

# Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8<sup>th</sup> and 9<sup>th</sup> Ed.
- The rest is made by me.

# Chap. 23 Electric potential



# Electric potential

Electric potential energy

$$\Delta U = U_f - U_i = -W$$

$$U = -W_\infty \quad (\text{infinity as a reference point})$$

Electric potential

$$V = \frac{U}{q}$$

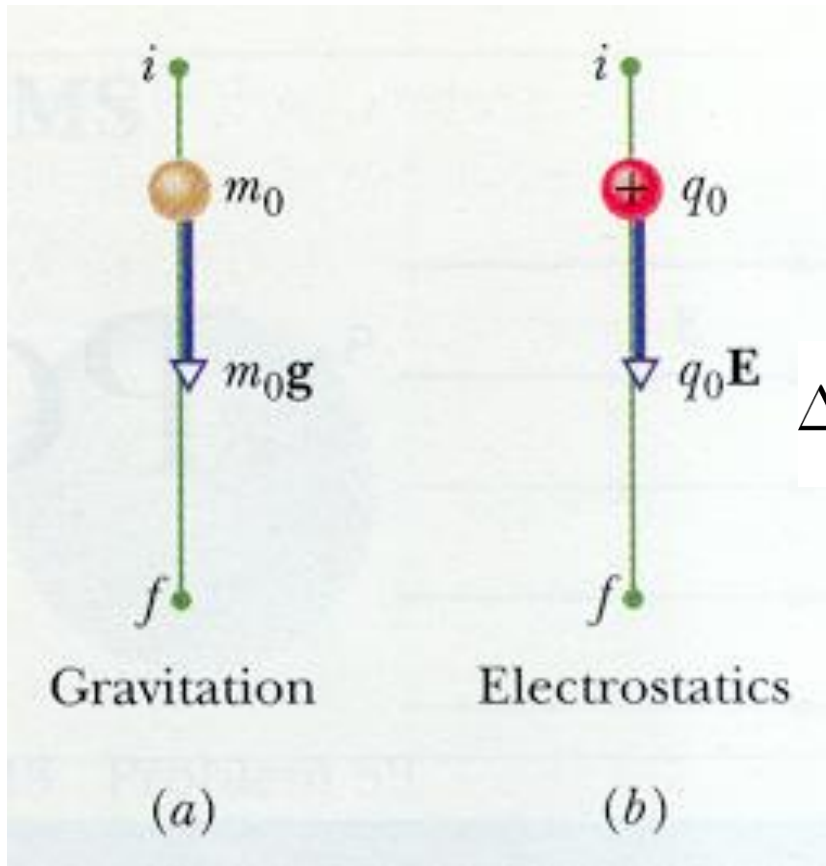
$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q}$$

$$V = -\frac{W_\infty}{q}$$

SI unit      1 volt = 1 V = 1 J/C

Electric field  $\frac{\text{N}}{\text{C}} = \frac{\text{N} \cdot \text{V} \cdot \text{C}}{\text{C} \cdot \text{J}} \frac{\text{J}}{\text{N} \cdot \text{m}} = \text{V/m}$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$



Both forces conservative

## Work done by an applied force

일과 에너지 정리에 의하면

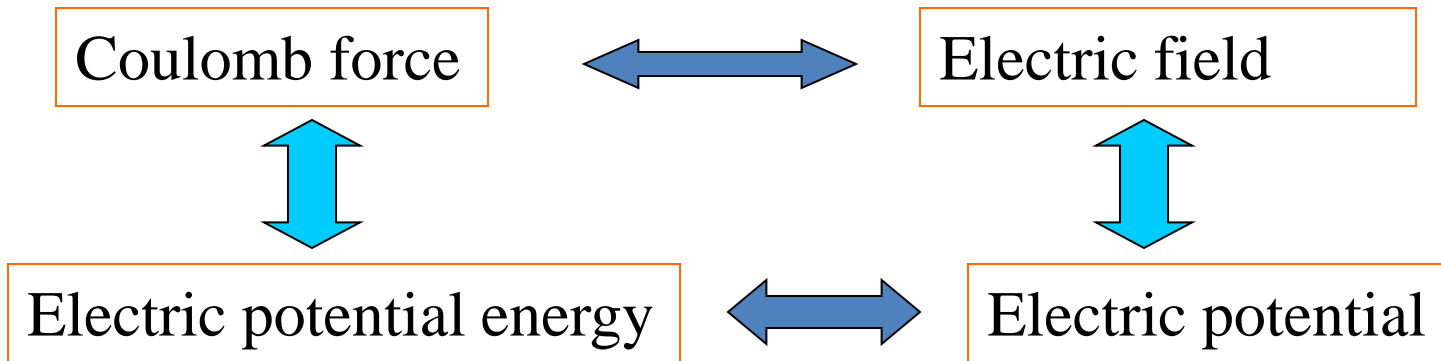
$$\Delta K = K_f - K_i = W_{\text{app}} + W$$

만일 운동 전후의 운동에너지가 같다면 ( $K_f = K_i$ )

$$W_{\text{app}} = -W$$

$$\Delta U = U_f - U_i = W_{\text{app}} = q\Delta V$$

# Electric potential energy and electric potential



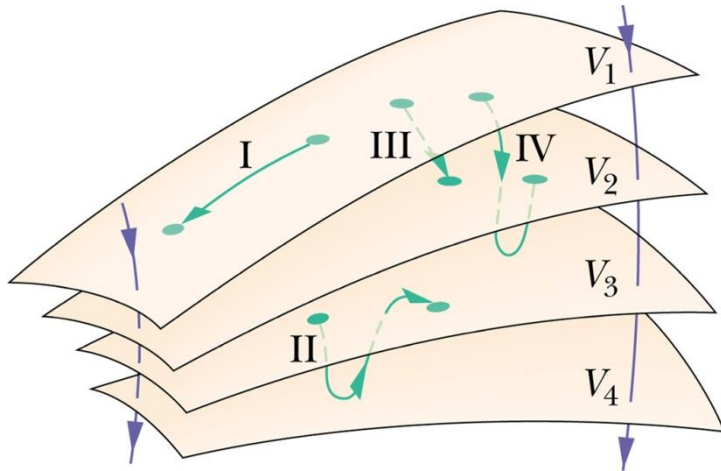
$$U_C (r) = - \int_{\infty}^r \mathbf{f}_C (\mathbf{r}) \cdot d \mathbf{s}$$

$$U_C (\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$V (r) = - \int_{\infty}^r \mathbf{E} (\mathbf{r}) \cdot d \mathbf{s}$$

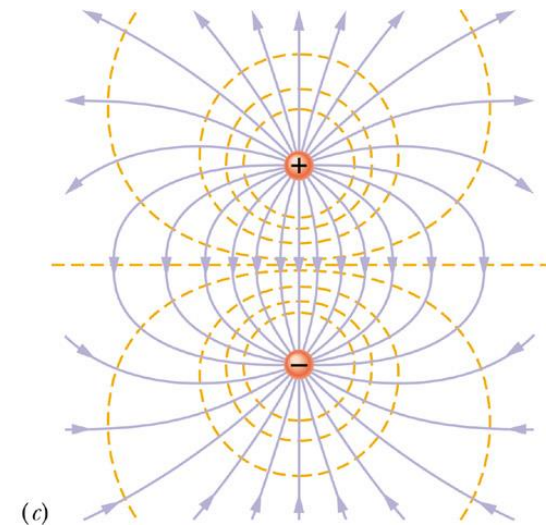
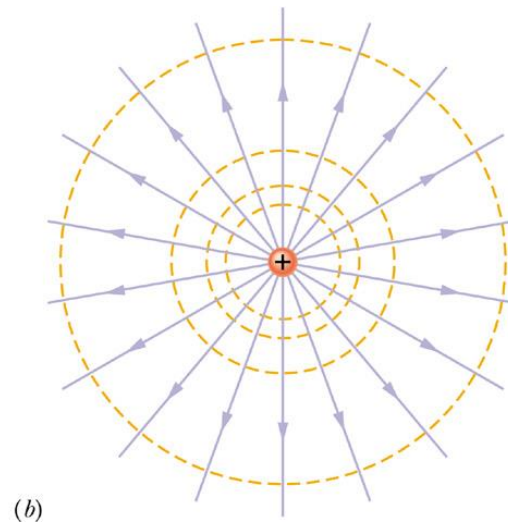
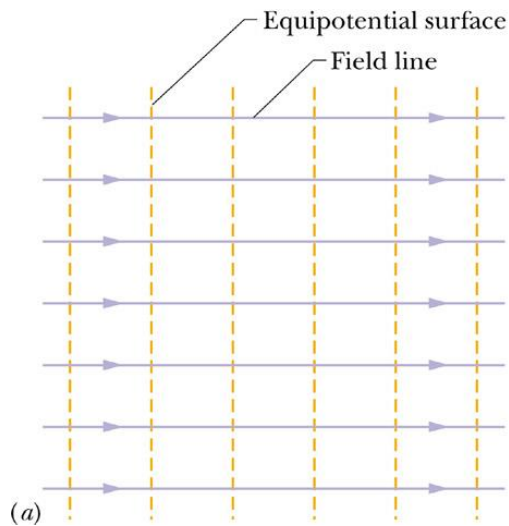
$$V (\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$$

# Equipotential surface

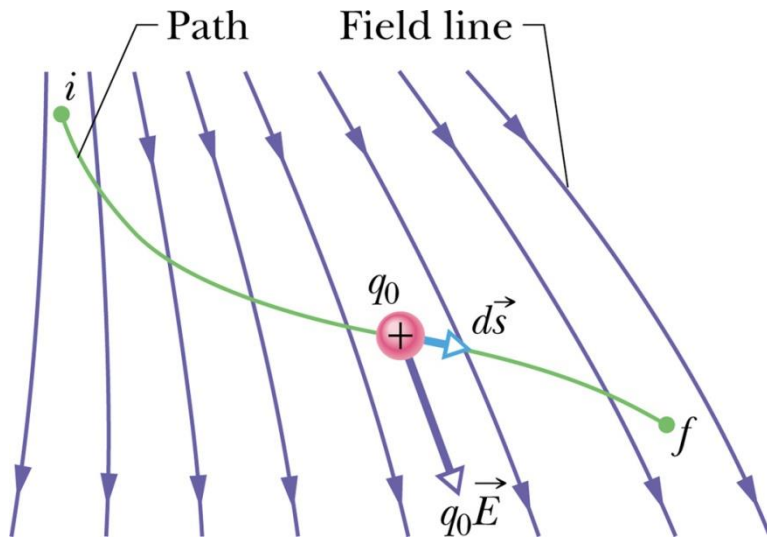


$$V_f - V_i = 0 \longrightarrow -\frac{W_{if}}{q} = 0$$

Equipotential surfaces are normal to the electric field.



# Electric potential from electric field

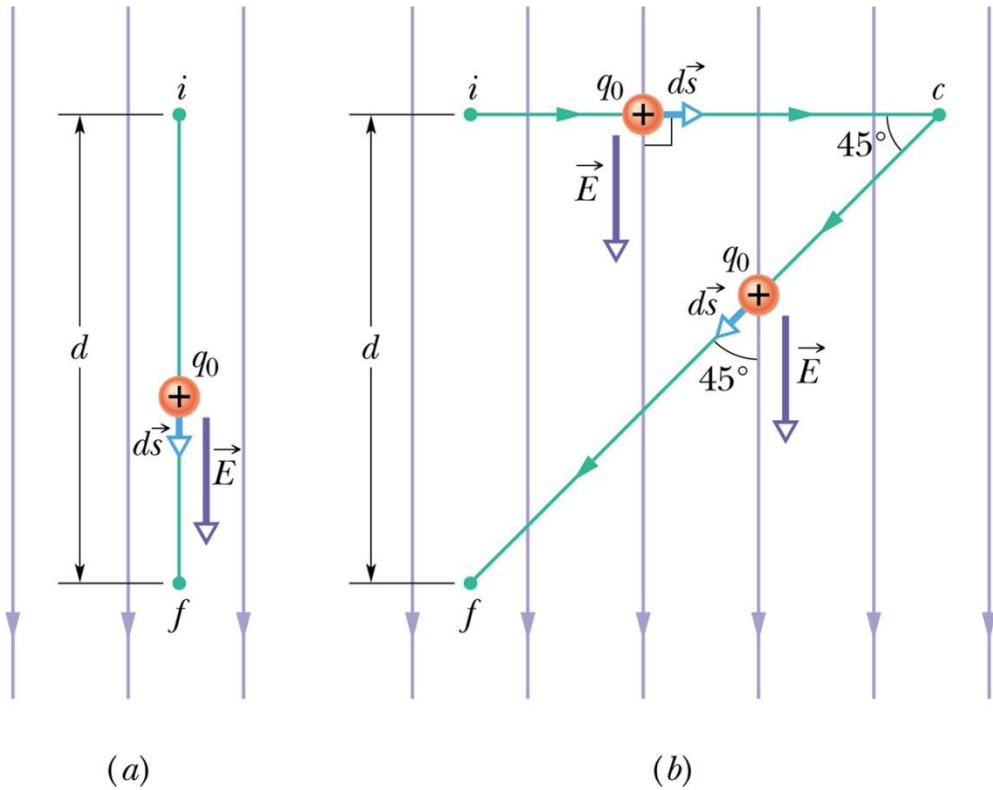


$$dW = \vec{F} \cdot d\vec{s} = q_0 \vec{E} \cdot d\vec{s}$$

$$W = -q_0 \int_i^f \vec{E} \cdot d\vec{s}$$

$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}$$

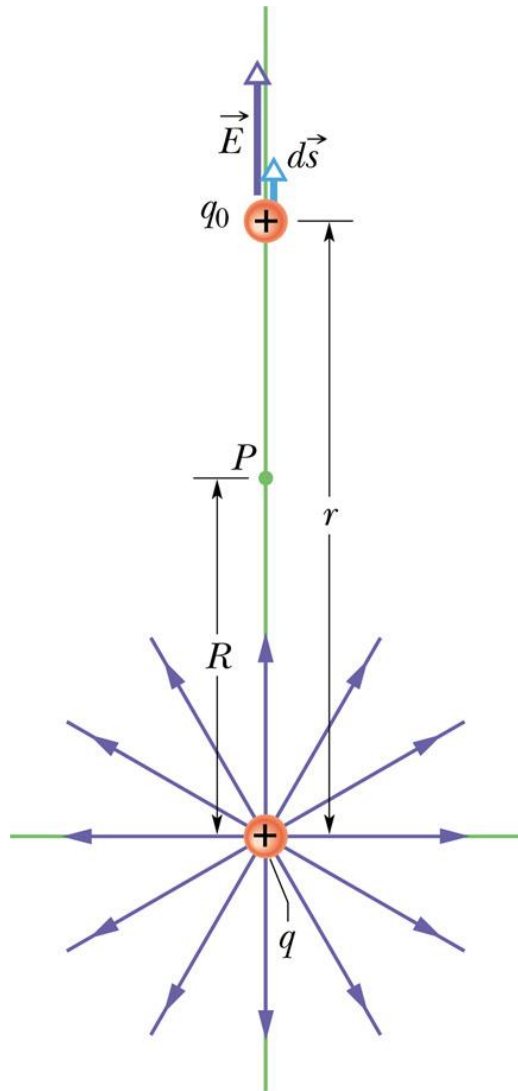
# Example



$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s} = - \int_i^f E ds = -Ed$$



# Electric potential by a point charge

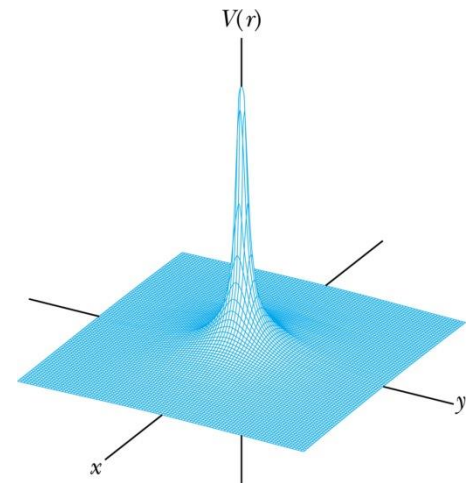


$$\vec{E} \cdot d\vec{s} = E \cos \theta ds = E dr$$

$$V_f - V_i = - \int_R^\infty E dr = - \int_R^\infty \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr$$

$$0 - V = - \frac{1}{4\pi\epsilon_0} \frac{1}{R}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$



Electric potentials from a point charge, electric dipole, line charge, surface charge, spherical charge (principle of superposition)

1) Point charge  $q$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

2) Many point charges

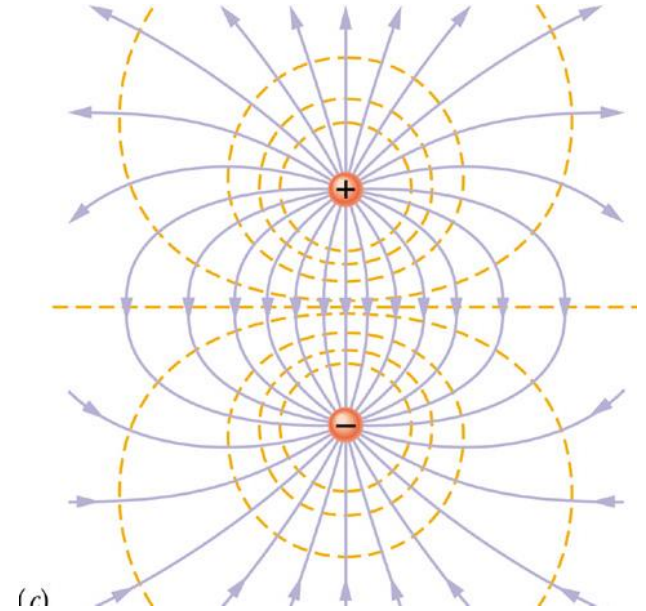
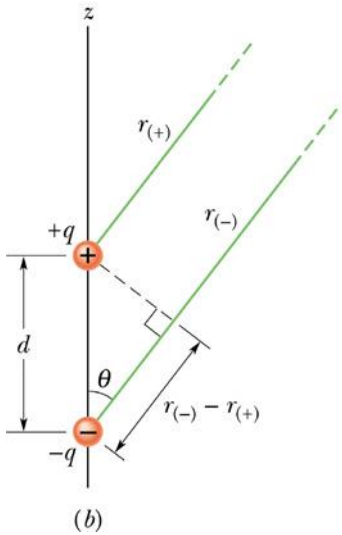
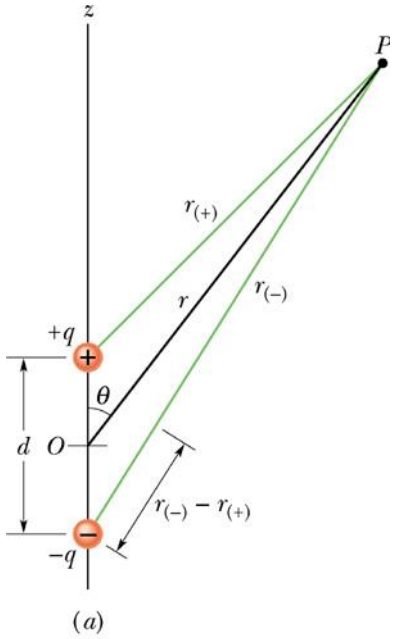
$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

3) Continuous charge distribution

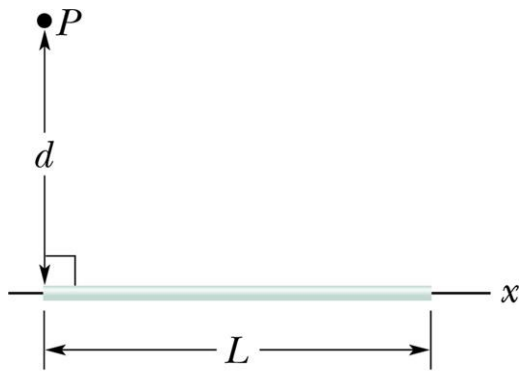
$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

# Electric dipole

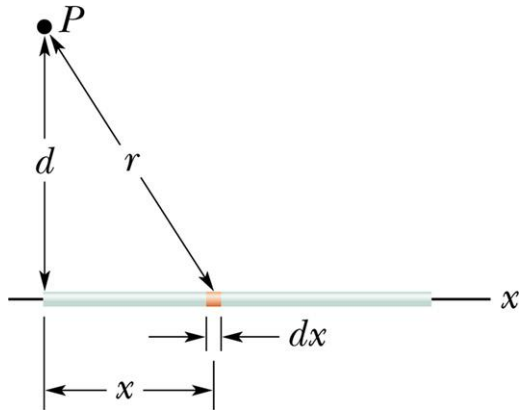
$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$



# Line charge



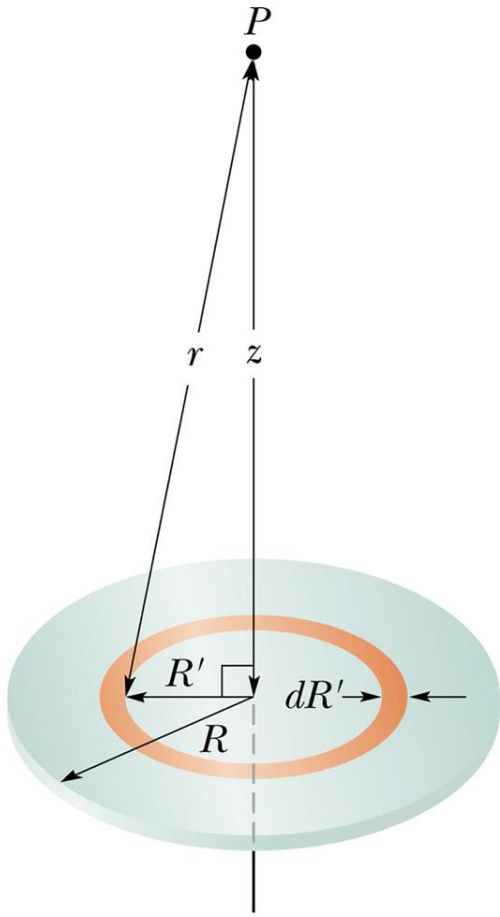
(a)



(b)

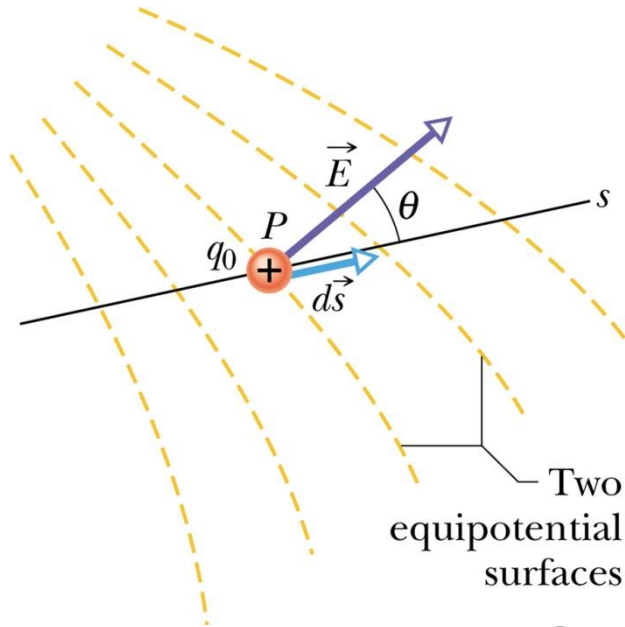
$$V = \frac{\lambda}{4\pi\epsilon_0} \ln \frac{L + \sqrt{L^2 + d^2}}{d}$$

# Surface charge



$$V = \frac{\sigma}{2\epsilon_0} \left( \sqrt{z^2 + R^2} - z \right)$$

# Electric field from electric potential



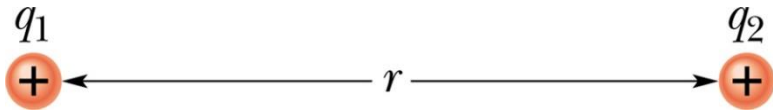
$$-q dV = q E \cos \theta ds \longrightarrow E \cos \theta = -\frac{dV}{ds}$$

$$E_s = -\frac{\partial V}{\partial s} \longrightarrow E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

일반적으로

$$\mathbf{E} = -\nabla V \equiv \left( -\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right)$$

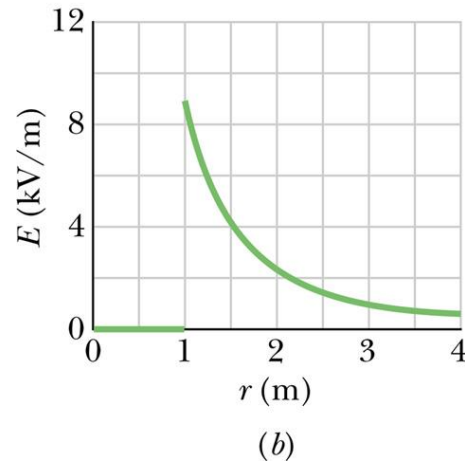
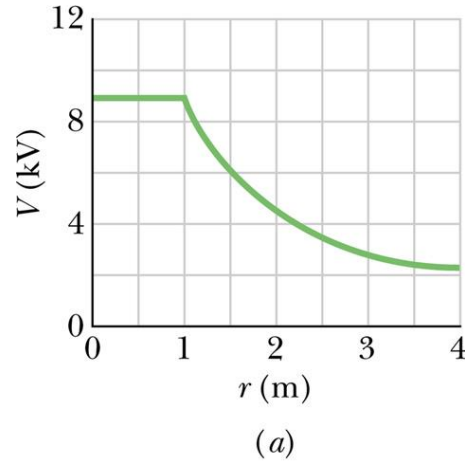
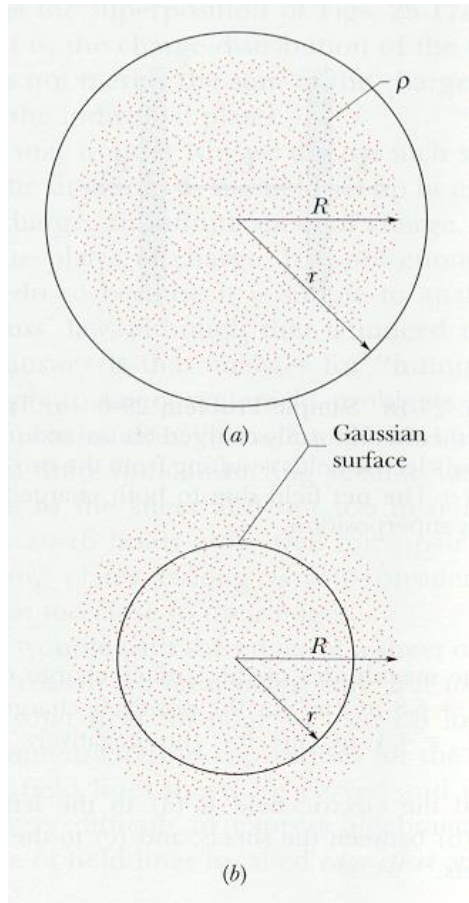
# Electric potential energy from point charges



$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r}$$

$$U = W = q_2 V = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

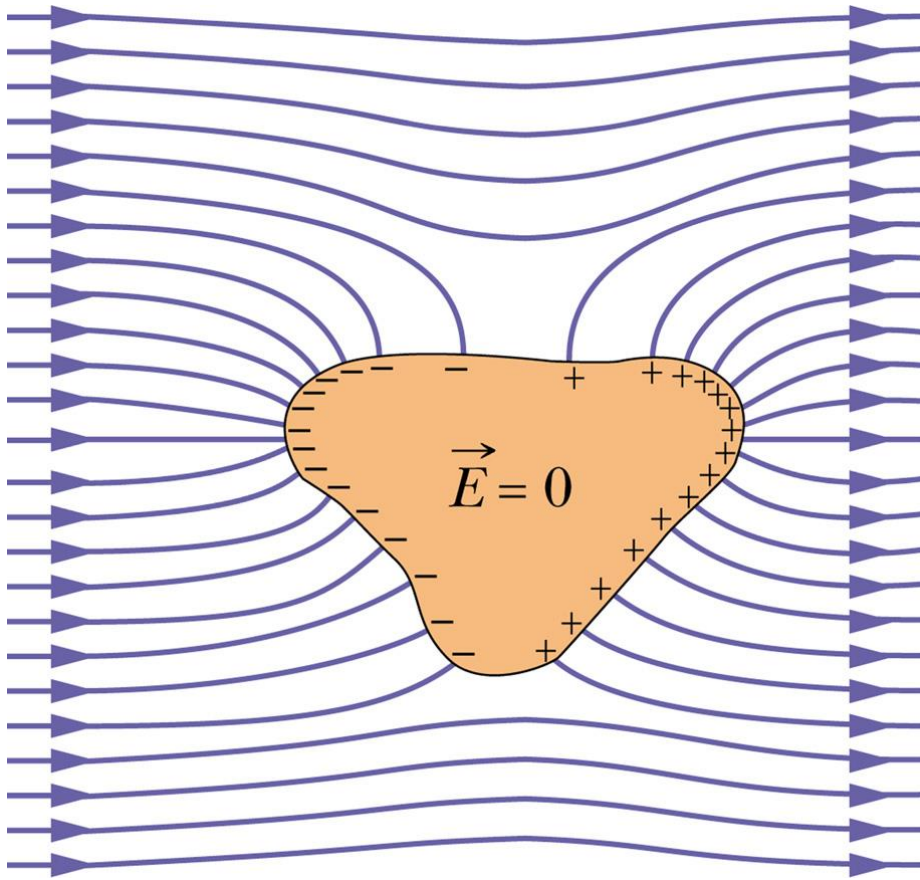
# Electric potential for an isolated charged conductor



$$V_f - V_i = - \int_i^f \vec{E} \cdot d\vec{s}$$



# Isolated conductor in an external E



Surface of a conductor is an equipotential surface, hence normal to the electric field.