KECE321 Communication Systems I

(Haykin Sec. 5.7 - Sec. 5.8)

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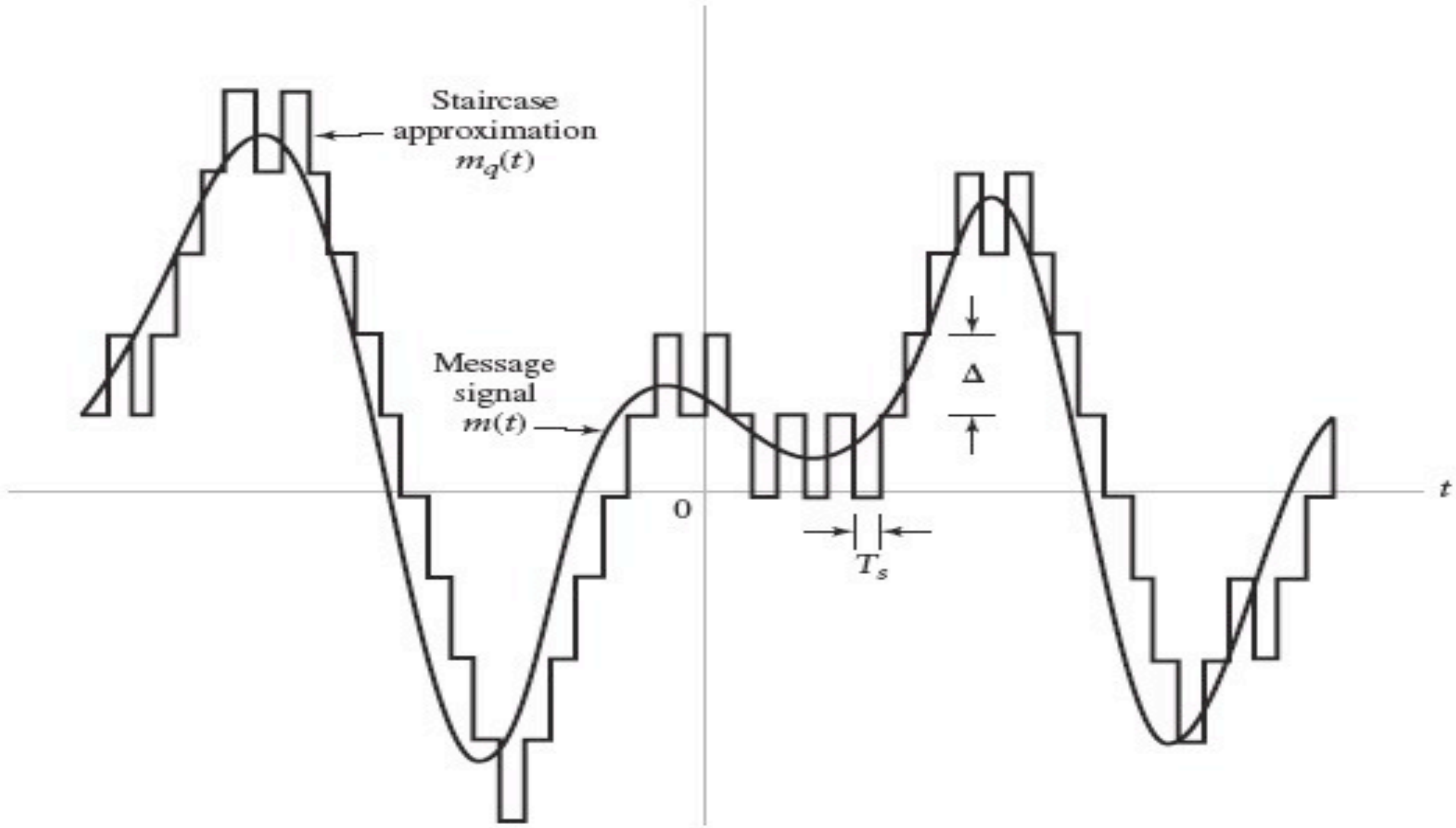
Delta Modulation

- An incoming message signal is oversampled to purposely increase the correlation between adjacent samples of the signal.
- The difference between the input signal and its approximation is quantized into only two levels - corresponding to positive and negative differences

$$
\begin{aligned}
e(nT_s) &= m(nT_s) - m_q(nT_s - T_s) \\
e_q(nT_s) &= \Delta \mathrm{sgn}[e(nT_s)] \\
m_q(nT_s) &= m_q(nT_s - T_s) + e_q(nT_s)
\end{aligned}
$$



(a)

Binary sequence at modulator output 10111101000000000111111010010101111010000000110111

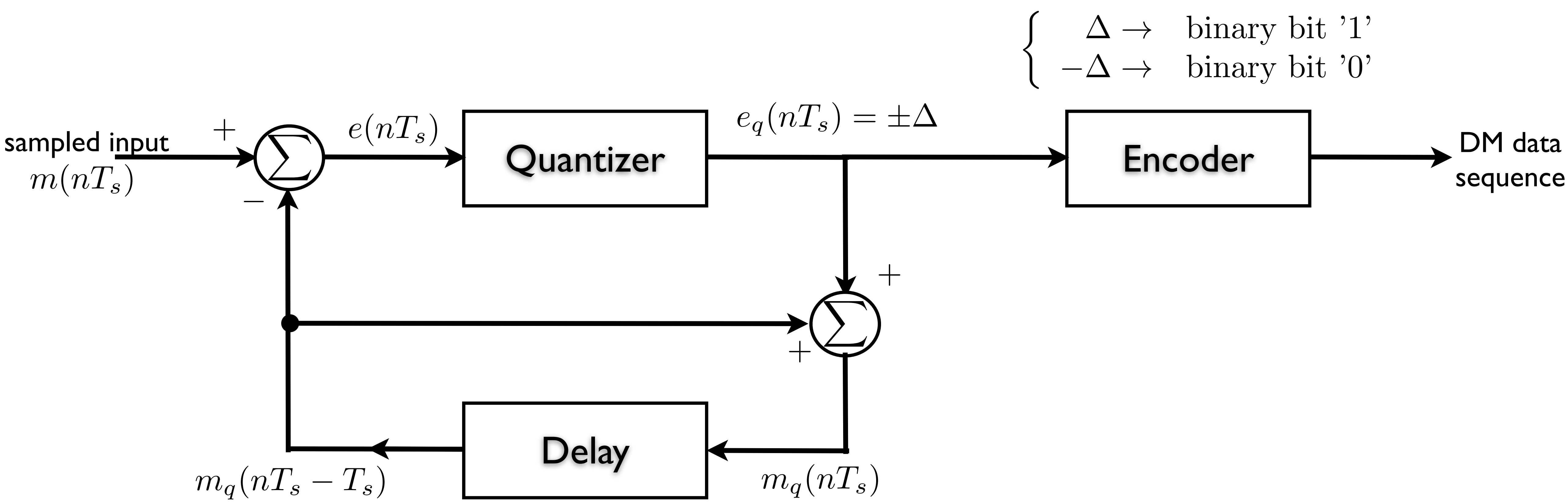
(b)

FIGURE 5.14 Illustration of delta modulation. (*a*) Analog waveform $m(t)$ and its staircase approximation $m_q(t)$. (*b*) Binary sequence at the modulator output.

[Ref: Haykin Textbook]

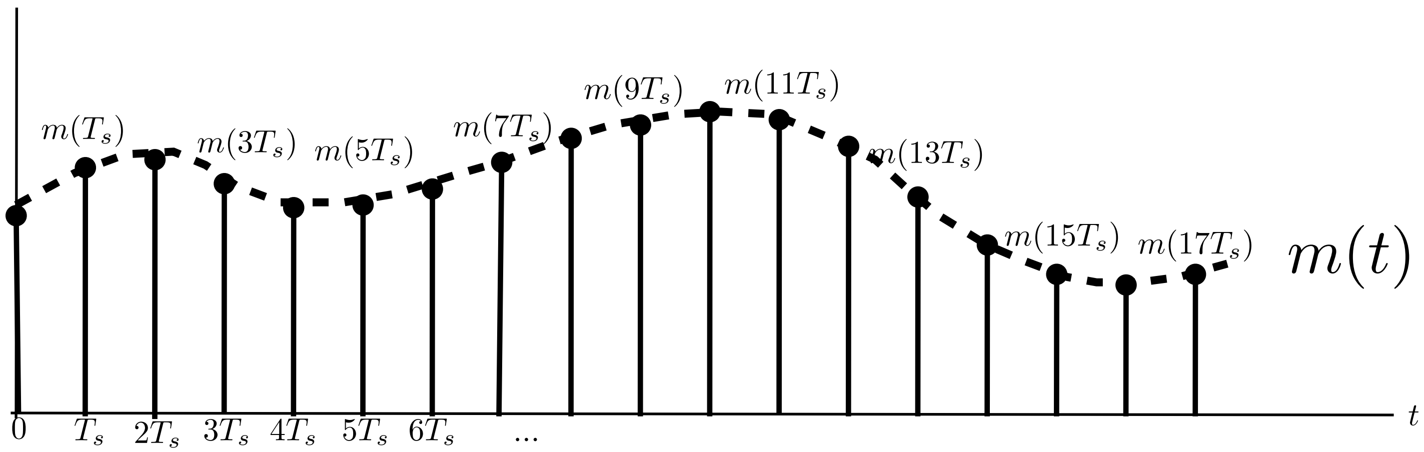
- System details
  - Comparator
    - Computes the difference between its two inputs
  - Quantizer
    - Consider of a hard limiter with an input-output characteristic that is a scaled version of the signum function
  - Accumulator
    - Operates on the quantizer output so as to produce an approximation to the message signal

$$
\begin{aligned}
m_q(nT_s) &= m_q(nT_s - T_s) + e_q(nT_s) \\
&= m_q(nT_s - 2T_s) + e_q(nT_s - T_s) + e_q(nT_s) \\
&\vdots \\
&= \sum_{i=1}^{n} e_q(iT_s)
\end{aligned}
$$



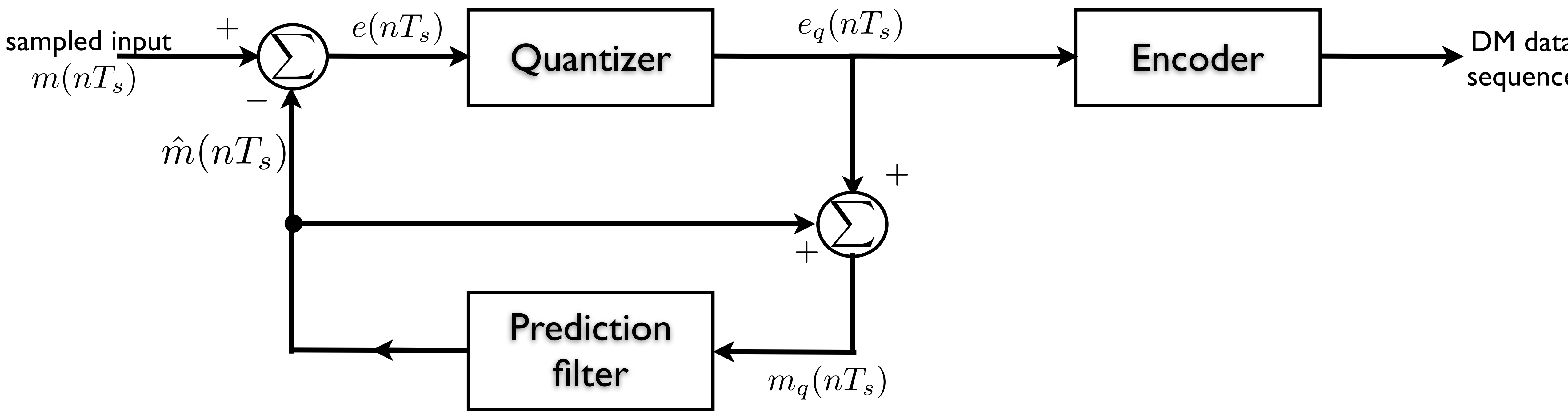
Differential Pulse-Code Modulation

- Assume the sampling rate is faster than the Nyquist rate $T_s < \frac{1}{2W}$ (or $f_s > 2W$)

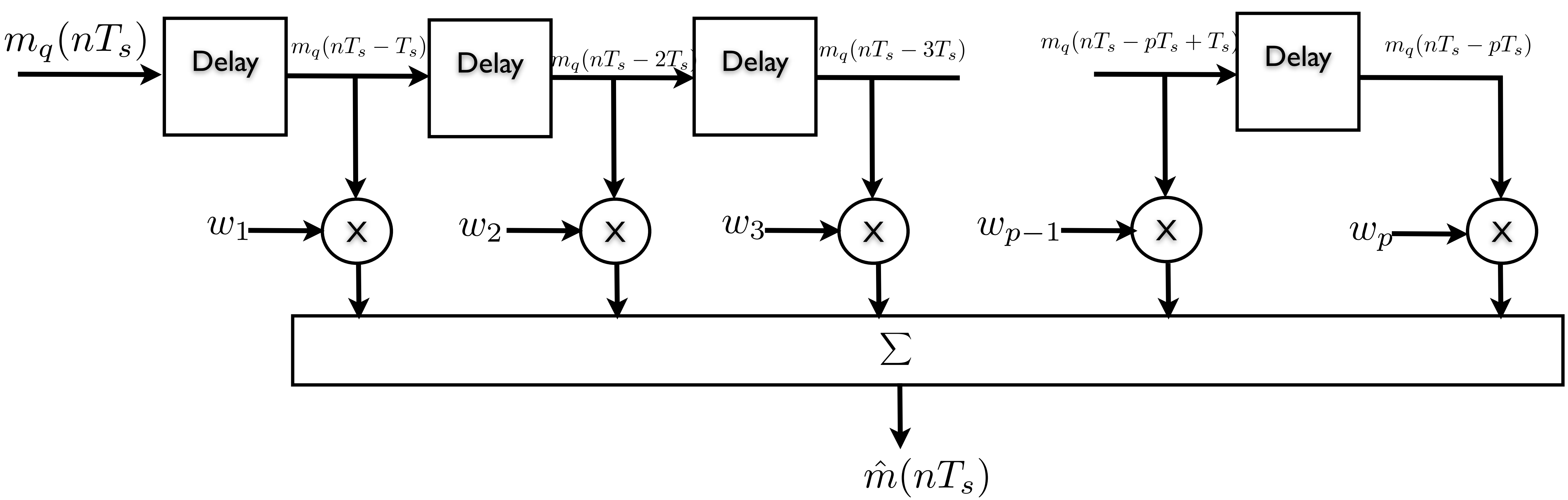


- Then sampled signals are highly correlated each other.
- Predict the future signal sample based on the previous sampled signals.
  - Differential pulse-code modulation

Transmitter of DPCM



- Prediction filter



- Input signal to the quantizer

$$e(nT_s) = m(nT_s) - \hat{m}(mT_s)$$

prediction error

predicted value

- Quantizer output

$$e_q(nT) = e(nT_s) + q(nT_s)$$

- Prediction filter input

$$\begin{aligned} m_q(nT_s) &= \hat{m}(nT_s) + e_q(nT_s) \\ &= \hat{m}(nT_s) + e(nT_s) + q(nT_s) \end{aligned}$$

$$\longrightarrow \quad m_q(nT_s) = m(nT_s) + q(nT_s)$$

which represents a quantized version of the message sample $m(nT_s)$. That is, irrespective of the properties of the prediction filter, the quantized signal $m_q(nT_s)$ at the prediction filter input differs from the sampled message signal $m(nT_s)$ by the quantization error $q(nT_s)$.

- Accordingly, if the prediction is good, the average power of the prediction error $e(nT_s)$ will be smaller than the average power of $m(nT_s)$, so that a quantizer with a given number of levels can be adjusted to produce a quantization error with a smaller average power than would be possible if $m(nT_s)$ were quantized directly using PCM.

  - Note the following:

$$E[(X_1 - X_2)^2] = E[X_1^2] + E[X_2^2] - 2E[X_1]E[X_2] \leq E[X_1^2] + E[X_2^2]$$

Receiver of DPCM