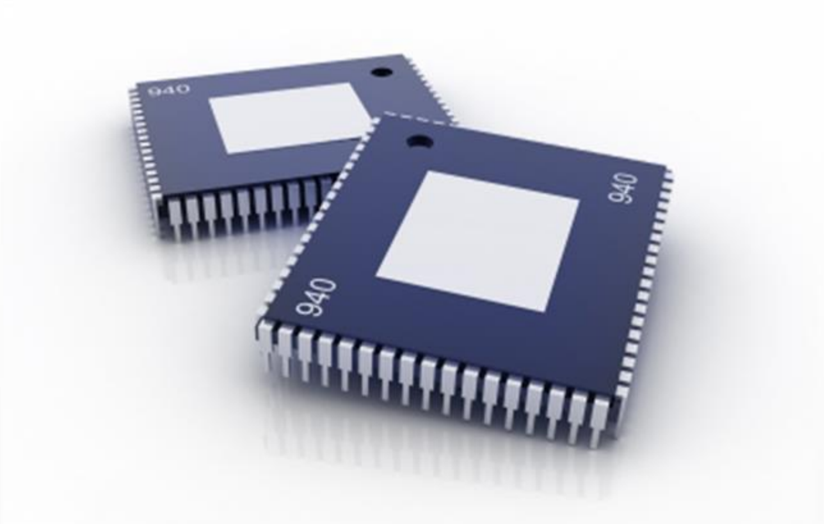

LECTURE 17

Soo-Won Kim

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18. Oversampling Converters

SQNR & SNR

$$V_{in(rms)} = \left[\frac{1}{T} \int_{-T/2}^{T/2} \left(\frac{V_{ref}}{2} \sin wt \right)^2 dt \right]^{1/2}$$

$$= \left[\frac{1}{T} \frac{V_{ref}^2}{8} \Bigg|_{-T/2}^{T/2} - \frac{1}{4w} \sin 2wt \Bigg|_{-T/2}^{T/2} \right]^{1/2}$$

$$= \frac{V_{ref}}{2\sqrt{2}}$$

Signal-to-quantization noise ratio (SQNR)

$$V_Q = V_{LSB} \cdot \frac{-t}{T} \left(-\frac{1}{2} T \le t \le \frac{1}{2} T \right)$$

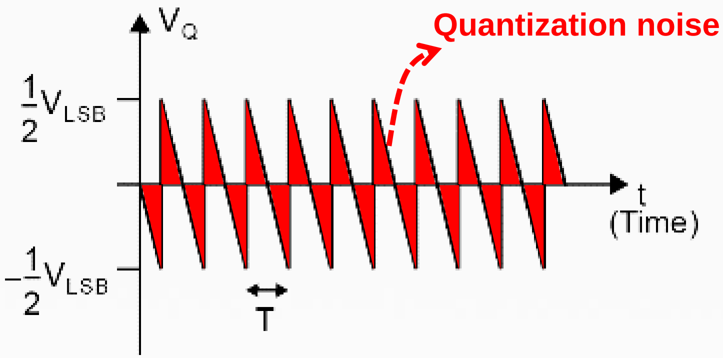


Fig. 15.6 Applying a ramp signal to the circuit in Fig. 15.5

$$SQNR = 20 \log \left(\frac{V_{in(rms)}}{V_{Q(rms)}} \right) \quad (15.15)$$

$$V_{Q(rms)} = \left[\frac{1}{T} \int_{-T/2}^{T/2} V_Q^2 dt \right]^{1/2}$$

$$= \left[\frac{1}{T} \int_{-T/2}^{T/2} (V_{LSB})^2 \left(\frac{-t}{T} \right)^2 dt \right]^{1/2} = \frac{V_{LSB}}{\sqrt{12}} \quad (15.11)$$

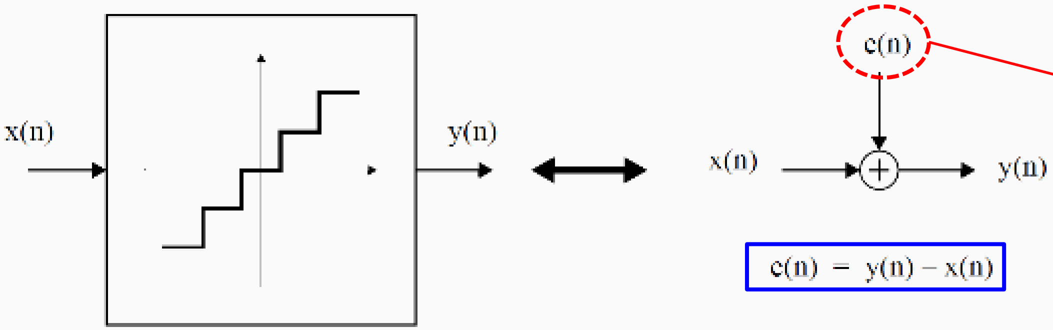
Root-mean-squared
: (+),(-)로 나오는 파형의 경우 파형의 크기를 구하기가 어려워 (-)를 양수화 하여 쉽게 크기를 구하기 위한 방법

$$SQNR = 20 \log \left(\frac{V_{in(rms)}}{V_{Q(rms)}} \right) = 20 \log \left(\frac{V_{ref} / 2\sqrt{2}}{V_{LSB} / \sqrt{12}} \right)$$

$$= 20 \log \left(2^N \right) = 6.02N + 1.76\ dB \quad (15.16)$$

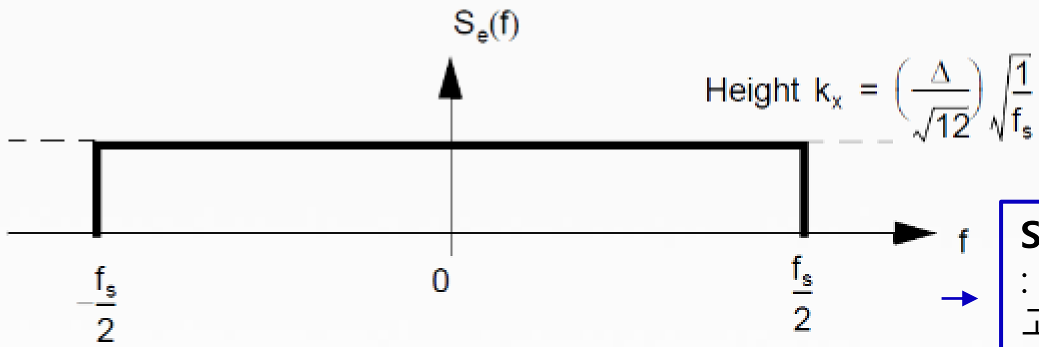
SQNR = Best possible SNR(signal to noise ratio)

Quantization Noise Modeling



Quantization error
= input과 output의 차이
→ **Related to the input signal**

Fig. 18.1 Quantizer and its linear model



→

Spectral density
: 단위 주파수에 얼마만큼의 에너지를 가지고 있는지를 나타낼 수 있는 방법

Fig. 18.2 Assumed spectral density of quantization noise

$$S_Q = \frac{1}{T}\int_{-T/2}^{T/2} V_Q^2 dt = \frac{1}{T}\int_{-T/2}^{T/2} \Delta^2 \left(\frac{-t}{T}\right)^2 dt = \frac{\Delta^2}{12}$$

$$(\Delta = V_{LSB}) \qquad (15.11)$$

$$\int_{-f_s/2}^{f_s/2} S_e^2(f)\,df = \int_{-f_s/2}^{f_s/2} k_x^2\,df = \frac{\Delta^2}{12} \qquad (18.1)$$

$$k_x = \left(\frac{\Delta}{\sqrt{12}}\right)\sqrt{\frac{1}{f_s}} \qquad (18.2)$$

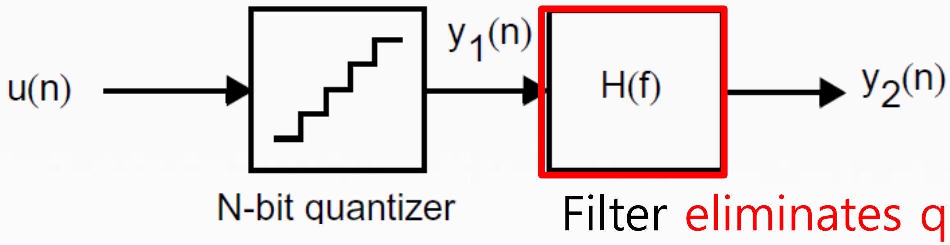
→ Independent of the f_s(Sampling frequency)

Oversampling Advantages

Oversampling occurs when $f_s > 2f_0$ ⟺ Nyquist-rate $f_s = 2f_0$

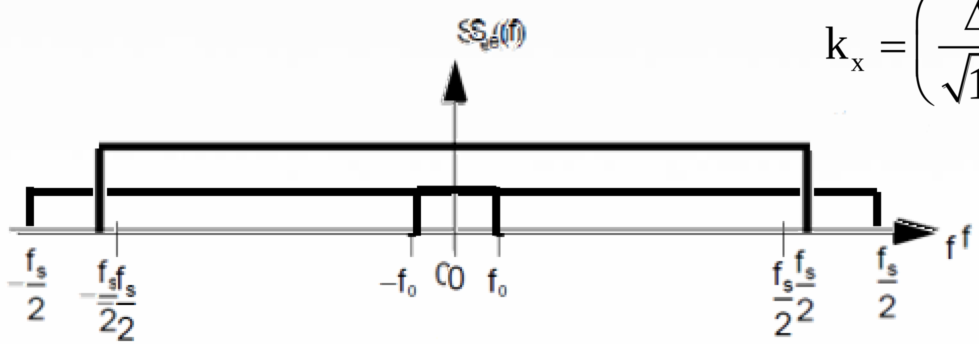
(Signal of interest = f_0, Sampling frequency = f_s)

Oversampling ratio, $OSR \equiv \frac{f_s}{2f_0}$ (18.8) → Every doubling of OSR, SNR improves by 3dB. (ex) OSR=4 → + 6.02dB



Filter eliminates quantization noise greater than f_0

$$k_x = \left(\frac{\Delta}{\sqrt{12}}\right)\sqrt{\frac{1}{f_s}} \qquad (18.2)$$



f_s ↑

Oversampling Advantages

Input signal power,

$$P_s = \left(\frac{V_{FS}}{2\sqrt{2}}\right)^2 = \frac{{V_{FS}}^2}{8} = \frac{\Delta^2 2^{2N}}{8} \qquad (18.9)$$

(assume, sin wave of peak-to-peak V_{FS}) $\quad \left(V_{FS} = \Delta 2^N\right)$

$$k_x = \left(\frac{\Delta}{\sqrt{12}}\right)\sqrt{\frac{1}{f_s}} \qquad (18.2)$$

Quantization noise power of input signal,

$$P_e = \int_{-f_s/2}^{f_s/2} S_e^2(f)\left|H(f)\right|^2 df = \int_{-f_0}^{f_0} k_x^2 df = \frac{2f_0}{f_s}\frac{\Delta^2}{12} = \frac{\Delta^2}{12}\left(\frac{1}{OSR}\right) \qquad (18.10)$$

$$OSR \equiv \frac{f_s}{2f_0} \qquad (18.8)$$

⇨ OSR ↑ (sampling frequency ↑), Quantization noise power ↓

Maximum SQNR

$$SQNR_{max} = 10\log\left(\frac{P_s}{P_e}\right) = 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log(OSR) \qquad (18.11)$$

$$= 6.02N + 1.76 + 10\log(OSR) \qquad (18.12)$$

⇨ **3 dB/octave** or equivalently **0.5 bits/octave**

Example

A 13-bit quantizer has an input frequency of 2MHz. Compare the SQNR if

1) Nyquist-rate (f_s = 4MHz)
2) Oversampling (f_s = 16MHz)

Input signal frequency f_0 = 2MHz
OSR = 4

Nyquist-rate

$$\mathrm{SQNR} = 20\log\left(\frac{V_{in(rms)}}{V_{Q(rms)}}\right)$$

$$= 20\log\left(\frac{V_{ref}/2\sqrt{2}}{V_{LSB}/\sqrt{12}}\right)$$

$$= 6.02N + 1.76\ \mathrm{dB}$$

$$= 80.02$$

Oversampling

$$\mathrm{SQNR} = 10\log\left(\frac{P_s}{P_e}\right)$$

$$= 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log(\mathrm{OSR})$$

$$= 6.02N + 1.76 + 10\log(\mathrm{OSR})$$

$$= 86.04$$

Oversampling improves overall SNR by 10log(OSR)

18. Oversampling Converters

18.1 Oversampling without Noise Shaping

18.2 Oversampling with Noise Shaping

18.3 System Architecture

18.4 Digital Decimation Filters

18.5 Higher-Order Modulators

18.6 Bandpass Oversampling Converters

18.7 Practical Considerations

18.8 Multi-Bit Oversampling Converters

18.9 Third-Order A/D Design Example

Delta-Sigma A/D converter system

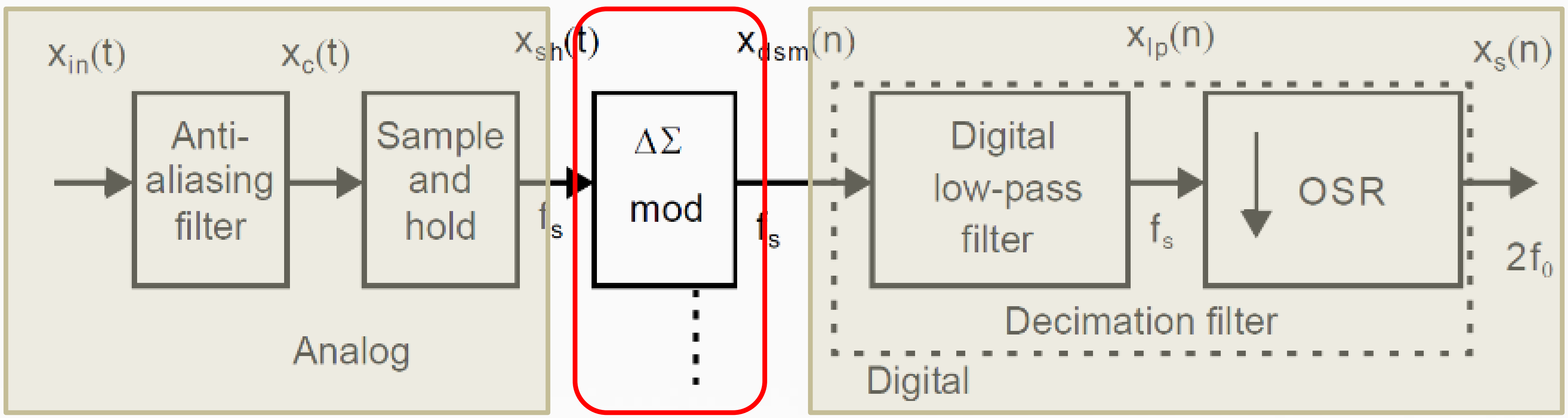
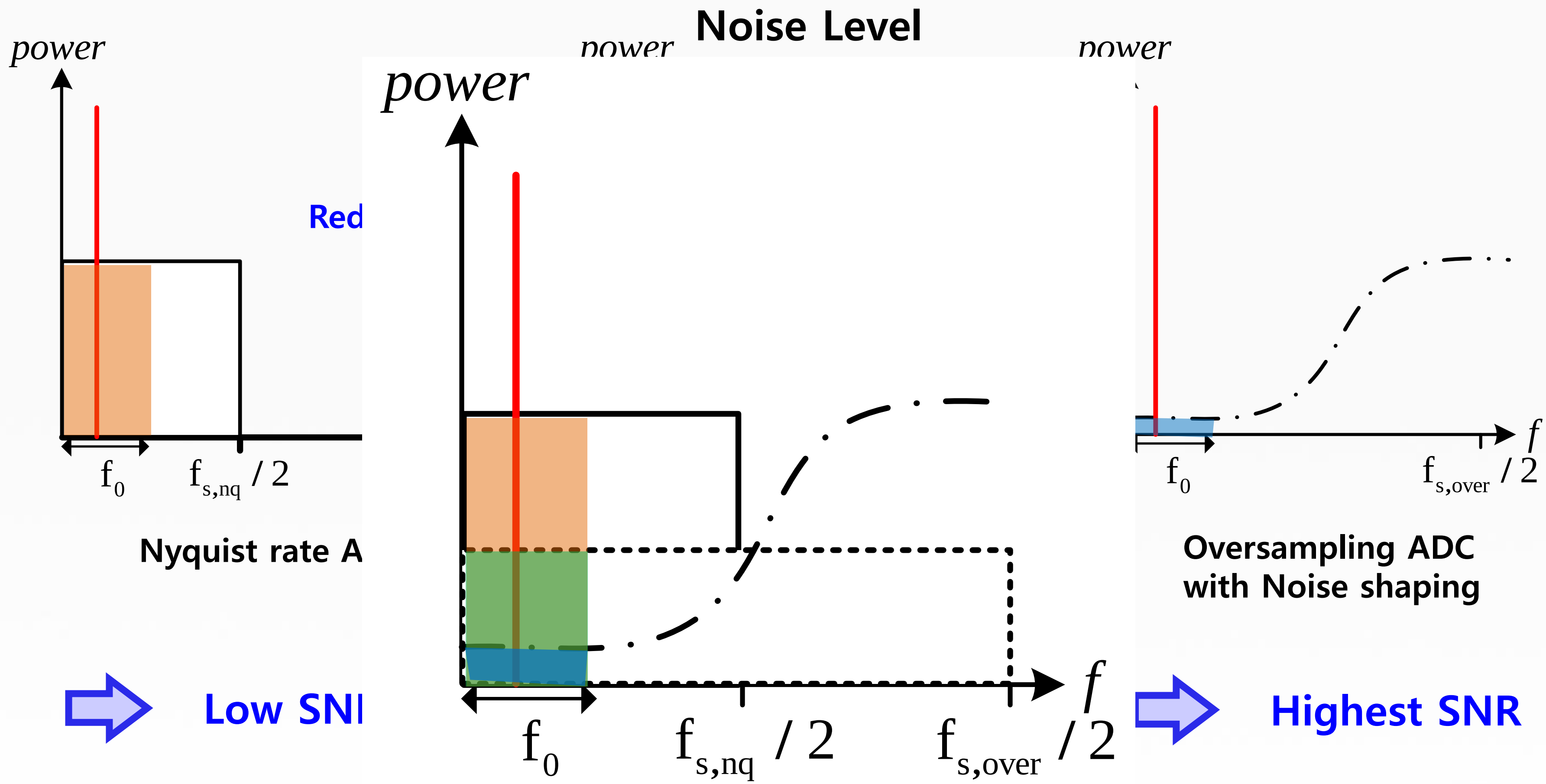


Fig. 18.4 Block diagram of an oversampling A/D converter

- Low input frequency bandwidth, high accuracy
- Low conversion rate

Oversampling with Noise Shaping

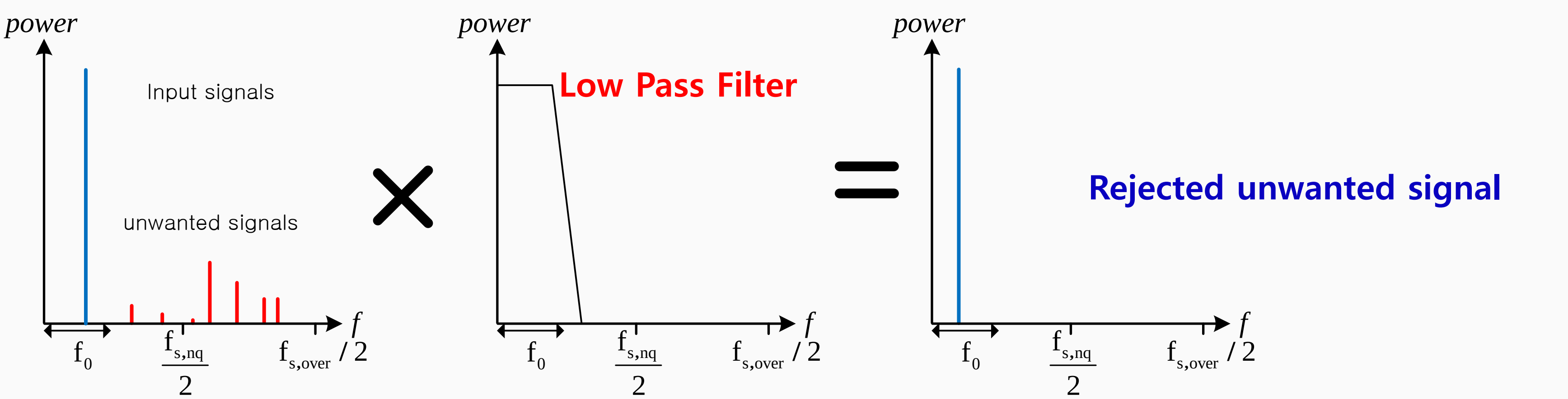
Noise shaping by High-pass filtering



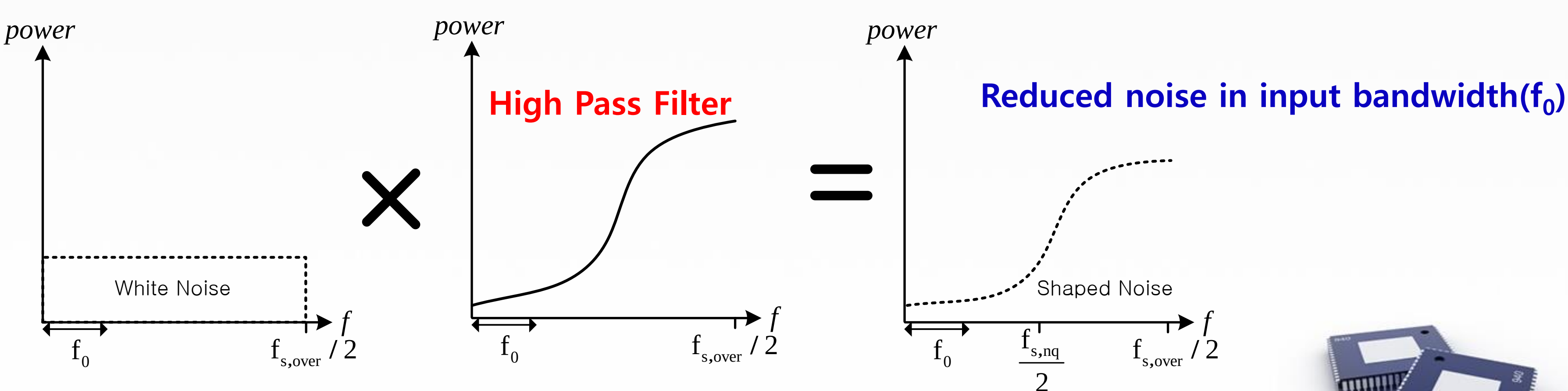
Reduce the noise

Signal / Noise transfer function

Signal X Filter = Filtered Signal



(a) Signal Transfer Function (S_{TF})



(b) Noise Transfer Function (N_{TF})

Noise-Shaped Delta-Sigma Modulator

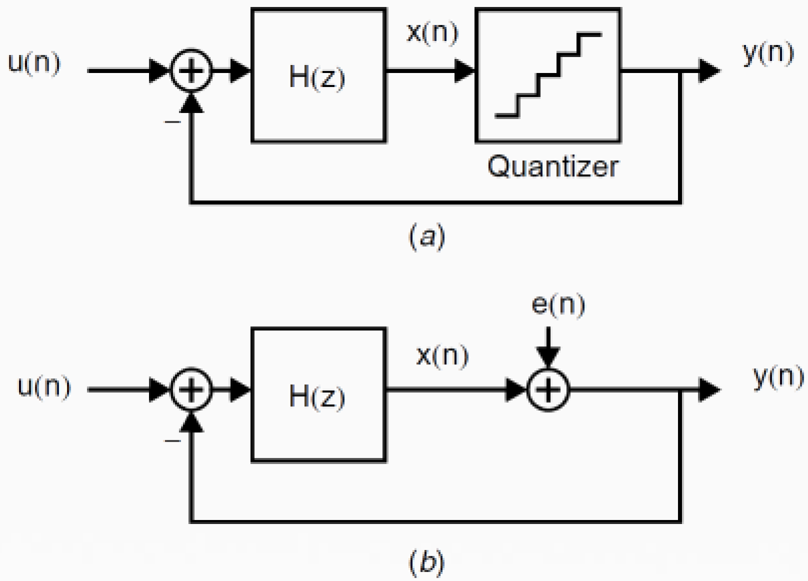
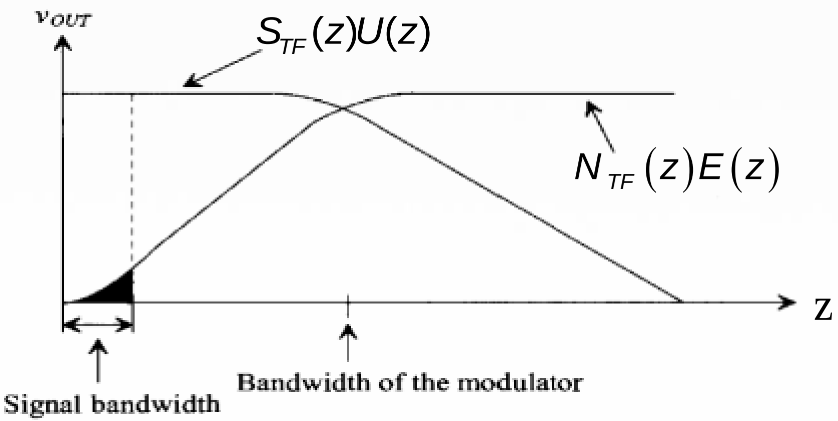


Fig. 18.5 A general ΔΣ modulator and its linear model.



Signal and Noise Transfer-Functions

$$Y(z) = \{U(z) - Y(z)\}H(z)$$

$$S_{TF}(z) \equiv \frac{Y(z)}{U(z)} = \boxed{\frac{H(z)}{1+H(z)}} \quad \text{LPF} \qquad (18.18)$$

$$Y(z) = E(z) - Y(z)H(z)$$

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \boxed{\frac{1}{1+H(z)}} \quad \text{HPF} \qquad (18.19)$$

$$Y(z) \equiv S_{TF}(z)U(z) + N_{TF}(z)E(z) \qquad (18.20)$$

Modulator pushes noise power out of the signal bandwidth

⇨ **Noise Shaping**

Integrator, H(z)

$$S_{TF}(z) \equiv \frac{Y(z)}{U(z)} = \boxed{\frac{H(z)}{1+H(z)}} \quad \textbf{LPF}$$

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \boxed{\frac{1}{1+H(z)}} \quad \textbf{HPF}$$

참고

$$Delay = z^{-1}$$

$$n\text{-th order } LPF = \frac{1}{\left(1-z^{-1}\right)^{N}}$$

$$n\text{-th order } HPF = \left(1-z^{-1}\right)^{N}$$

$$S_{TF}(z) \equiv \frac{Y(z)}{U(z)} = \frac{H(z)}{1+H(z)} = \frac{1}{1-z^{-1}}$$

$$\rightarrow 1+\frac{1}{H(z)} = 1-z^{-1}$$

$$\boxed{H(z) = -z}$$

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \frac{1}{1+H(z)} = 1-z^{-1}$$

$$\rightarrow 1+H(z) = \frac{1}{1-z^{-1}}$$

$$\boxed{H(z) = \frac{z^{-1}}{1-z^{-1}}}$$

Integrator, H(z)

$$S_{TF}(z) \equiv \frac{Y(z)}{U(z)} = \boxed{\frac{H(z)}{1+H(z)}} \ \text{LPF}$$

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \boxed{\frac{1}{1+H(z)}} \ \text{HPF}$$

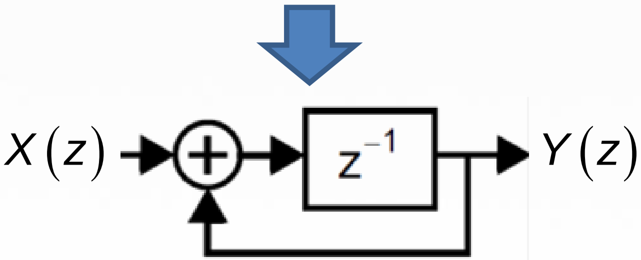
참고

$$Delay = z^{-1}$$

$$n\text{-th order } LPF = \frac{1}{\left(1-z^{-1}\right)^{N}}$$

$$n\text{-th order } HPF = \left(1-z^{-1}\right)^{N}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-z^{-1}} = \frac{1}{z-1}$$



Integrator

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{H(z)}{1+H(z)} = \frac{\frac{1}{z-1}}{1+\frac{1}{z-1}} = \frac{1}{z} = \boxed{z^{-1}}$$

$$N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1+H(z)} = \frac{1}{1+\frac{1}{z-1}} = \boxed{1-z^{-1}}$$

First-Order Noise Shaping

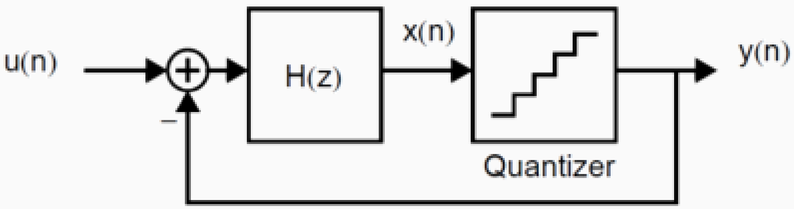


Fig. 18.5(a)

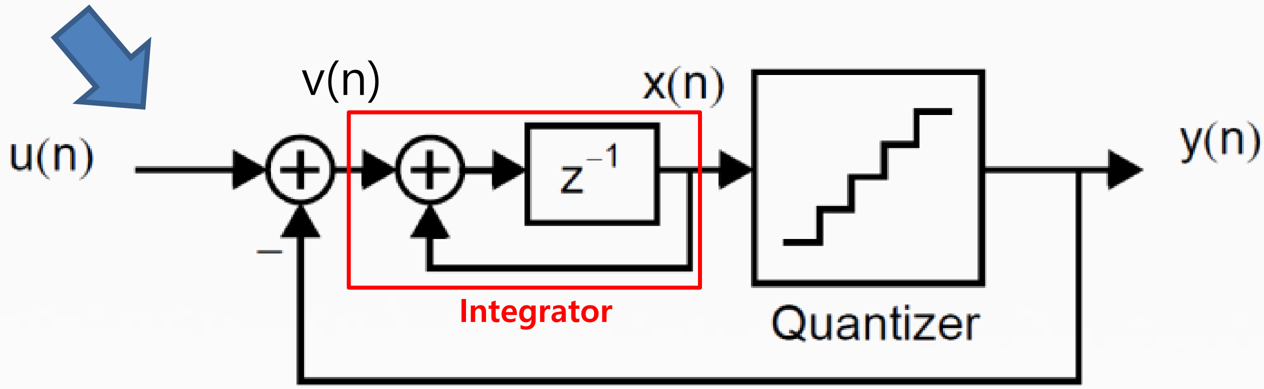


Fig. 18.6 A first-order noise-shaped interpolative modulator

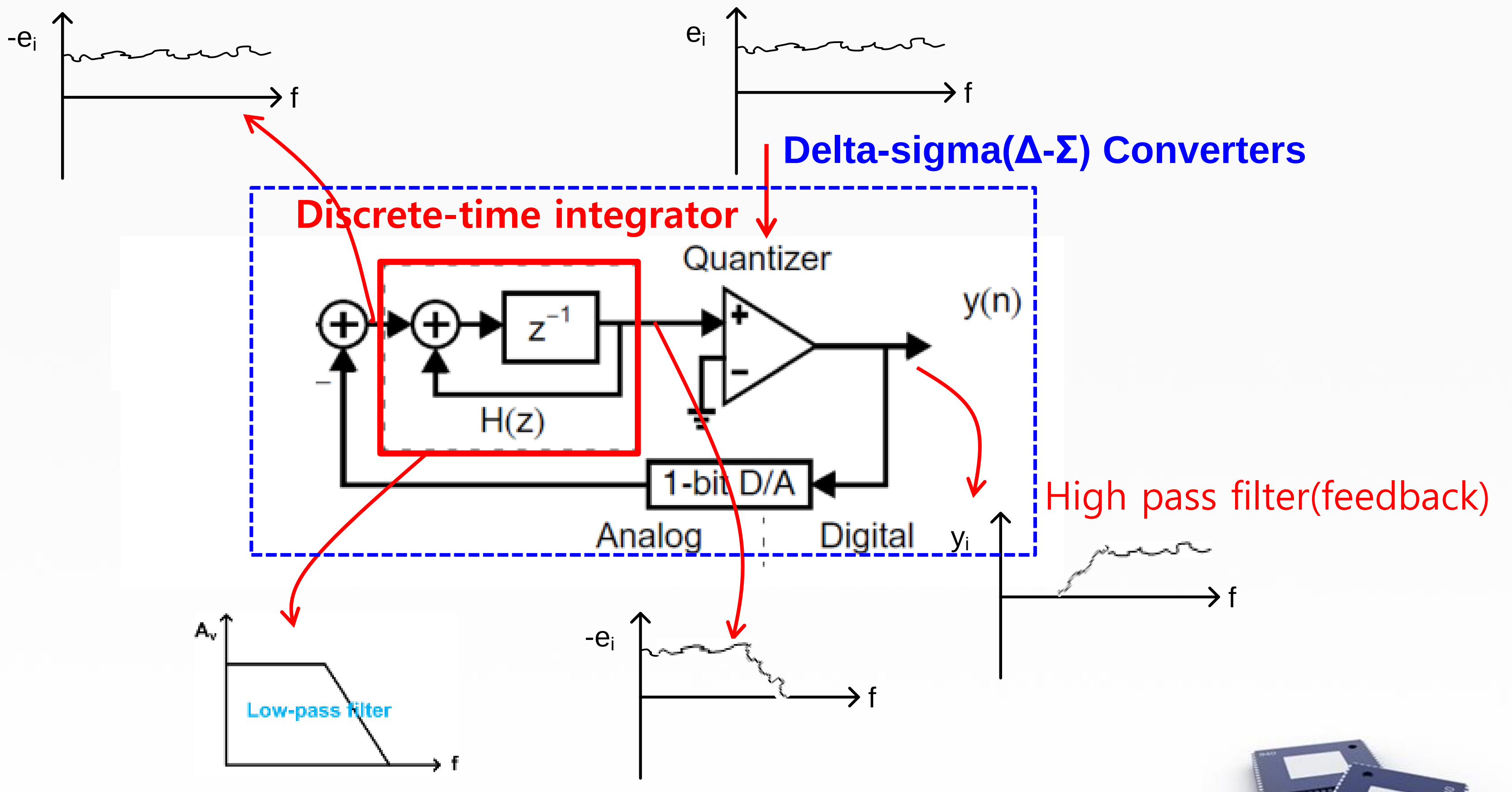
- **Time domain View**

✓ Average value of integrator input v(n)= 0 (i.e., average value of u(n) – y(n) = 0)

✓ Average value of u(n) ≈ average value of y(n)

First-Order Noise Shaping

- **Frequency Domain View**



First-Order Noise Shaping

$$k_x = \left(\frac{\Delta}{\sqrt{12}}\right)\sqrt{\frac{1}{f_s}} \quad (18.2)$$

- **Frequency Domain View**

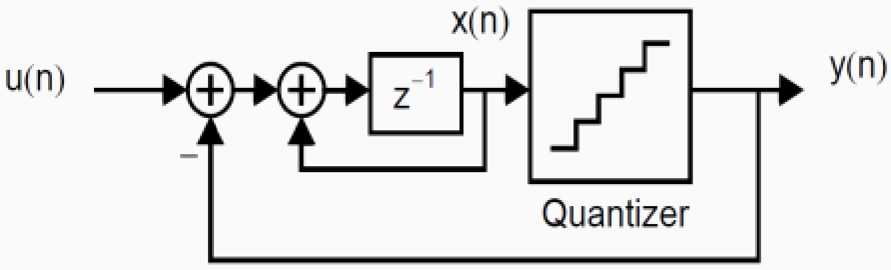


Fig. 18.6 A first-order noise-shaped interpolative modulator

Signal and Noise Transfer-Functions

$$S_{TF}(z) \equiv \frac{Y(z)}{U(z)} = \frac{1/(z-1)}{1+1/(z-1)} = z^{-1} \quad (18.22)$$

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \frac{1}{1+1/(z-1)} = (1-z^{-1}) \quad (18.23)$$

Calculation of SQNR to fine the enhancement by Noise shaping, $(z = e^{jwT} = e^{j2\pi f/f_s})$

$$N_{TF}(f) = 1 - e^{-j2\pi f/f_s} = \frac{e^{j\pi f/f_s} - e^{-j\pi f/f_s}}{2j} \times 2j \times e^{-j\pi f/f_s} = \sin\left(\frac{\pi f}{f_s}\right) \times 2j \times e^{-j\pi f/f_s} \quad (18.24)$$

$$P_e = \int_{-f_0}^{f_0} S_e^2(f)\left|N_{TF}(f)\right|^2 df = \int_{-f_0}^{f_0}\left(\frac{\Delta^2}{12}\right)\frac{1}{f_s}\left[2\sin\left(\frac{\pi f}{f_s}\right)\right]^2 df \cong \frac{\Delta^2\pi^2}{36}\left(\frac{1}{OSR}\right)^2 \quad (18.27)$$

$$SQNR_{max} = 10\log\left(\frac{P_s}{P_e}\right) = 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log\left[\frac{3}{\pi^2}(OSR)^3\right] \quad (18.28)$$

$$= 6.02N + 1.76 - 5.17 + 30\log(OSR) \quad (18.29)$$

First-Order Noise Shaping

Oversampling without noise shaping

$$SQNR_{max} = 6.02N + 1.76 + 10\log(OSR) \qquad (18.12)$$

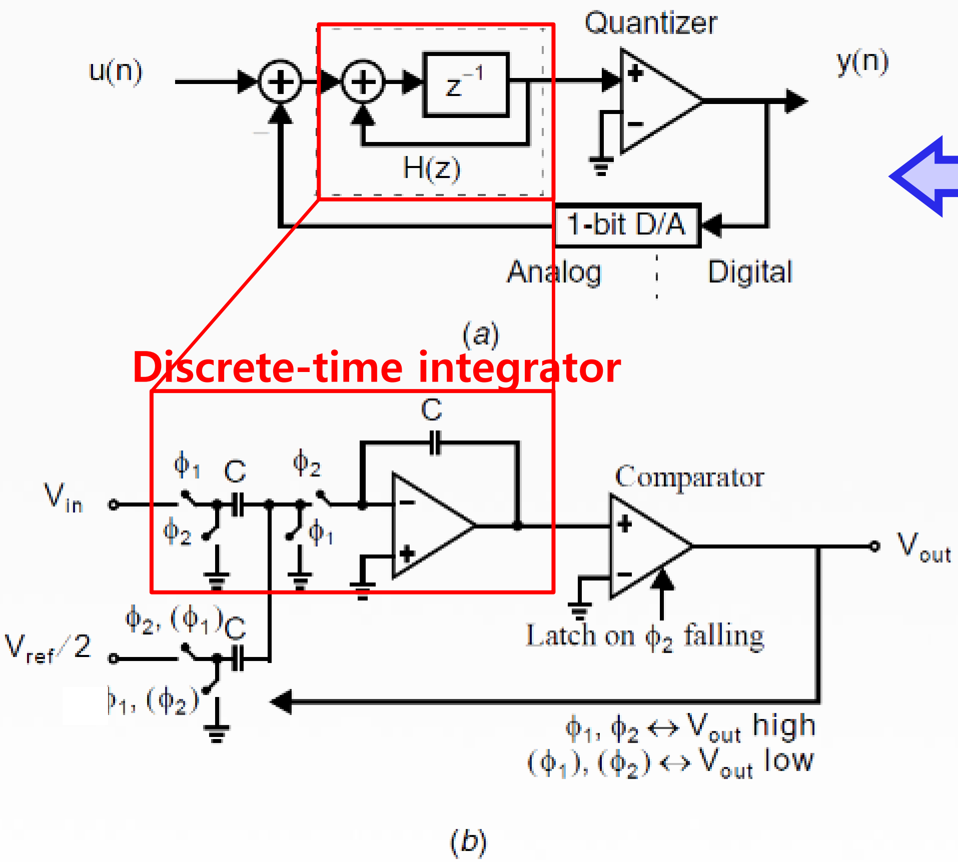
⇨ **3 dB/octave or equivalently 0.5 bits/octave**

Oversampling with noise shaping

$$SQNR_{max} = 6.02N + 1.76 - 5.17 + 30\log(OSR) \qquad (18.29)$$

⇨ **9 dB/octave or equivalently 1.5 bits/octave**

Switched-Capacitor Realization of a First-Order A/D Converter



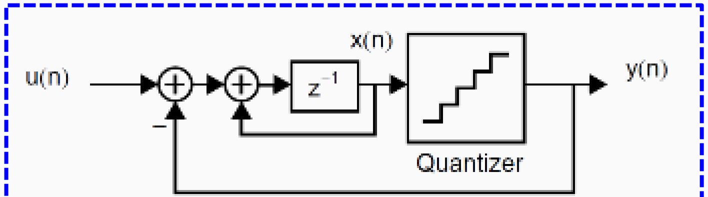


Fig. 18.6 A first-order noise-shaped interpolative modulator

Fig. 18.7 First-order A/D modulator : (a) block diagram; (b) switched-capacitor implementation

Operation of First-Order A/D Converter

- V_{out} = High

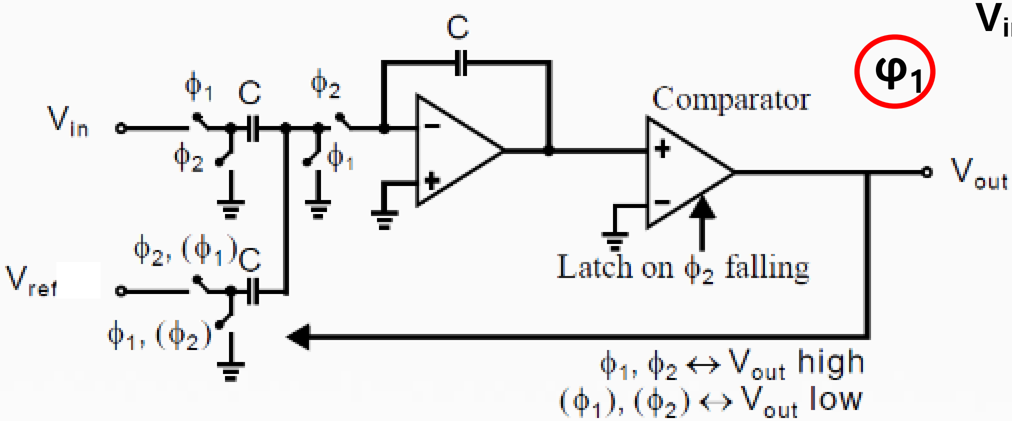
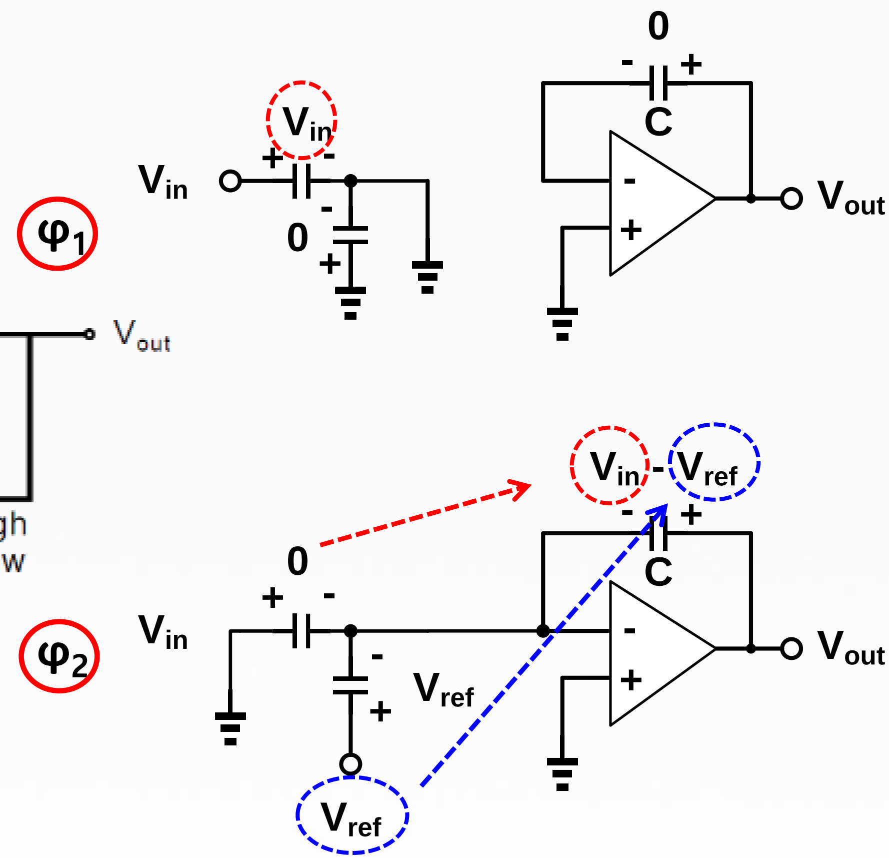


Fig. 18.7 First-order A/D modulator
; switched-capacitor implementation



V_{out} = High → Subtracted $V_{in} - V_{ref}$

Operation of First-Order Modulation

- V_{out} = Low

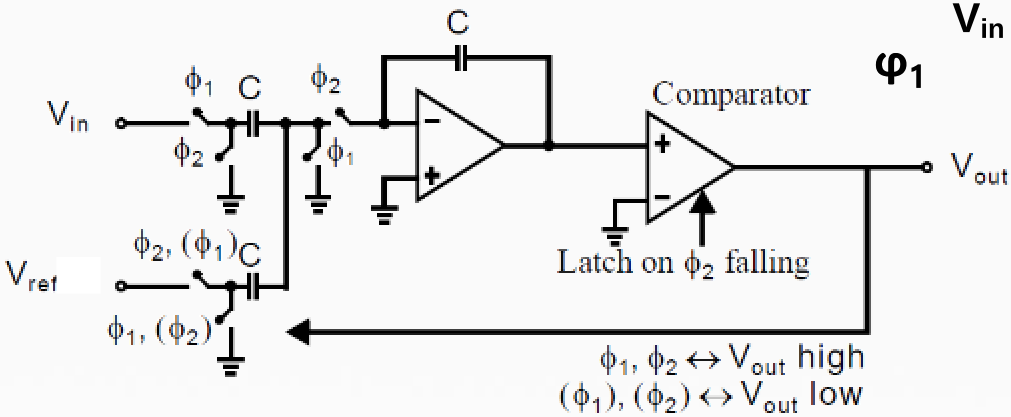
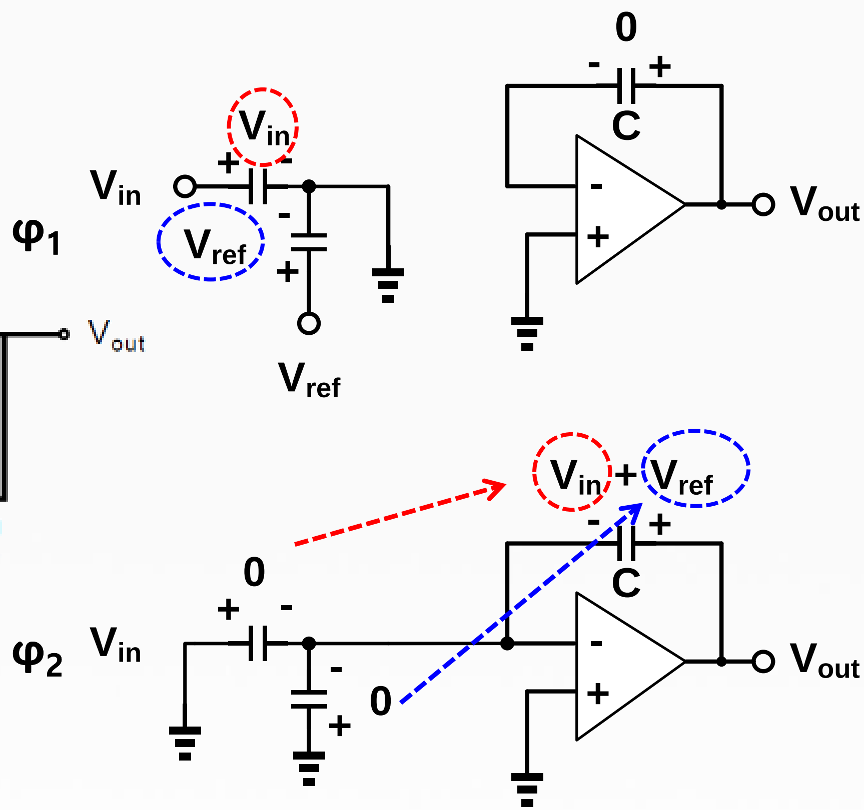


Fig. 18.7 First-order A/D modulator
; switched-capacitor implementation



V_{out} = High → Added $V_{in} + V_{ref}$

First-Order Noise Shaping

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \boxed{\frac{1}{1+H(z)}} \quad \text{HPF} \qquad (18.19)$$

$$H(z) = \frac{X(z)}{V(z)} = \frac{1}{z-1} \qquad (18.21)$$

H(z) should have a pole at DC (i.e., z = 1)

$$X(z)(z-1) = V(z)$$

$$zX(z) - X(z) = V(z) \quad \Rightarrow \quad x(n+1) - x(n) = v(n)$$

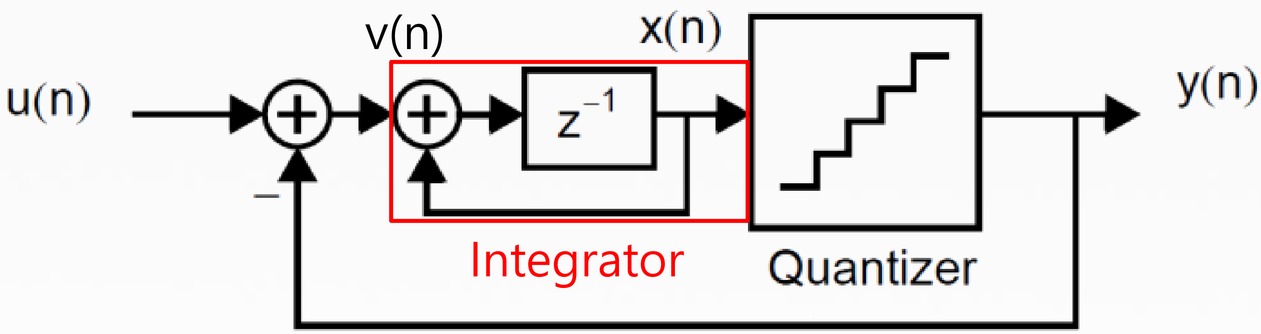


Fig. 18.6 A first-order noise-shaped interpolative modulator

- **Time domain View**
  - ✓ Average value of integrator input = 0 (i.e., average value of u(n) – y(n) = 0)
  - ✓ Average value of u(n) ≈ average value of y(n)

Operation of First-Order A/D Converter

First-order Δ-Σ ADC

Input = 0.82

Discrete-time integrator

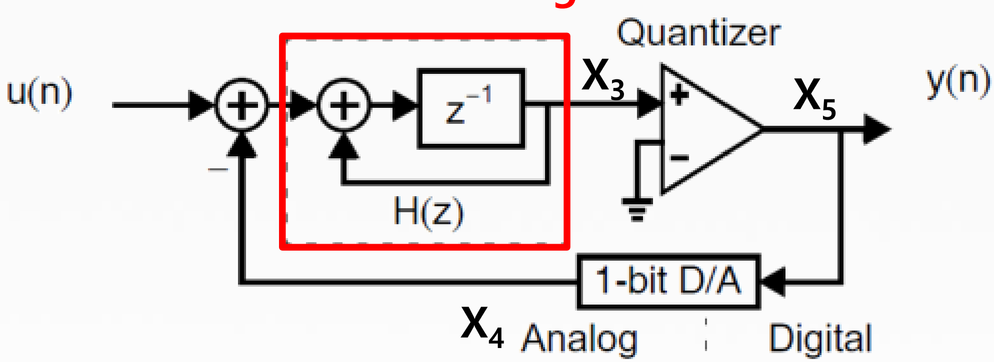
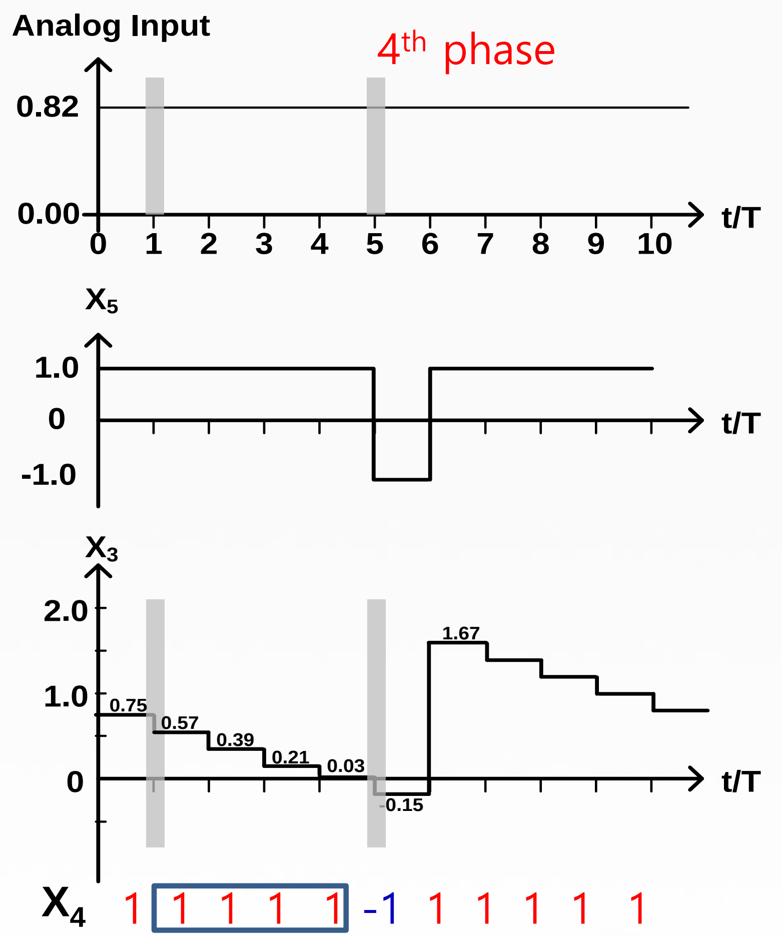


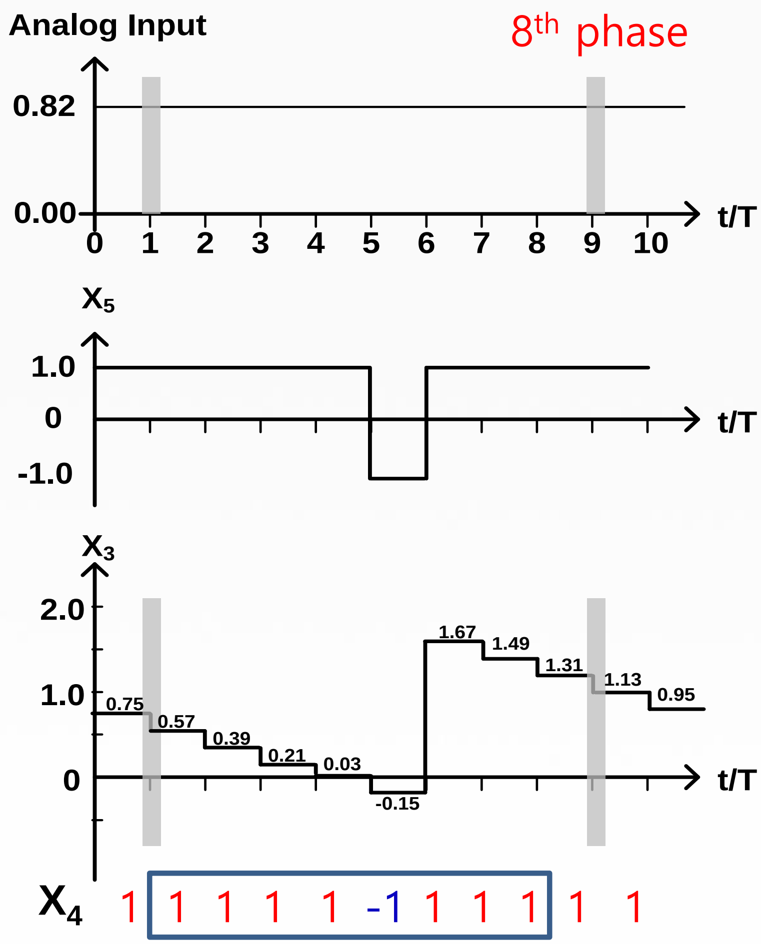
Fig. 18.7 First-order A/D modulator



Digital filter → $\frac{1+1+1+1}{4} = 1$ (+0.18)

Operation of First-Order A/D Converter

First-order Δ-Σ ADC Input = 0.82

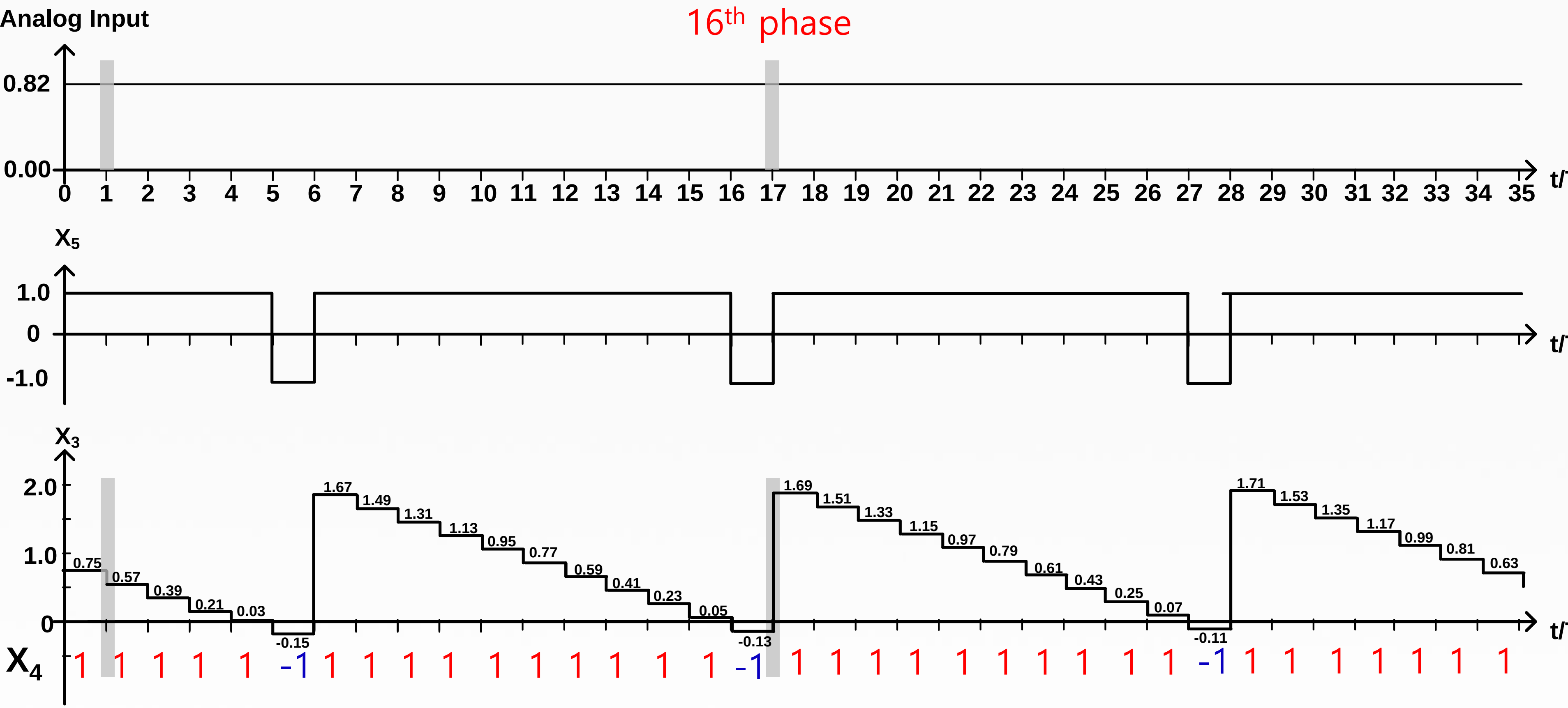


Digital filter → $\frac{1+1+1+1-1+1+1+1}{8} = 0.75$

(-0.07)

Operation of First-Order A/D Converter

First-order Δ-Σ ADC Input = 0.82

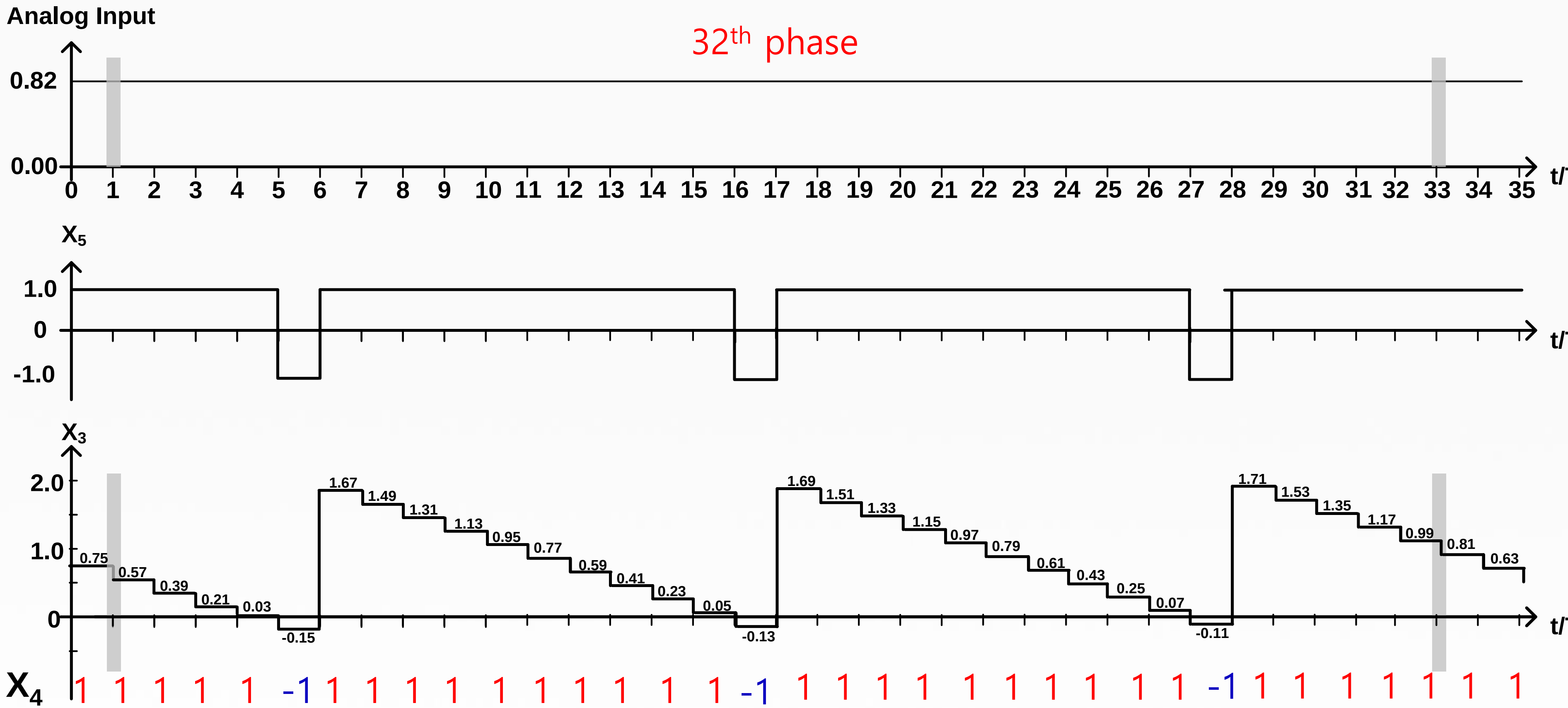


Digital filter → $\dfrac{1+1+1+1+1-1+1+1+1+1+1+1+1+1+1+1-1}{16} = 0.88$

(+0.06)

Operation of First-Order A/D Converter

First-order Δ-Σ ADC Input = 0.82



Digital filter → 0.8125 (-0.0075)

High-order Noise Shaping

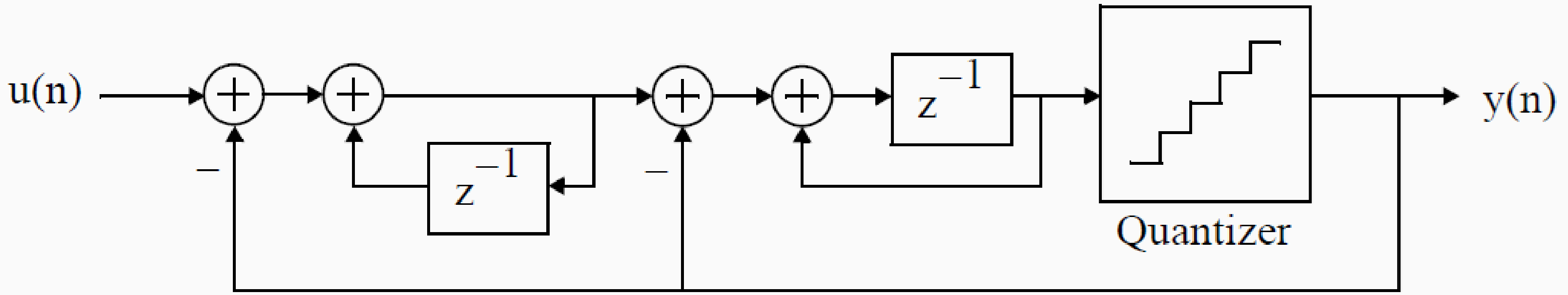
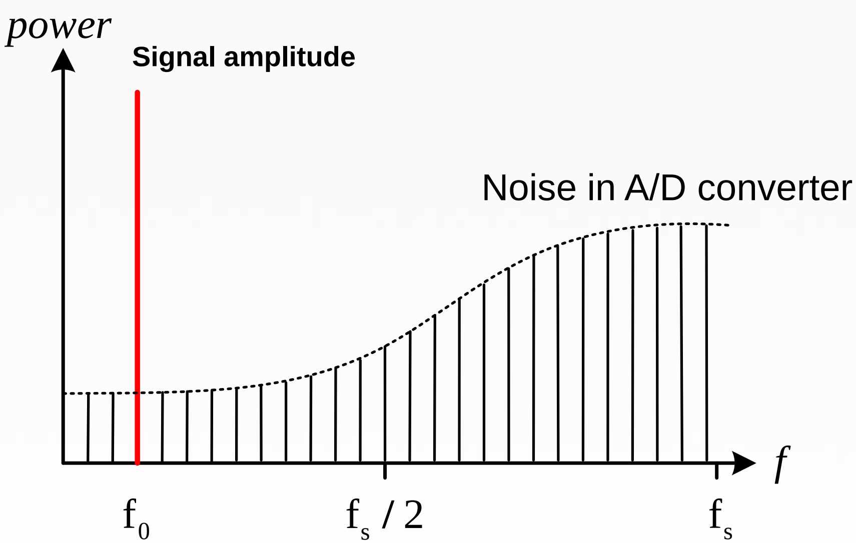
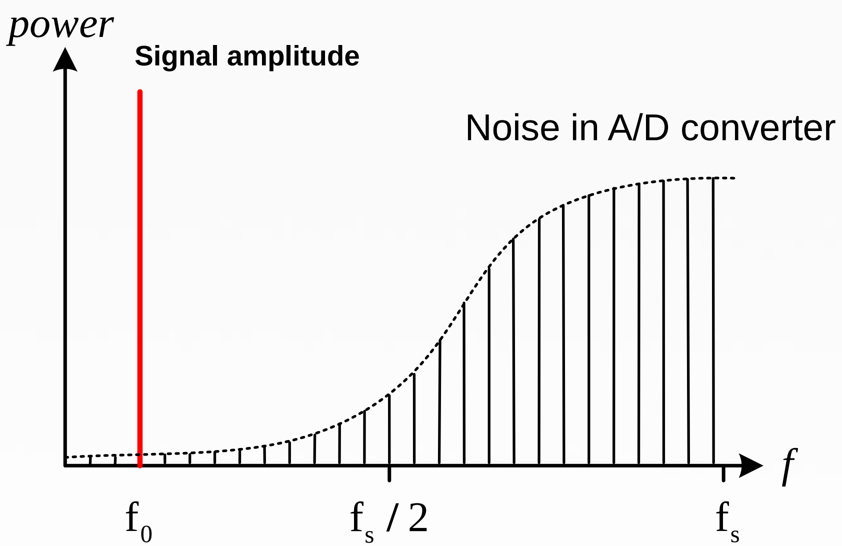


Fig. 18.9 Second-order ΔΣ modulator.



With 1st order noise shaping



With 2nd order noise shaping

$N_{TF}(f)$ = high-pass filter

High-order Noise Shaping

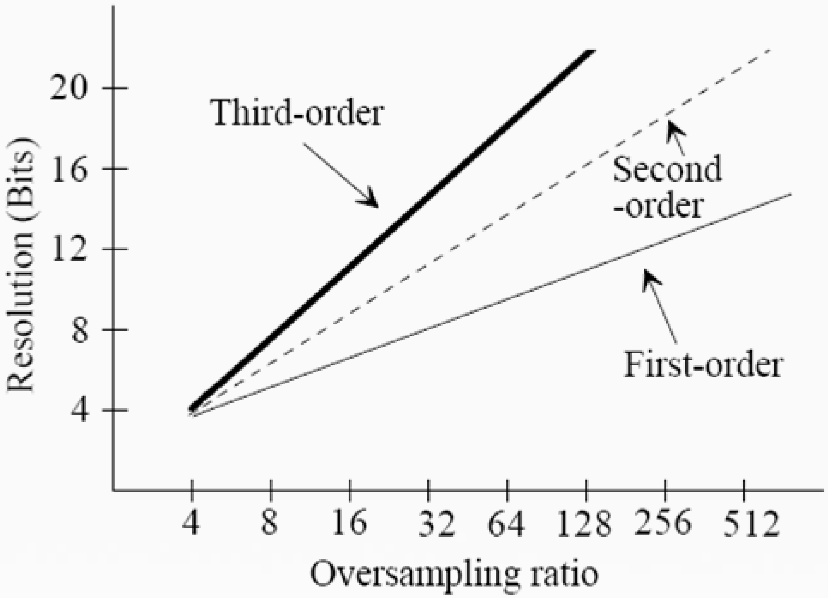


Fig. Comparison of resolution

- **As order increases,**
  - **Noise ↓, Resolution ↑**
  - **Stability issue ↑**

Delta-Sigma A/D converter system

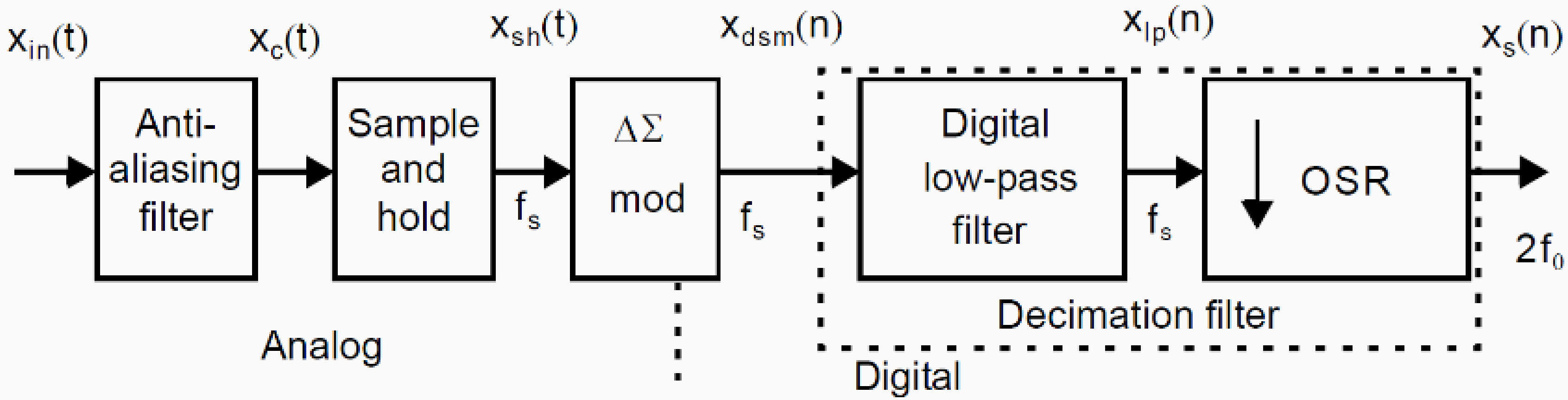
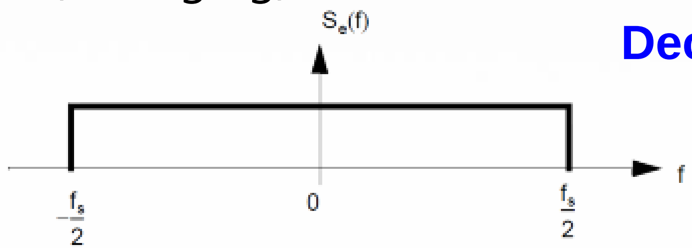
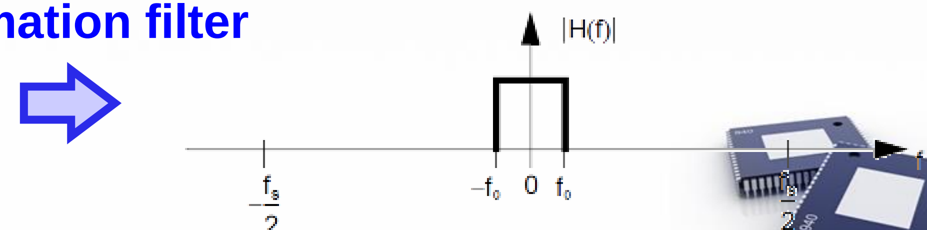


Fig. 18.4 Block diagram of an oversampling A/D converter

- ✓ **Anti-aliasing filter** : 0.5·f_s 보다 높은 입력의 주파수 term들을 filtering 한다
- ✓ **Decimation filter** : f_s(high)로 처리한 출력을 f_0(low)으로 filtering 한다 (averaging)



Decimation filter



Appendix : Input Signal Power Calculation

$$P_s \propto |V|^2$$

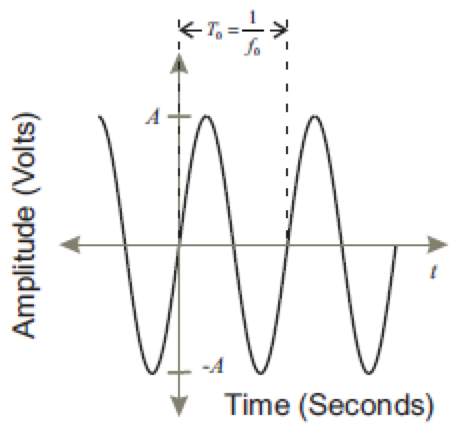
$$\sin(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2}$$

$$P_s = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \left[\left(\frac{V_{FS}}{2}\right)\sin(2\pi f_0 t)\right]^2 dt$$

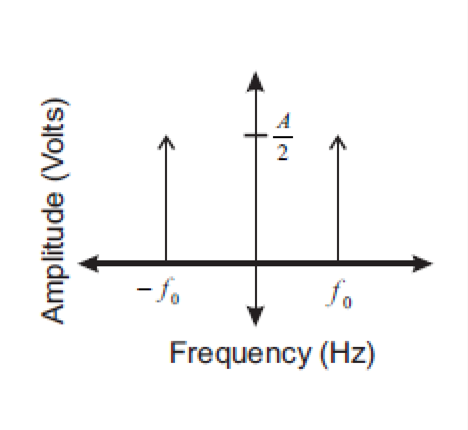
$$= \frac{1}{T_0}\int_{0}^{\frac{T_0}{2}} \left[\left(\frac{V_{FS}}{2}\right)^2 \frac{1-\cos(4\pi f_0 t)}{2}\right]^2 dt$$

$$= \frac{1}{T_0}\left[\left(\frac{V_{FS}}{2}\right)^2 t - \frac{V_{FS}^{\ 2}}{16\pi f}\sin(4\pi f_0 t)\right]_{t=0}^{t=\frac{T_0}{2}}$$

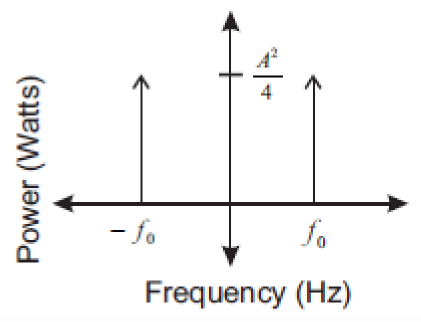
$$= \frac{V_{FS}^{\ 2}}{8} = \left(\frac{V_{FS}}{2\sqrt{2}}\right)^2$$



<sine wave>



<frequency domain representation>



<power spectrum>