cases} j2\pi\left(f + \frac{1}{2}B_T\right)\tilde{S}(f), & -\frac{1}{2}B_T \le f \le \frac{1}{2}B_T \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

- For $-\frac{1}{2}B_T \le f \le \frac{1}{2}B_T$, we have

$$\frac{d}{dt}\tilde{s}(t)$$

$$\tilde{S}_1(f) = j2\pi f \tilde{S}(f) + j\pi B_T \tilde{S}(f)$$

- Taking the inverse Fourier transform yields

$$\tilde{s}_1(t) = \frac{d}{dt}\tilde{s}(t) + j\pi B_T \tilde{s}(t)$$

- Noting that

$$\tilde{s}(t) = A_c \exp\left(j2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

- we can rewrite $\tilde{s}_1(t)$ as

$$\tilde{s}_1(t) = j\pi A_c B_T \left[1 + \left(\frac{2k_f}{B_T}\right) m(t)\right] \exp\left(j2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

- Finally,

$$\begin{aligned} s_1(t) &= \Re[\tilde{s}_1(t)\exp(j2\pi f_c t)] \\ &= \pi A_c B_T \left[1 + \left(\frac{2k_f}{B_T}\right) m(t)\right] \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau + \frac{\pi}{2}\right) \end{aligned}$$

- Then the envelope detector recovers the message signal $m(t)$, except for a bias. Specifically, under the ideal conditions, the output of the envelope detector is given by

$$\nu_1(t) = \pi A_c B_T \left[1 + \left(\frac{2k_f}{B_T}\right) m(t)\right]$$

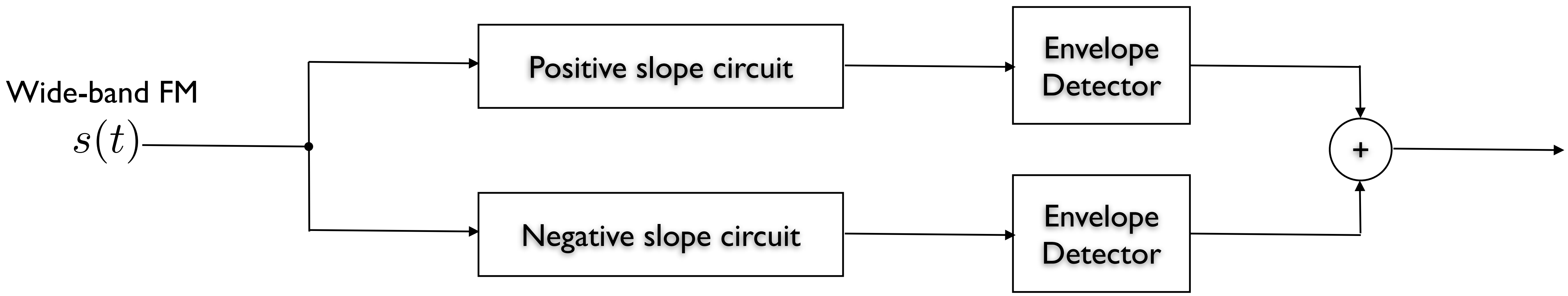
- To remove the bias, we may use a second slope circuit followed by an envelope detector of its own which gives the output signal at the envelope detector as

$$\nu_2(t) = \pi A_c B_T \left[1 - \left(\frac{2k_f}{B_T}\right) m(t)\right]$$

- Now summing $\nu_1(t)$ and $\nu_2(t)$ removes the bias term such as

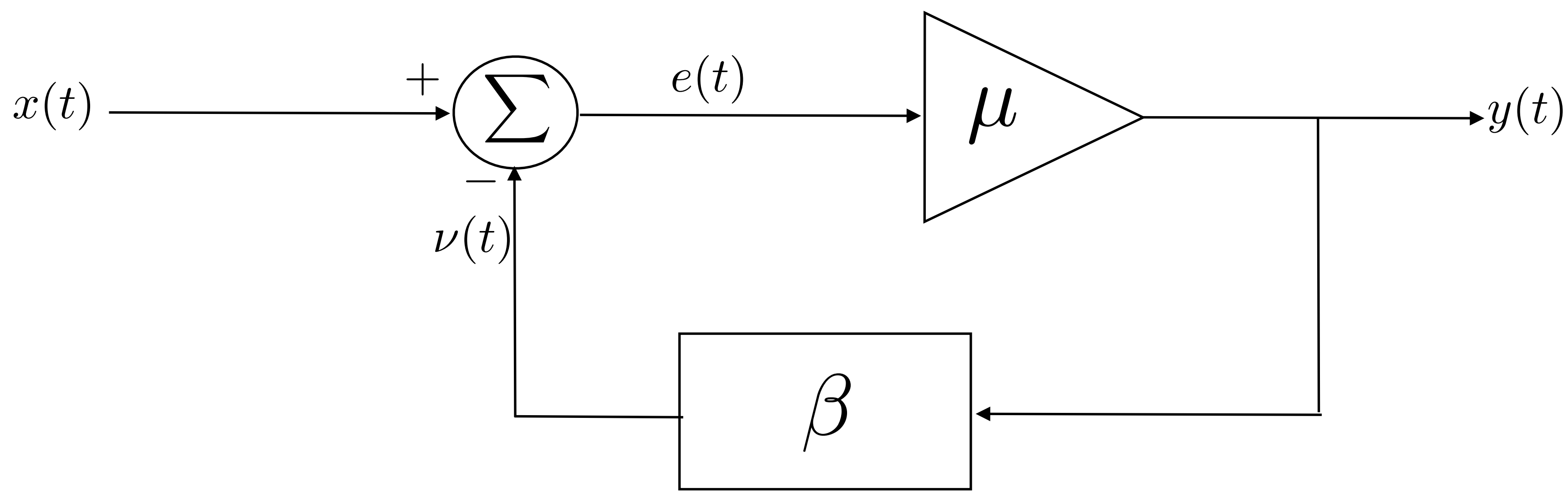
$$\nu(t) = \nu_1(t) + \nu_2(t) = cm(t)$$

where c is a certain constant.



Negative Feedback System

- Negative feedback system



$$[x(t) - \beta y(t)]\mu = y(t) \longleftrightarrow [X(f) - \beta Y(f)]\,\mu = Y(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{\mu}{1 + \mu\beta}$$

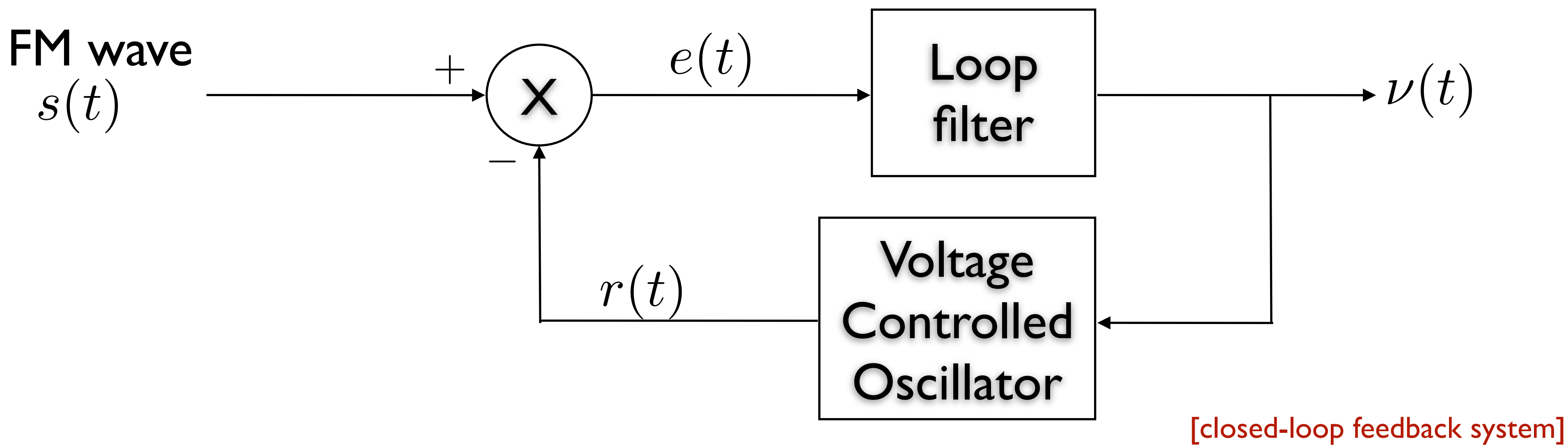
open-loop gain

For $\mu\beta >> 1$,

$$H(f) \approx \frac{1}{\beta}$$

Phase-Locked Loop (PLL)

- Block diagram of the PLL



- PLL consists of
  - VCO
  - Multiplier
  - Loop filter

- VCO
  - The frequency of the VCO is set precisely at the unmodulated carrier frequency f_c of the incoming FM wave $s(t)$.
  - The VCO output has a 90-degree phase-shift with respect to the unmodulated carrier wave.
- Suppose then that the incoming FM wave is defined by

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)]$$

where $\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$

  - FM wave produced by the VCO

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]$$

where $\phi_2(t) = 2\pi k_\nu \int_0^t \nu(\tau)\, d\tau$

- At the output of the multiplier, there are two components

1. double-frequency term

$$k_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$$

where k_m is the *multiplier gain.*

2. difference-frequency term

$$k_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$

- With the loop filter designed to suppress the high-frequency components in the multiplier's output, we may henceforth discard the double-frequency term. Doing this, we may reduce the signal applied to the loop filter to

$$e(t) = k_m A_c A_v \sin[\phi_e(t)]$$

where $\phi_e(t)$ is the phase error defined by

$$\phi_e(t) = \phi_1(t) - \phi_2(t) = \phi_1(t) - 2\pi k_v \int_0^t \nu(\tau)\, d\tau$$

- Phase-lock

  - When the phase error $\phi_e(t)$ is zero, the phase-lock loop is said to be in *phase-lock*.

  - It is said to be near-phase-lock when the phase error $\phi_e(t)$ is small compared with one radian, under which condition we may use the approximation

$$\sin[\phi_e(t)] \approx \phi_e(t)$$

- Correspondingly we may approximate the error signal as

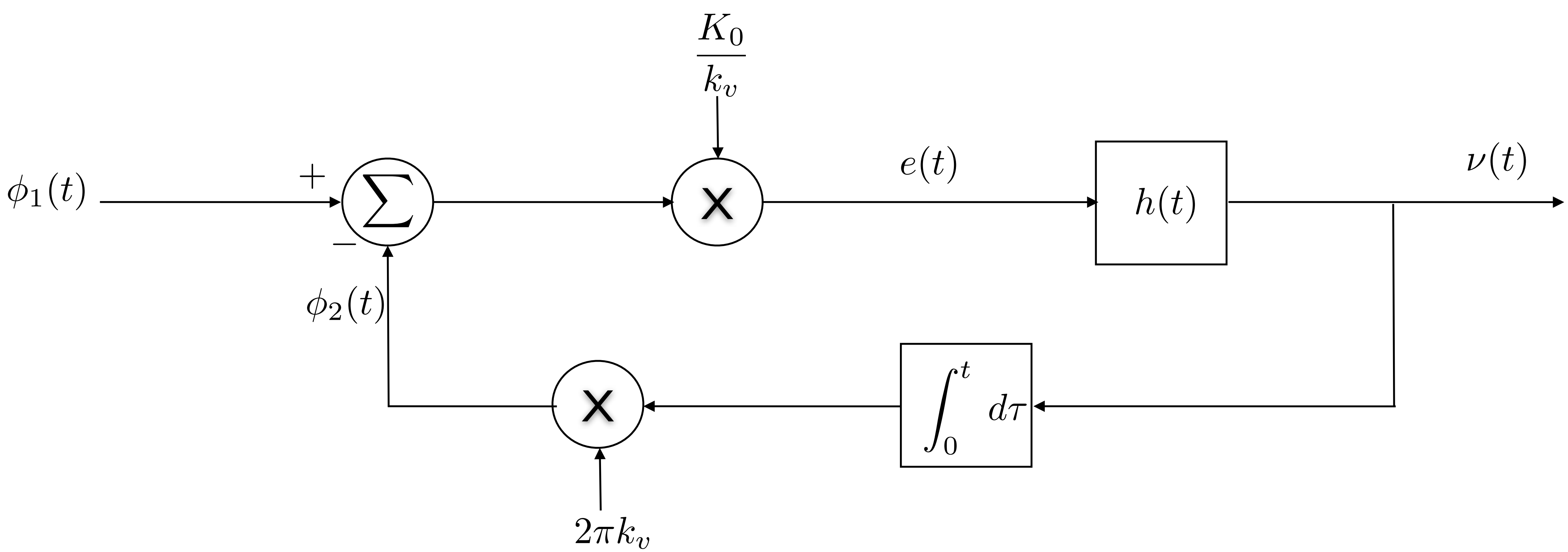
$$e(t) \approx k_m A_c A_v \phi_e(t) = \frac{K_0}{k_v}\phi_e(t)$$

where the new parameter $K_0 = k_m k_v A_c A_v$ is called the *loop-gain* parameter of the phase-lock loop.

- Let $h(t)$ be the impulse response of the loop filter. Then the output of the loop filter is

$$\nu(t) = \int_{-\infty}^{\infty} e(\tau)h(t-\tau)\, d\tau$$

- Linearized feedback model of the PLL



- Some observations from the linearized feedback model of the PLL

1. The feedback path is defined solely by the scaled integrator which is the VCO's contribution to the model. Correspondingly, the inverse of this feedback path is described in the time domain by the scaled differentiator:

$$\nu(t) = \frac{1}{2\pi k_v}\left(\frac{d\phi_2(t)}{dt}\right)$$

2. The closed-loop time-domain behavior of the phase-lock loop is described by the overall output $\nu(t)$ produced in response to the angle $\phi_1(t)$ in the incoming FM wave $s(t)$.

3. The magnitude of the open-loop transfer function of the phase-locked loop is controlled by the loop-gain parameter K_0 .

- Assuming that the loop-gain parameter K_0 is large compared with unity, application of the feedback theorem to the model of the linearized feedback of the PLL teaches us that the closed-loop transfer function (i.e., closed-loop time-domain behavior) of the PLL is effectively determined by the inverse of the transfer function (i.e., time-domain behavior) of the feedback path.

- Accordingly, in light of the feedback theorem we may related the overall output $\nu(t)$ to the input angle

$$\nu(t) \approx \frac{1}{2\pi}\left(\frac{d\phi_1(t)}{dt}\right)$$

- Permitting K_0 to assume a large have has the effect of making the phase error $\phi_e(t)$ approaches zero. Under this condition, we have $\phi_1(t) \approx \phi_2(t)$.

- From the linearized feedback model of the PLL, we can obtain the following:

$$\begin{aligned}
\nu(t) &= \frac{1}{2\pi k_v}\left(\frac{d\phi_2(t)}{dt}\right) = \frac{1}{2\pi k_v}\left(\frac{d\phi_1(t)}{dt} - \frac{d\phi_e(t)}{dt}\right) \\
&= \frac{1}{2\pi k_v}\left(\frac{d\phi_1(t)}{dt}\right) = \frac{1}{2\pi k_v}\frac{d}{dt}\left(2\pi k_f \int_0^t m(\tau)\, d\tau\right) \\
&= \frac{k_f}{k_v} m(\tau)
\end{aligned}$$

- When the system operates in the phase-lock mode or near phase lock and the loop-gain parameter K_0 is large compared with unity, frequency demodulation of the incoming FM wave $s(t)$ is accomplished.