Wireless Communications (ITC731)

Lecture Note 13

28-May-2013

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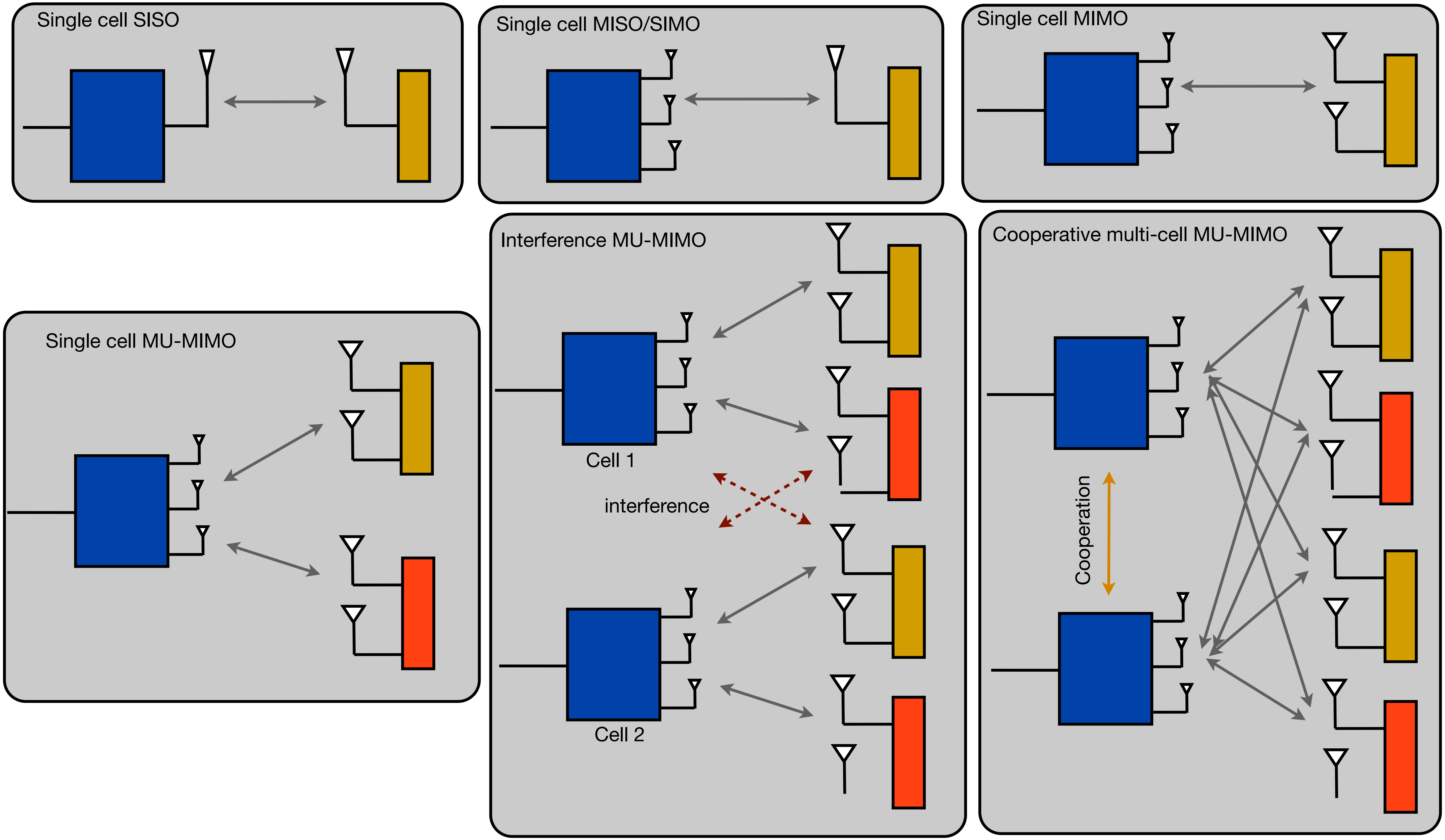
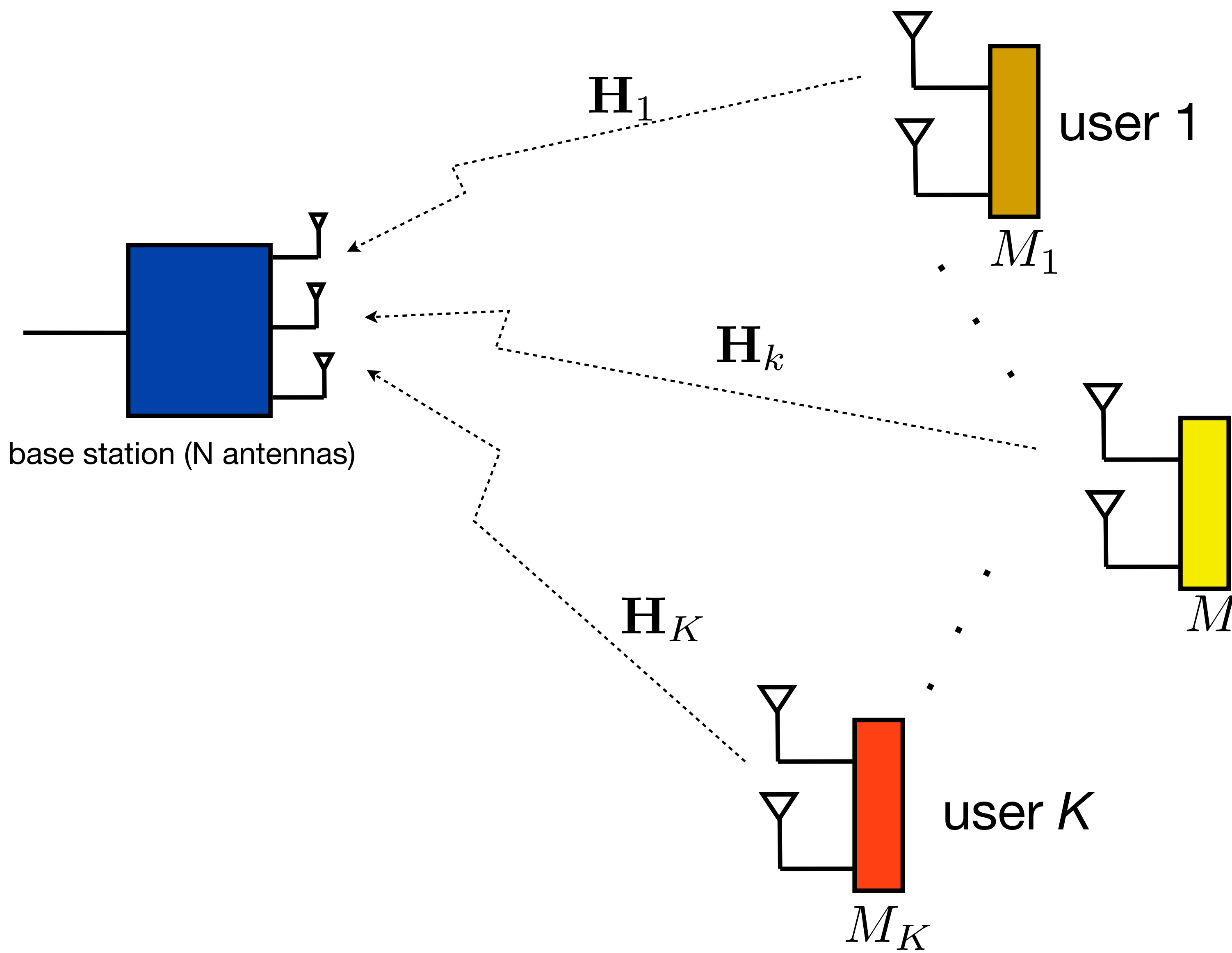
Summary

- Multi-User MIMO

MIMO Configuration



Uplink Multi-User MIMO (MU-MIMO)

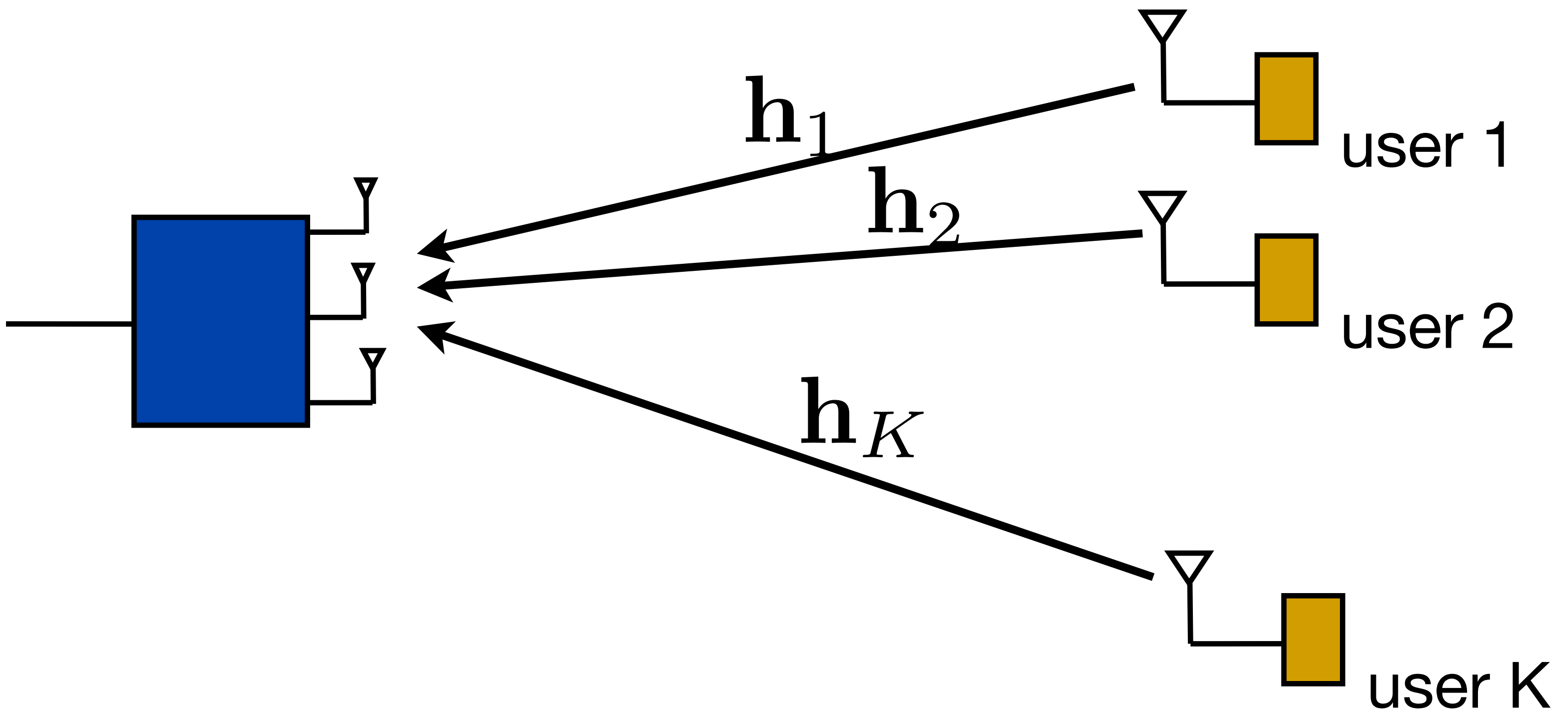


User k has M_k antennas.

At the same time!
At the same frequency!
All the users communicate simultaneously!

Uplink MU-MIMO Signal Model with Single TX Antenna

- Received signal at BS



$$\mathbf{h}_k = [h_{1k}\; h_{2k}\; \cdots h_{n_R k}]^T$$

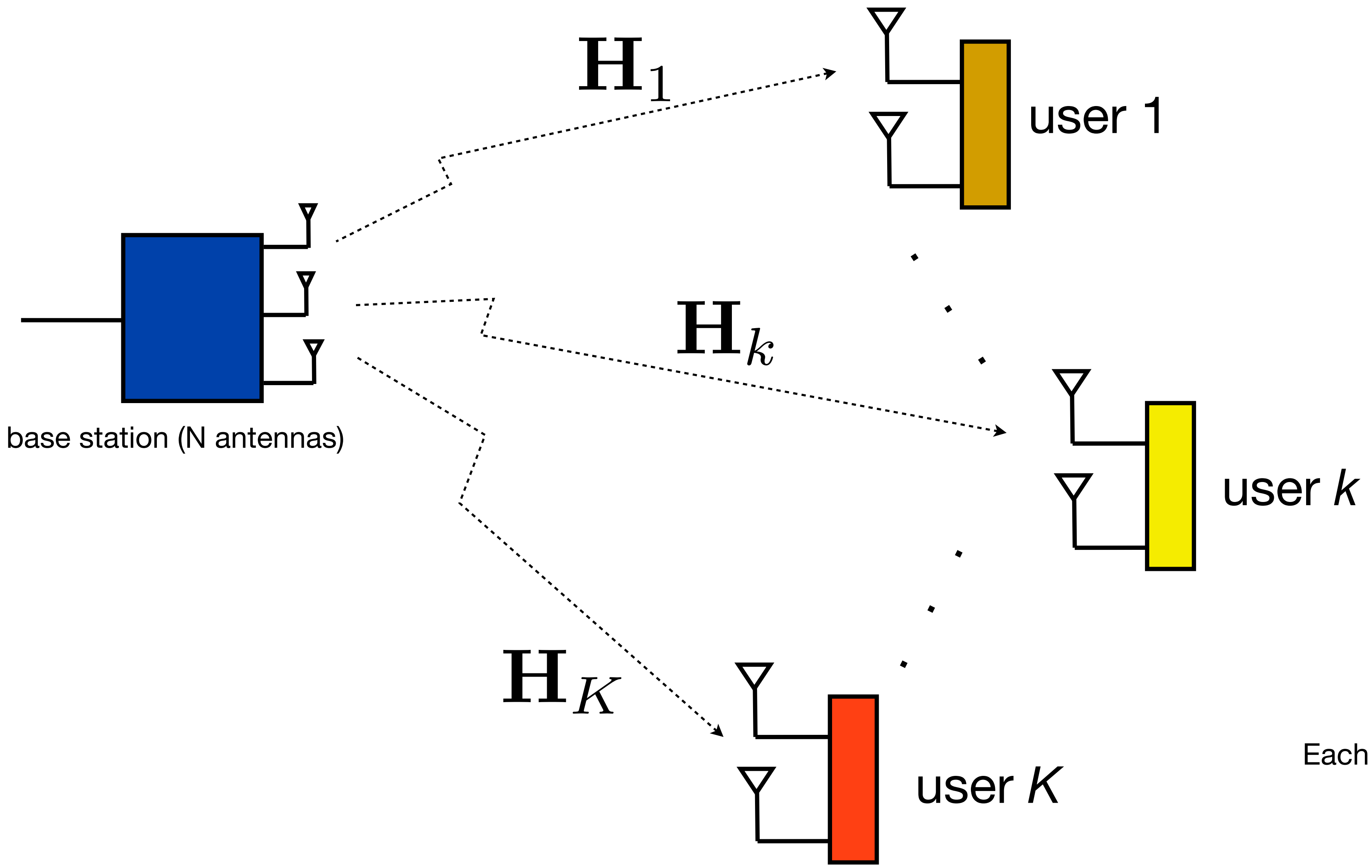
$$
\begin{aligned}
r_1 &= h_{11}x_1 + h_{12}x_2 + \cdots + h_{1K}x_{n_K} + n_1 \\
r_2 &= h_{21}x_1 + h_{22}x_2 + \cdots + h_{2K}x_{n_K} + n_2 \\
&\vdots \\
r_{n_R} &= h_{n_R1}x_1 + h_{n_R2}x_2 + \cdots + h_{n_RK}x_{n_K} + n_{n_R}
\end{aligned}
$$

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

The problem is the same as the single user MIMO.

The condition for the signal detection: $n_R \geq K$

Downlink MU-MIMO (MIMO Broadcast Channels)



Each user has M antennas.

Signal Model for Downlink MU-MIMO

Denote the signal vector for k user as:

$$\mathbf{x}_k = [x_{k1} \; x_{k2} \; \cdots \; x_{kN}]^T$$

Transmit signal for user k:

$$\mathbf{x}_k = \underset{[N \times M]}{\mathbf{W}_k} \underset{[M \times 1]}{\mathbf{s}_k} \Longrightarrow M \text{ data streams}$$

$\mathbf{W}_k$ is called the precoding matrix for user k.

Total transmit signal from BS:

$$\mathbf{X} = \sum_{k=1}^{K} \mathbf{x}_k = \sum_{k=1}^{K} \mathbf{W}_k \mathbf{s}_k$$

Received signal at user k:

$$
\begin{aligned}
\underbrace{\mathbf{y}_k}_{[M \times 1]} &= \mathbf{H}_k \mathbf{X} + \mathbf{n}_k \\
&= \mathbf{H}_k \sum_{j=1}^{K} \mathbf{x}_j + \mathbf{n}_k \\
&= \underbrace{\mathbf{H}_k}_{[M \times N]} \underbrace{\left(\sum_{j=1}^{K} \mathbf{W}_j \mathbf{s}_j \right)}_{[N \times 1]} + \underbrace{\mathbf{n}_k}_{[M \times 1]} \\
&= \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \left(\sum_{j=1, j \neq k}^{K} \mathbf{H}_k \mathbf{W}_j \mathbf{s}_j \right) + \mathbf{n}_k
\end{aligned}
$$

Multi-user interference (MUI)

$$\mathbf{H}_k \mathbf{W}_j : \; M \times M \text{ matrix}$$

$[M \times N] \quad [N \times M]$

Zero-forcing condition:

$$\mathbf{H}_k \mathbf{W}_j = \begin{cases} \mathbf{0}_M, & k \neq j \\ \mathbf{I}_M, & k = j \end{cases}$$

Consider the total channel matrix given as:

$$\mathbf{H}_{KM\times N} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} = \begin{bmatrix} h^1_{11} & h^1_{12} & \cdots & h^1_{1N} \\ h^1_{21} & h^1_{22} & \cdots & h^1_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ h^1_{M1} & h^1_{M2} & \cdots & h^1_{MN} \\ h^2_{11} & h^2_{12} & \cdots & h^2_{1N} \\ h^2_{21} & h^2_{22} & \cdots & h^2_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ h^2_{M1} & h^2_{M2} & \cdots & h^2_{MN} \\ \vdots & \vdots & \vdots & \vdots \\ h^K_{11} & h^K_{12} & \cdots & h^K_{1N} \\ h^K_{21} & h^K_{22} & \cdots & h^K_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ h^K_{M1} & h^K_{M2} & \cdots & h^K_{MN} \end{bmatrix} \begin{matrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \mathbf{H}_K \end{matrix}$$

Also define the aggregated precoding matrix as:

$$\mathbf{W}_{N\times KM} = [\mathbf{W}_1\ \mathbf{W}_2\ \cdots\ \mathbf{W}_K]$$

$$= \left[\begin{array}{cccc|cccc|c|cccc}
w^1_{11} & w^1_{12} & \cdots & w^1_{1M} & w^2_{11} & w^2_{12} & \cdots & w^2_{1M} & \cdots & w^K_{11} & w^K_{12} & \cdots & w^K_{1M} \\
w^1_{21} & w^1_{22} & \cdots & w^1_{2M} & w^2_{21} & w^2_{22} & \cdots & w^2_{2M} & \cdots & w^K_{21} & w^K_{22} & \cdots & w^K_{2M} \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots\vdots & \vdots & \cdots & \vdots & \vdots & \\
w^1_{N1} & w^1_{N2} & \cdots & w^1_{NM} & w^2_{N1} & w^2_{N2} & \cdots & w^2_{NM} & \cdots & w^K_{N1} & w^K_{N2} & \cdots & w^K_{NM}
\end{array}\right]$$

$$\mathbf{W}_1 \qquad \mathbf{W}_2 \qquad \mathbf{W}_K$$

Note that

$$\mathbf{H}_{KM\times N}\mathbf{W}_{N\times KM} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} [\mathbf{W}_1\ \mathbf{W}_2\ \cdots\ \mathbf{W}_K]$$

$$= \begin{bmatrix} \mathbf{H}_1\mathbf{W}_1 & \mathbf{H}_1\mathbf{W}_2 & \cdots & \mathbf{H}_1\mathbf{W}_K \\ \mathbf{H}_2\mathbf{W}_1 & \mathbf{H}_2\mathbf{W}_2 & \cdots & \mathbf{H}_2\mathbf{W}_K \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{H}_K\mathbf{W}_1 & \mathbf{H}_K\mathbf{W}_2 & \cdots & \mathbf{H}_K\mathbf{W}_K \end{bmatrix}$$

where $\mathbf{H}_i\mathbf{W}_j$: $M \times M$ matrix

Zero-forcing condition: $\mathbf{H}_k \mathbf{W}_j = \mathbf{0}_{M\times M} \quad (k \neq j)$

$$\mathbf{H}_k \mathbf{W}_k = \mathbf{I}_{M\times M}$$

$$\mathbf{H}_{KM\times N}\mathbf{W}_{N\times KM} = \begin{bmatrix} \mathbf{H}_1\mathbf{W}_1 & \mathbf{H}_1\mathbf{W}_2 & \cdots & \mathbf{H}_1\mathbf{W}_K \\ \mathbf{H}_2\mathbf{W}_1 & \mathbf{H}_2\mathbf{W}_2 & \cdots & \mathbf{H}_2\mathbf{W}_K \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{H}_K\mathbf{W}_1 & \mathbf{H}_K\mathbf{W}_2 & \cdots & \mathbf{H}_K\mathbf{W}_K \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 1 & \cdots & \cdots & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & \cdots & 0 & \cdots & 1 \end{bmatrix} = \mathbf{I}_{KM\times KM}$$

Then the received signal for all the user can be written as

$$\underset{[KM \times 1]}{\mathbf{Y}} = \mathbf{H}_{KM \times N} \mathbf{W}_{N \times KM} \underset{[KM \times 1]}{\mathbf{S}} + \mathbf{N}$$

where

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_K \end{bmatrix}$$

$$\mathbf{y}_k = \begin{bmatrix} y_{k1} \\ y_{k2} \\ \vdots \\ y_{kM} \end{bmatrix} \qquad \mathbf{s}_k = \begin{bmatrix} s_{k1} \\ s_{k2} \\ \vdots \\ s_{kM} \end{bmatrix}$$

Hence, the zero-forcing solution becomes

$$\mathbf{W}_{N\times KM} = \mathbf{H}^{H}_{KM\times N}\left(\mathbf{H}_{KM\times N}\mathbf{H}^{H}_{KM\times N}\right)^{-1}$$

Then,

$$\begin{aligned}\mathbf{Y} &= \mathbf{H}_{KM\times N}\mathbf{H}^{H}_{KM\times N}\left(\mathbf{H}_{KM\times N}\mathbf{H}^{H}_{KM\times N}\right)^{-1}\mathbf{S}+\mathbf{N} \\ &= \mathbf{S}+\mathbf{N}\end{aligned}$$

The received signal at user k becomes

$$\mathbf{y}_k = \mathbf{s}_k + \mathbf{n}_k$$

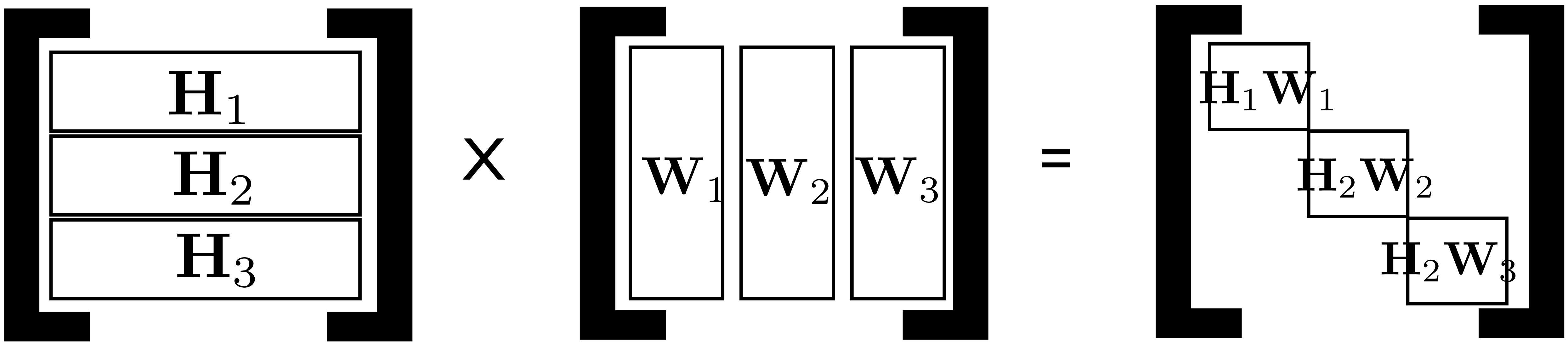
Sum-Rate Performance

$$
\begin{aligned}
C_s &= \sum_{k=1}^{K} C_k \\
&\approx \sum_{k=1}^{K} M \log_2(1+\gamma_k) \\
&\sim MK \log_2(1+\gamma)
\end{aligned}
$$

Degree of freedom: MK

MU-MIMO for Zero MUI

Example of 3 User MU-MIMO



Design $\mathbf{W}$ to suppress MUI.

zero MUI condition:

$$\underset{[M \times N]}{\mathbf{H}_k} \underset{[N \times M]}{\mathbf{W}_m} = 0 \quad \text{for all } k \neq m$$

Block Diagonalization

Step 1: Precoding matrix design for MUI elimination

$$\mathbf{H}_k \mathbf{W}_m = 0 \quad \text{for all } k \neq m$$

- Define the null channel matrix for $\mathbf{W}_m$ given as $\tilde{\mathbf{H}}_j$

$$\underset{[M(K-1) \times N]}{\tilde{\mathbf{H}}_m} = [\mathbf{H}_1^T \ \cdots \ \mathbf{H}_{m-1}^T \ \mathbf{H}_{m+1}^T \ \cdots \ \mathbf{H}_K^T]$$

Then, $\mathbf{W}_m$ should be then null space of $\tilde{\mathbf{H}}_j$

Singular value decomposition (SVD) of $\tilde{\mathbf{H}}_m$

$$\underset{[M(K-1)\times N]}{\tilde{\mathbf{H}}_m} = \overset{[M(K-1)\times M(K-1)]}{\tilde{\mathbf{U}}_m} \underset{[M(K-1)\times N]}{\tilde{\mathbf{D}}_m} \left[\underset{[N\times M(K-1)]}{\tilde{\mathbf{V}}_m^{(1)}} \;\; \underset{[N\times(N-M(K-1))]}{\tilde{\mathbf{V}}_m^{(0)}} \right]^H$$

where $\tilde{\mathbf{V}}_m^{(0)}$ is the null space of $\tilde{\mathbf{H}}_m$

Then

$$\mathbf{W}_m = \tilde{\mathbf{V}}_m^{(0)}$$

Received signal vector at user k

$$\begin{aligned} \mathbf{y}_k &= \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \underbrace{\sum_{j=1, j\neq k} \mathbf{H}_k \mathbf{W}_j}_{\to 0} + \mathbf{n}_k \\ &= \mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)} \mathbf{s}_k + \mathbf{n}_k \end{aligned}$$

Example: $N=6,\ M=2,\ K=3$ for $m=1$

$$\tilde{\mathbf{H}}_1 = \begin{bmatrix} \mathbf{H}_2 \\ \mathbf{H}_3 \end{bmatrix}$$

$$= \begin{bmatrix} | & | & | & | \\ \tilde{\mathbf{u}}_1 & \tilde{\mathbf{u}}_2 & \tilde{\mathbf{u}}_3 & \tilde{\mathbf{u}}_4 \\ | & | & | & | \end{bmatrix} \begin{bmatrix} \tilde{\lambda}_1 & 0 & 0 & 0 \\ 0 & \tilde{\lambda}_2 & 0 & 0 \\ 0 & 0 & \tilde{\lambda}_3 & 0 \\ 0 & 0 & 0 & \tilde{\lambda}_4 \end{bmatrix} \begin{bmatrix} - & \tilde{v}_1^{(1)H} & - \\ - & \tilde{v}_2^{(1)H} & - \\ - & \tilde{v}_3^{(1)H} & - \\ - & \tilde{v}_4^{(1)H} & - \\ - & \tilde{v}_1^{(0)H} & - \\ - & \tilde{v}_2^{(0)H} & - \end{bmatrix}$$

$$\mathbf{W}_1 = \tilde{\mathbf{V}}_1^{(0)}$$

Step 2: Receiver filter $\mathbf{G}_m$ design for ISI elimination

- Define the effective channel matrix for user m as

$$\underset{[M \times (N - M(K-1))]}{\mathbf{H}_m^{eff}} \triangleq \underset{[M \times N]}{\mathbf{H}_m} \underset{[N \times (N - M(K-1))]}{\mathbf{W}_m} = \mathbf{H}_m \tilde{\mathbf{V}}_m^{(0)}$$

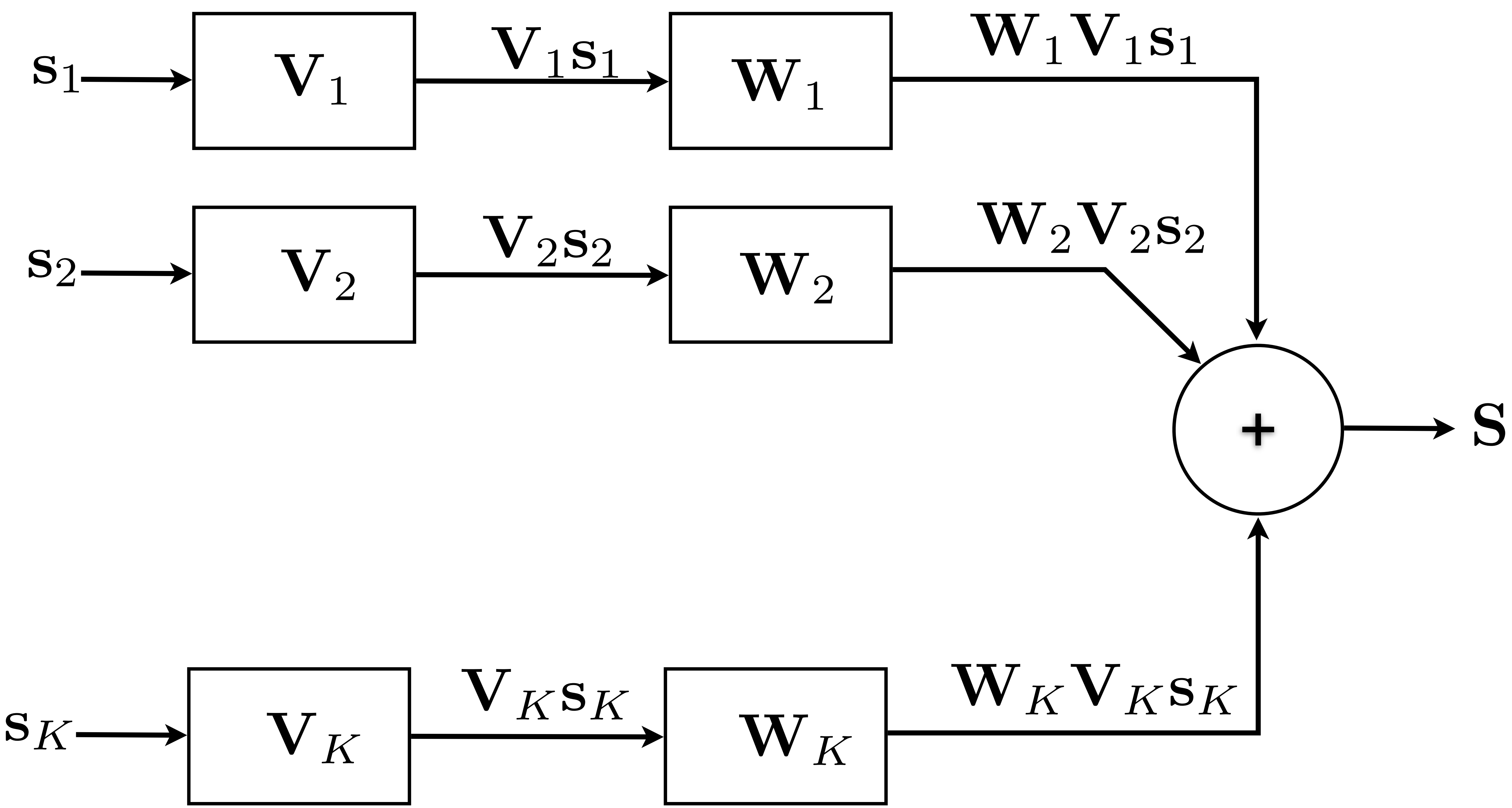
- SVD based eigen-beamforming (Optimal scheme)

$$\underset{[M \times (N - M(K-1))]}{\mathbf{H}_m^{eff}} = \underset{[M \times M]}{\mathbf{U}_m} \underset{[M \times (N - M(K-1))]}{\mathbf{D}_m} \underset{[(N - M(K-1) \times (N - M(K-1))]}{\mathbf{V}_m^H}$$

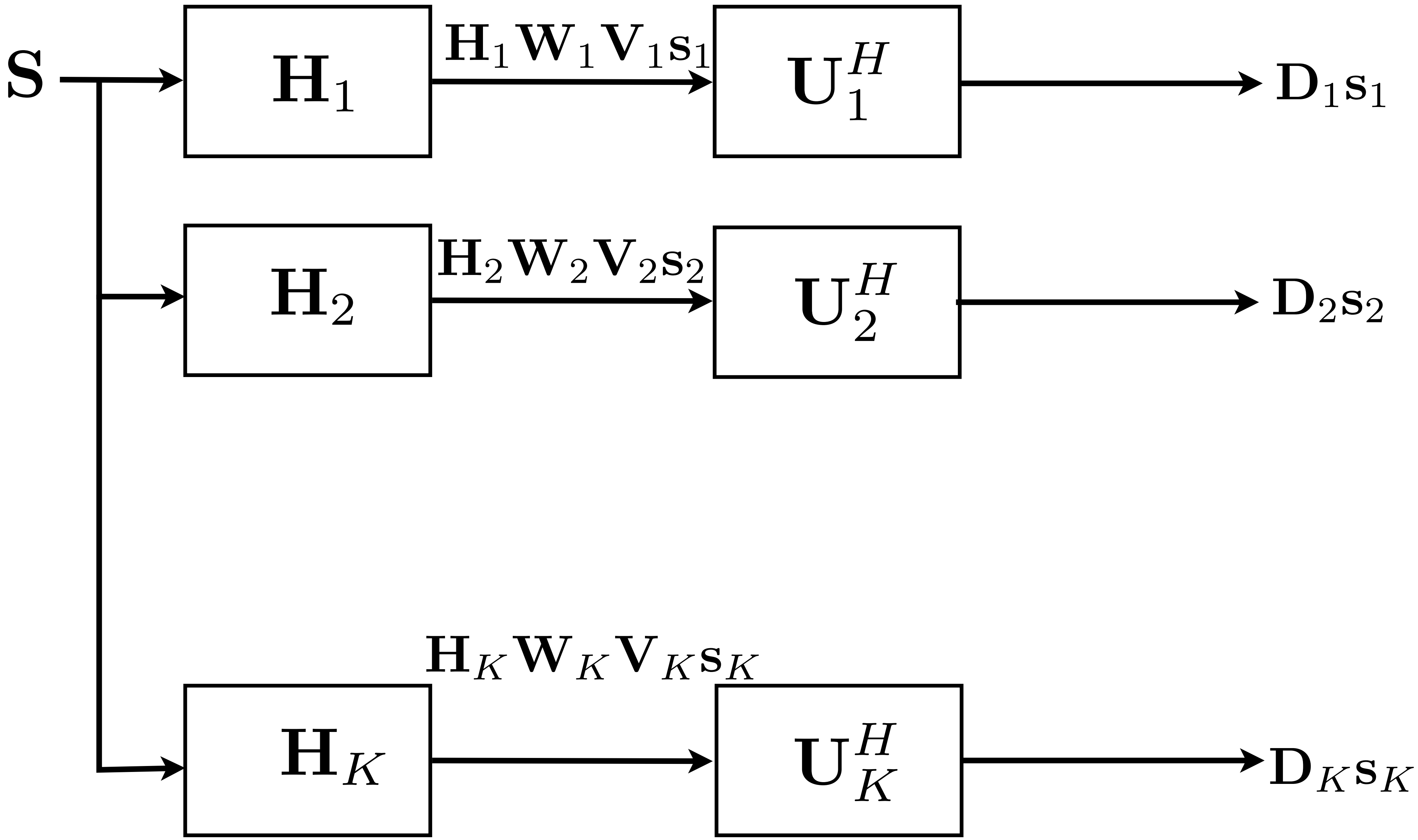
- Received signal vector and ISI cancellation

$$\mathbf{y}_m = \mathbf{H}_m \tilde{\mathbf{V}}_m^{(0)} \mathbf{s}_m + \mathbf{n}_m = \mathbf{U}_m \mathbf{D}_m \mathbf{V}_m^H \mathbf{s}_m + \mathbf{n}_m$$

Transmit signal



$$\mathbf{S} = \sum_{k=1}^{K} \mathbf{W}_k \mathbf{V}_k \mathbf{s}_k$$



$$\mathbf{H}_k\mathbf{W}_k\mathbf{V}_k\mathbf{s}_k = \mathbf{U}_k^H\mathbf{D}_k\mathbf{V}_k^H\mathbf{V}_k\mathbf{s}_k = \mathbf{U}_k^H\mathbf{D}_k\mathbf{s}_k$$

$$\mathbf{U}_K^H\mathbf{H}_k\mathbf{W}_k\mathbf{V}_k\mathbf{s}_k = \mathbf{U}_k^H\mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^H\mathbf{V}_k\mathbf{s}_k = \mathbf{D}_k\mathbf{s}_k$$

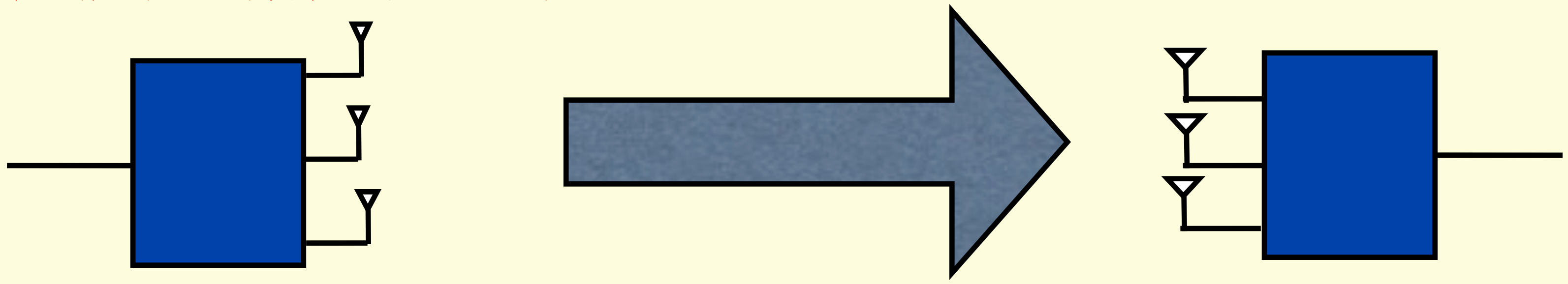
Sum-Rate of BD MU-MIMO

$$R_s = \max_{P_k} \sum_{k=1}^{K} \log_2 \det \left(\mathbf{I} + \frac{\mathbf{D}\mathbf{D}^H P_k}{\sigma^2} \right)$$

Optimal power loading P_k can be found by water-filling!

Closed-Loop and Open-Loop MIMO

Open-Loop MIMO

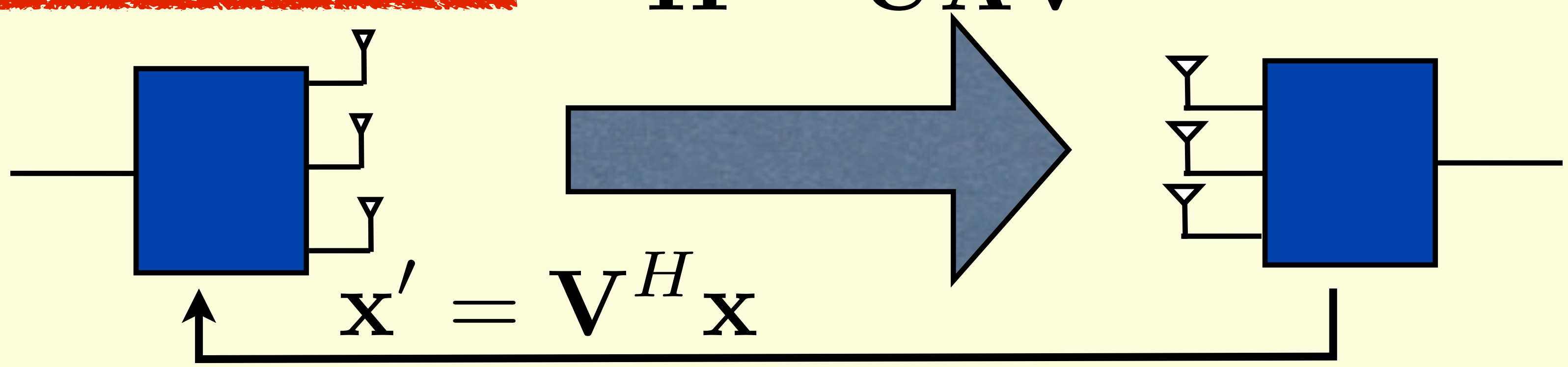


- ML detection
- QR detection
- MMSE detection

TX does not need to know the channel information for the transmission.

Closed-Loop MIMO

$$\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$$

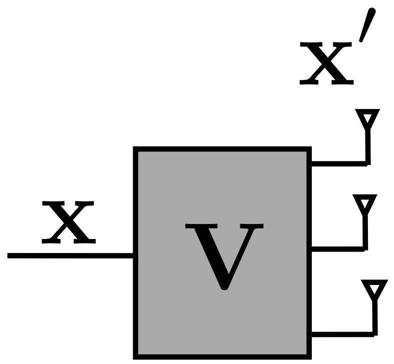


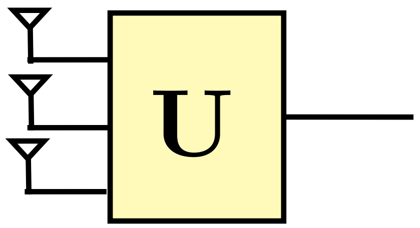
$$\mathbf{x}' = \mathbf{V}^H\mathbf{x}$$

- SVD (eigen-beamforming)

TX does needs to know the channel information for the transmission.

Limited Feedback in Closed-Loop MIMO Scheme





$\mathbf{V}$ is called the precoding matrix.

$\mathbf{V}$ can be obtained from $\mathbf{H}$.

Instead of obtaining the precoding matrix with floating point, we set up the precoding matrix with finite value and the transmitter sends the index of the precoding matrix.

BS sends the pilot signals.

↓

Each user estimates the channel, that is, $\mathbf{h}_k$ for user k .

↓

Each user quantizes its channel to the quantization vector that is closest to its channel.

MU-MIMO with a User with Single Antenna

- N transmit antennas at BS (or AP)
- K users (K receiver multiple-antenna broadcast channels)
- Each user has a single antenna ($M = 1$)

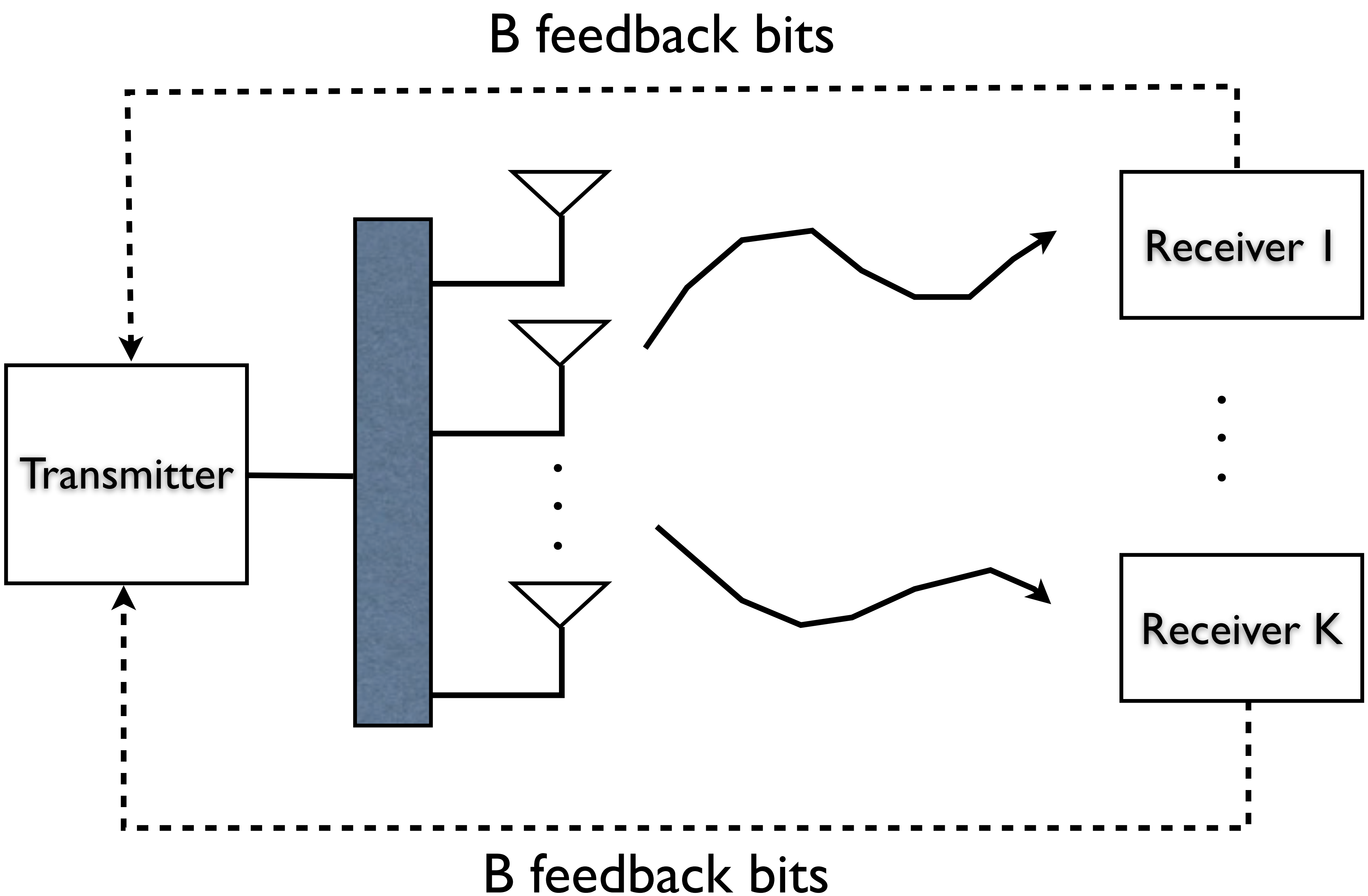
$$y_k = \mathbf{h}_k^T \mathbf{x} + n, \quad k = 1, ..., K$$

$$\begin{aligned} \mathbf{h}_k &= [h_{k1}\, h_{k2}\, \cdots\, h_{kN}]^T \\ \mathbf{x} &= [x_1\, x_2\, \cdots\, x_N]^T \end{aligned}$$

with power constraint $E[||\mathbf{x}||^2] \leq P$

Codebook vector

$$\mathcal{C} \triangleq \{\mathbf{w}_1, \cdots, \mathbf{w}_{2^B}\}$$



User k computes quantization index

$$F_k = \arg \max_{j=1,..,2^B} |\mathbf{h}_k^T \mathbf{w}_j|^2$$
$$= \arg \min_{j=1,2,\ldots,2^B} \sin^2 \left(\angle(\mathbf{h}_k^T, \mathbf{w}_j) \right)$$

Assume that user k selects $\mathbf{v}_k \in \mathcal{C}$

SINR at user k

$$\text{SINR}_k = \frac{\frac{P}{N} |\mathbf{h}_k^T \mathbf{v}_k|^2}{\sigma^2 + \sum_{j \neq k} \frac{P}{M} |\mathbf{h}_k^T \mathbf{v}_j|^2}$$

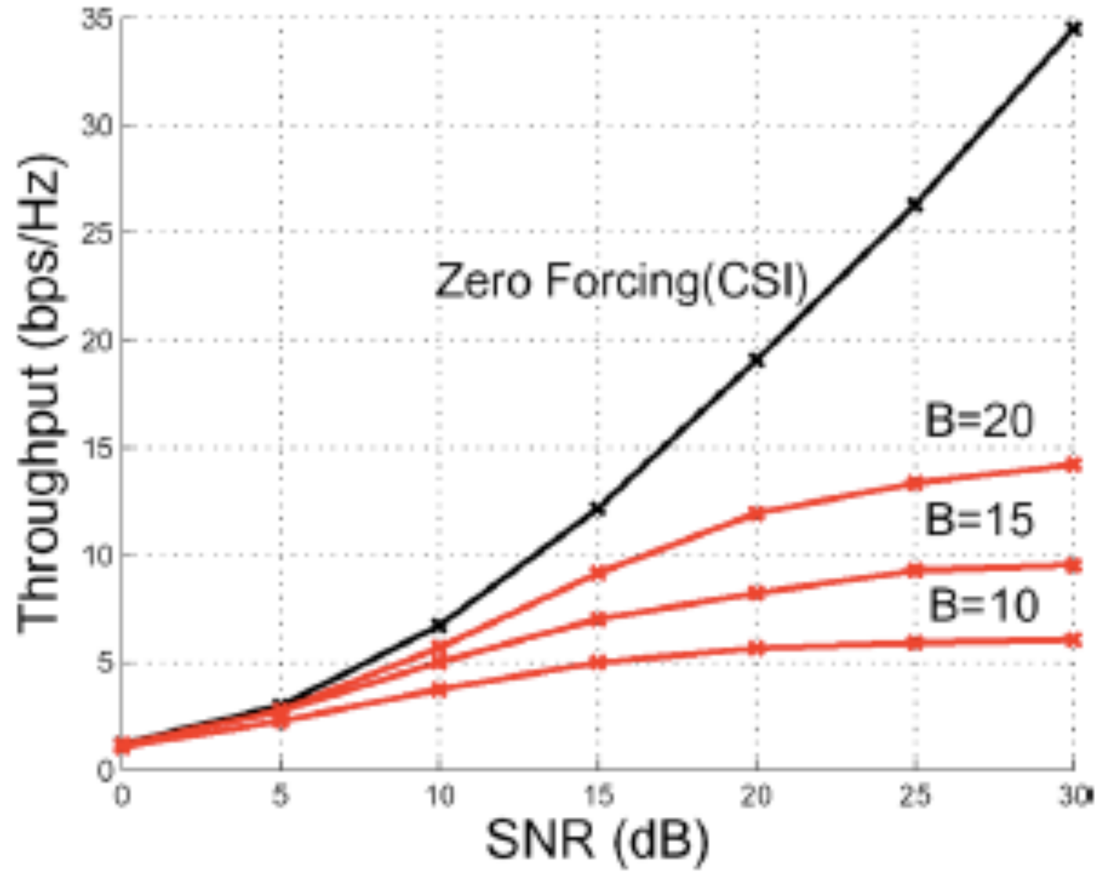


Fig. 3. 5 × 5 channel with fixed number of feedback bits.

$$B = \frac{M-1}{3} P_{dB} \text{ bits/user}$$

[N. Jindal, "MIMO broadcast channels with finite-rate feedback", IEEE Trans. on Inf. Theory, Vol. 52, No. 11, pp. 5045-5060, Nov. 2006]