Mobile Communications (KECE425)

Lecture Note 18

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Integration Table

- Following integration result is useful in obtaining the BER of diversity systems over fading channels.

$$
\begin{aligned}
I_L(\bar{\gamma}) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{\sin^2\phi + \bar{\gamma}}\right)^L d\phi \\
&= \left[\frac{1-\mu}{2}\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1+\mu}{2}\right]^k,
\end{aligned}
$$

where $\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$

- Moment generating function (MGF) of RV γ

$$\mathcal{M}_\gamma(s) = \int_0^\infty e^{s\gamma} p_\gamma(\gamma)\, d\gamma$$

where $p_\gamma(\gamma)$ is the PDF of γ and $\gamma \geq 0$.

- MGF for exponential PDF

$$\mathcal{M}_\gamma(s) = \frac{1}{\bar{\gamma}} \int_0^\infty e^{s\gamma} e^{-\gamma/\bar{\gamma}}\, d\gamma = (1 - s\bar{\gamma})^{-1}$$

- BER of BPSK over Rayleigh channel

$$P_b(e) = \int_0^\infty Q(\sqrt{2\gamma}) p_\gamma(\gamma)\, d\gamma = \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{\gamma}{\sin^2\theta}\right) p_\gamma(\gamma)\, d\theta\, d\gamma$$

$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^\infty \exp\left(-\frac{\gamma}{\sin^2\theta}\right) p_\gamma(\gamma)\, d\gamma d\theta$$

$$= \mathcal{M}_\gamma\left(-\frac{1}{\sin^2\theta}\right)$$

$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \mathcal{M}_\gamma\left(-\frac{1}{\sin^2\theta}\right) d\theta$$

$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}}{\sin^2\theta}\right)^{-1} d\theta \quad \Longleftarrow \mathcal{M}_\gamma(s) = (1 - s\bar{\gamma})^{-1}$$

for Rayleigh channel

$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{\sin^2\theta + \bar{\gamma}}\right) d\theta = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

- Hence we can set up the following integration table

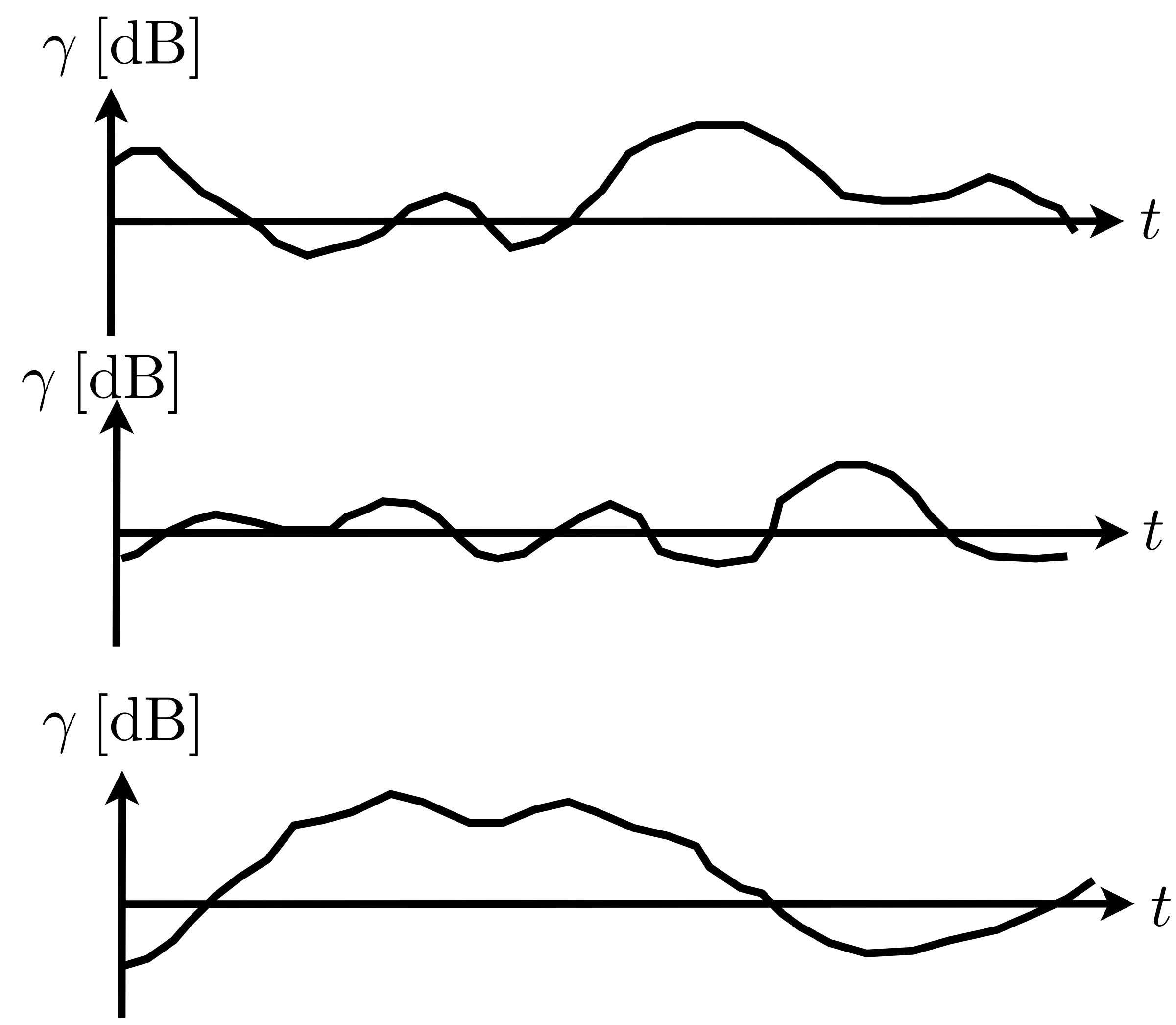
$$P_b(e) \quad = \quad \int_0^{\infty} Q(\sqrt{2\gamma}) p_\gamma(\gamma)\, d\gamma = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

Part III. Diversity Techniques

- Receive diversity
- Transmit diversity
- Transmit-Receive diversity

Concept and Intuition of Diversity Systems

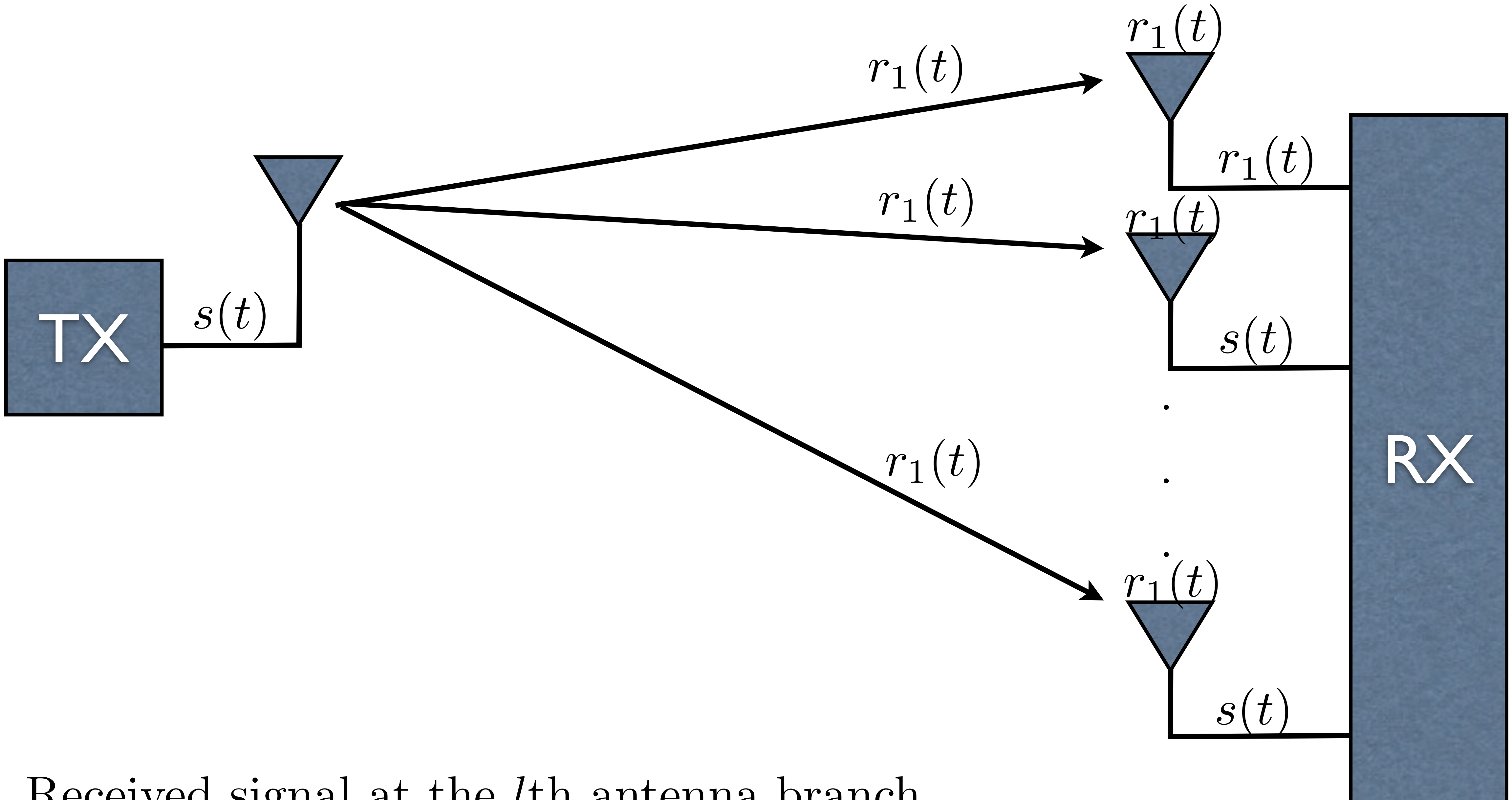
- Concept
  - Receiving redundantly the same information bearing signals over two or more fading channels
- Intuition
  - Take advantage of low probability of occurrence of deep fades in all diversity branches



Receiver Diversity

- Maximal ratio combining
- Equal gain combining
- Selection combining
- Switched combining

Channel Model in Receiver Antenna Diversity Systems



- Received signal at the lth antenna branch

$$r_l(t) = s(t)h(t) + n(t)$$

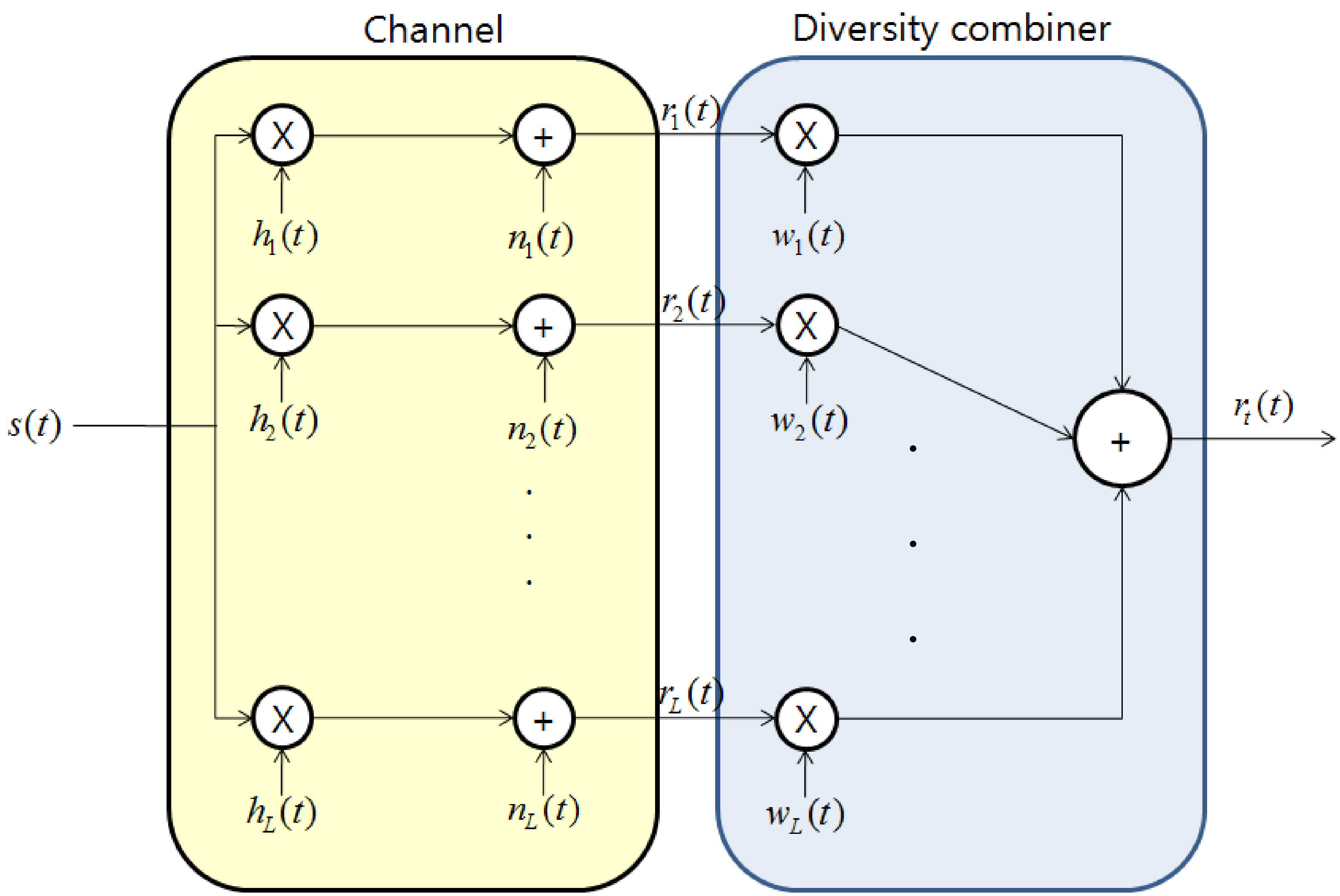
Linear Diversity Combining

- Linear diversity combining:

$$
\begin{aligned}
r_t(t) &= \sum_{l=1}^{L} w_l(t) r_l(t) \\
&= \sum_{l=1}^{L} w_l(t) h_l(t) s(t) + \sum_{l=1}^{L} w_l(t) n_l(t) \\
&= \sum_{l=1}^{L} w_l(t) h_l(t) s(t) + \sum_{l=1}^{L} w_l(t) n_l(t)
\end{aligned}
$$

Linear Receiver Antenna Diversity Combining

- Block diagram



Combined received signal

$$
\begin{aligned}
r_t(t) &= \sum_{l=1}^{L} w_l(t) r_l(t) \\
&= \sum_{l=1}^{L} w_l(t)(h_l(t)s(t) + n_l(t))
\end{aligned}
$$

Maximal Ratio Combining

- MRC maximizes the combined signal-to-noise ratio.
  - We want to find the optimum weight vector to maximize the SNR of $r_t(t)$ the combined signal.

- Combined signal

$$r_t(t) \quad = \quad \sum_{l=1}^{L} w_l r_l(t) = \sum_{l=1}^{L} w_l h_l(t) s(t) + \sum_{l=1}^{L} w_l n_l((t)$$

  - SNR of the combined signal

$$\gamma_t = \frac{|\sum_{l=1}^{L} w_l h_l|^2 E_s}{\sum_{l=1}^{L} |w_l|^2 N_0} = \frac{E_s}{N_0} \frac{|\sum_{l=1}^{L} w_l h_l|^2}{\sum_{l=1}^{L} |w_l|^2}$$

  - Cauchy-Schwarz inequality:

$$\left|\sum_{l=1}^{L} w_l h_l\right|^2 \leq \left|\sum_{l=1}^{L} w_l\right|^2 \left|\sum_{l=1}^{L} h_l\right|^2$$

Equality hold iff $w_l = c h_l^*(t)$ with an arbitrary constant value of c.

- Maximal ratio combining

$$\gamma_t \leq \frac{E_s}{N_0}\sum_{l=1}^{L}|h_l|^2 = \frac{E_s}{N_0}\sum_{l=1}^{L}\alpha_l^2 = \sum_{l=1}^{L}\gamma_l$$

where $\gamma_l = \dfrac{\Omega_l E_s}{N_0}$, that is, SNR at each branch.

- Maximal ratio combining

$$\begin{aligned} \gamma_t &= \sum_{l=1}^{L}\gamma_l \\ w_l &= c h_l^*(t) \text{ for an arbitrary constant value of } c \end{aligned}$$

Received Output SNR of MRC

- Received output SNR of MRC

$$\gamma_t = \sum_{l=1}^{L} \gamma_l$$

  - Average received output SNR

$$\bar{\gamma}_t = \sum_{l=1}^{L} \bar{\gamma}_l$$

    - If $\Omega_l = \Omega$ for $l = 1, \ldots, L$ (identical channels) and hence , $\bar{\gamma}_l = \bar{\gamma}$

$$\bar{\gamma}_t = L\bar{\gamma}$$

Average BER/SER of MRC

- Average BER/SER

$$P(e) = \int_0^{\infty} P(e|\gamma_t) p_{\gamma_t}(\gamma_t) d\gamma_t$$

  - For example for BPSK,

$$P(e|\gamma_t) = Q\left(\sqrt{2\gamma_t}\right) = Q\left(\sqrt{2\sum_{l=1}^{L}\gamma_l}\right)$$

  - or it can be written as

$$P(e) = \underbrace{\int_0^{\infty} \cdots \int_0^{\infty}}_{L\text{-fold}} P(e|\gamma_1, \cdots, \gamma_L) p_{\gamma_1, \cdots, \gamma_L}(\gamma_1, \cdots, \gamma_L)\, d\gamma_1 \cdots d\gamma_L$$

  - For independent channels,

$$p_{\gamma_1, \cdots, \gamma_L}(\gamma_1, \cdots, \gamma_L) = \prod_{l=1}^{L} p_{\gamma_l}(\gamma_l)$$

- Average BER of BPSK using MRC

$$
\begin{aligned}
Q(\sqrt{2\gamma_t}) &= Q\left(\sqrt{2\sum_{l=1}^{L}\gamma_l}\right) \\
&= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left[-\frac{\sum_{l=1}^{L}\gamma_l}{\sin^2\phi}\right] d\phi \\
&= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{l=1}^{L} \exp\left[-\frac{\gamma_l}{\sin^2\phi}\right] d\phi
\end{aligned}
$$

- Average BER of BPSK using MRC

$$
\begin{aligned}
P(e) &= \int_0^{\infty}\cdots\int_0^{\infty}\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{l=1}^{L}\exp\left[-\frac{\gamma_l}{\sin^2\phi}\right]\prod_{l=1}^{L} p_{\gamma_l}(\gamma_l) d\gamma_1\cdots d\gamma_L \\
&= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left\{\int_0^{\infty} e^{-\frac{\gamma_1}{\sin^2\phi}} p_{\gamma_1}(\gamma_1) d\gamma_1 \cdots \int_0^{\infty} e^{-\frac{\gamma_L}{\sin^2\phi}} p_{\gamma_L}(\gamma_L)\, d\gamma_L\right\} \\
&= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{l=1}^{L} M_{\gamma_l}\left(-\frac{1}{\sin^2\phi}\right) d\phi
\end{aligned}
$$

- For i.i.d. Rayleigh channels, MGF of SNR at each brand can be written as

$$M_{\gamma_l}(s) = (1 - \bar{\gamma}s)^{-1}$$

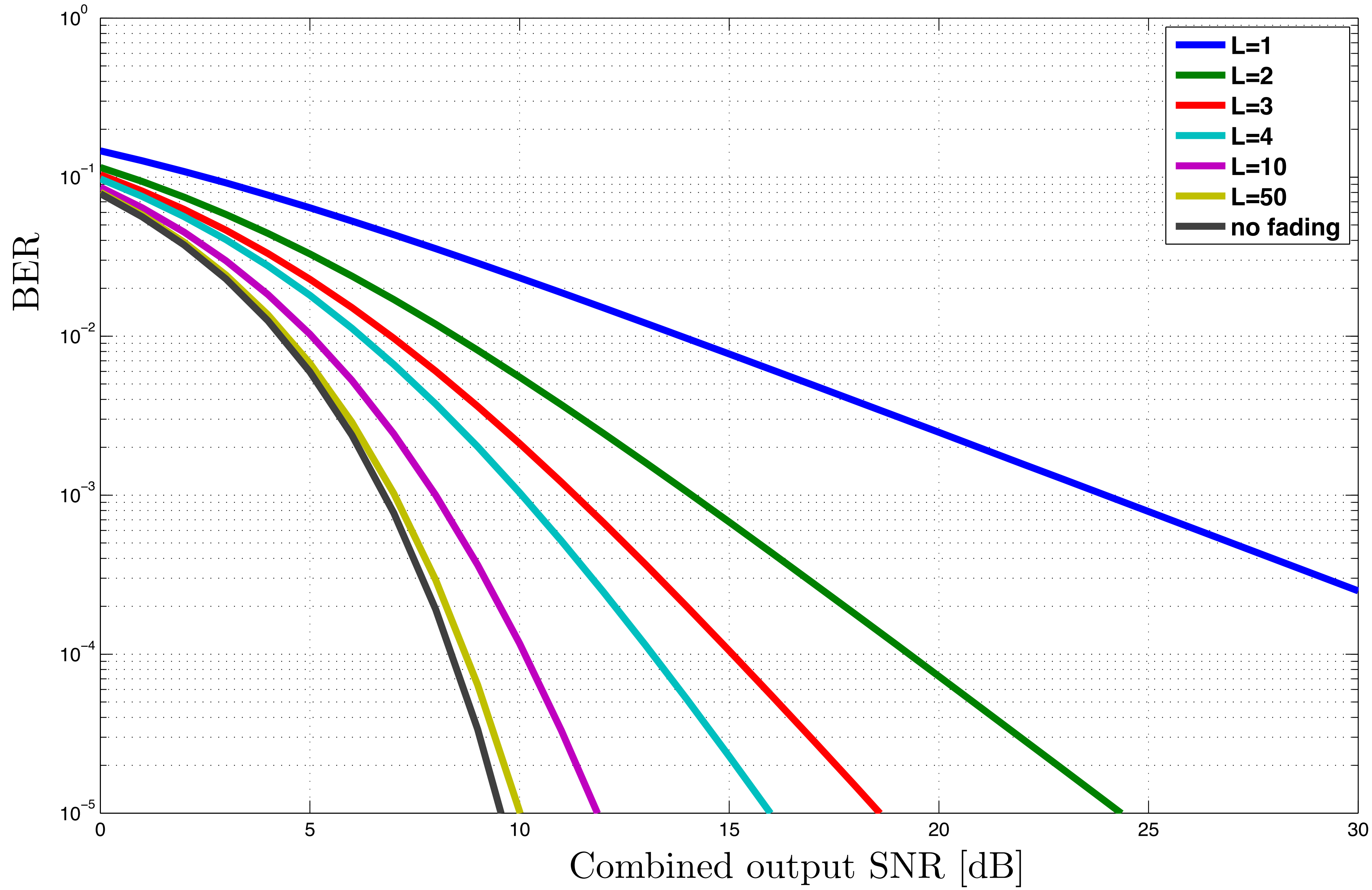
  - Then BER of BPSK using MRC can be derived in closed-form as

$$\begin{aligned} P(e) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}}{\sin^2\phi}\right)^{-L} d\phi \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{\sin^2\phi + \bar{\gamma}}\right)^{L} d\phi \\ &= I_L(\bar{\gamma}) \\ &= \left[\frac{1-\mu}{2}\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1+\mu}{2}\right]^k, \end{aligned}$$

where $\mu = \sqrt{\dfrac{\bar{\gamma}}{1+\bar{\gamma}}}$

BER of BPSK over Rayleigh Channels using MRC

- Average BER of BPSK using MRC versus combined output average SNR over Rayleigh fading channels



Equal Gain Combining

- Let us rewrite the channel gain as

$$h_l(t) = \alpha_l(t)e^{j\theta_l(t)}$$

- Then, EGC compensates only the phase distortion of the channel so that the weight is given as

$$w_l = e^{-j\theta_l(t)}$$

- The combined signal is

$$r_c(t) = [r_1(t) \;\; r_2(t) \;\; \cdots r_L(t)] \cdot [w_1 \;\; w_2 \;\; w_L]$$

$$= s(t)\sum_{l=1}^{L}\alpha_l + \sum_{n=1}^{L} e^{-j\theta_l(t)} n_l(t)$$

- Combined output SNR of EGC

$$\gamma_{\mathrm{EGC}} = \frac{\left(\sum_{l=1}^{L} \alpha_l\right)^2 E_s}{\sum_{l=1}^{L} N_l}$$

N_l : AWGN power spectral density on the lth path

E_s : Energy (in joules) per symbol

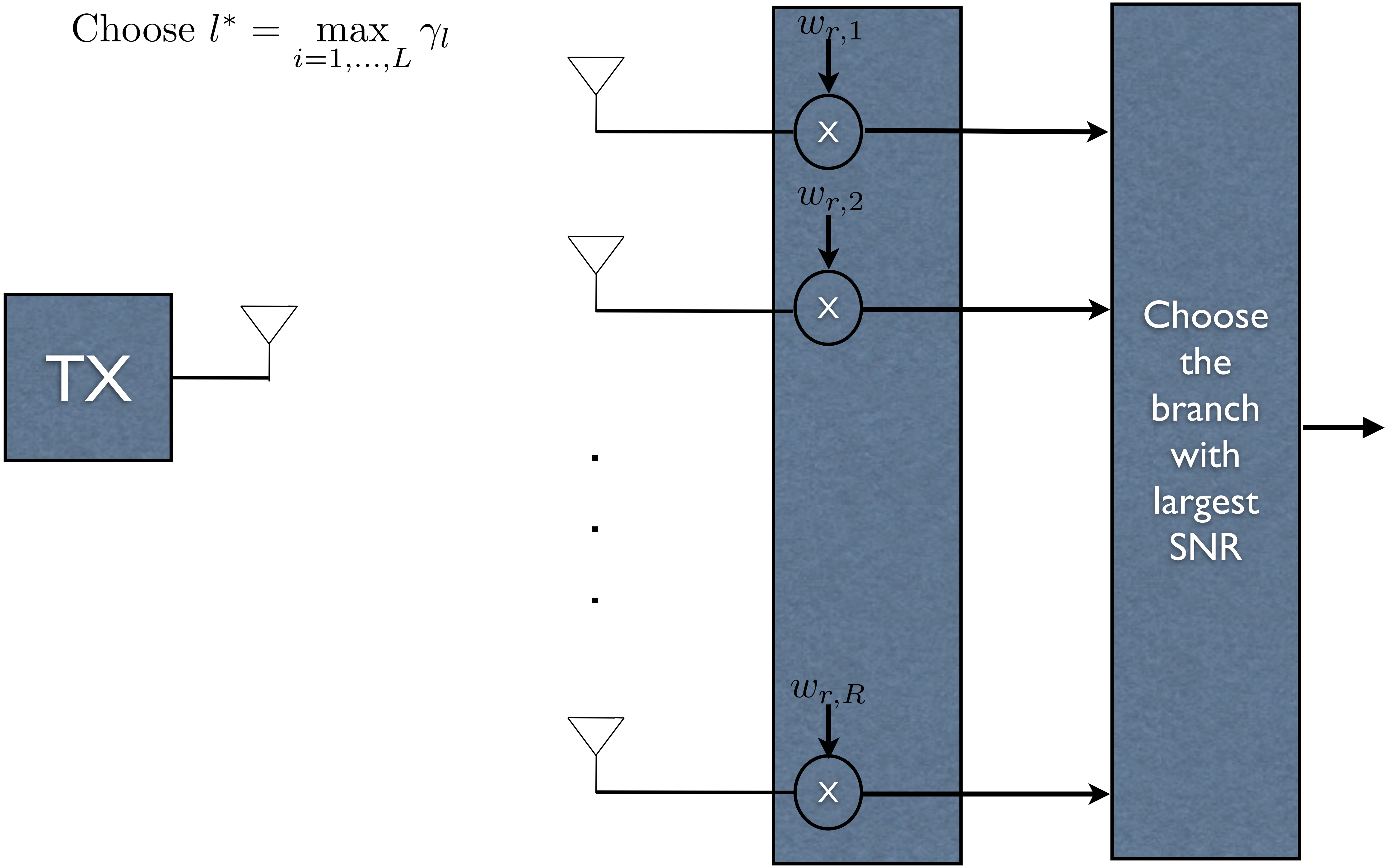
- For i.i.d. Rayleigh case, the average output SNR can be shown as

$$\bar{\gamma}_{\mathrm{EGC}} = \bar{\gamma}\left[1 + (L-1)\frac{\pi}{4}\right]$$

Selection Combining

- Selection combining chooses the branch with the largest SNR

$$\text{Choose } l^* = \max_{i=1,\ldots,L} \gamma_l$$



Combined output SNR

$$\gamma_{\mathrm{sc}} = \max\{\gamma_1, \gamma_2, \cdots, \gamma_L\}$$

CDF

$$P_{\gamma_{\mathrm{sc}}}(\gamma) = \Pr[\gamma_1 \leq \gamma,\ \gamma_2 \leq x, \cdots, \gamma_L \leq x]$$

$$= \prod_{l=1}^{L} P_{\gamma_l}(\gamma) \quad \text{(independent case)}$$

$$= \prod_{l=1}^{L} \left[1 - e^{-\gamma/\bar{\gamma}_l}\right] \quad \text{(independent Rayleigh case)}$$

$$= \left[1 - e^{-\gamma/\bar{\gamma}}\right]^L \quad \text{(i.i.d. Rayleigh case)}$$

- PDF for $L = 2$

$$p_{\gamma_{\mathrm{sc}}}(\gamma) = \frac{L}{\bar{\gamma}}\left[1 - e^{-\gamma/\bar{\gamma}}\right]^{L-1} e^{-\gamma/\bar{\gamma}}$$

- PDF for $L = 2$

$$p_{\gamma_{\mathrm{sc}}}(\gamma) \quad = \quad \frac{2}{\bar{\gamma}}\left[1 - e^{-\gamma/\bar{\gamma}}\right] e^{-\gamma/\bar{\gamma}} = \frac{2}{\bar{\gamma}}\left[e^{-\gamma/\bar{\gamma}} - e^{-2\gamma/\bar{\gamma}}\right]$$

- MGF for $L = 2$

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- MGF for $L = 2$

$$\mathcal{M}_{\gamma_{\mathrm{sc}}}(s) \quad = \quad \frac{2}{1 - \bar{\gamma}s} - \frac{2}{2 - \bar{\gamma}s} = \frac{2}{(1 - \bar{\gamma}s)(2 - \bar{\gamma}s)}$$

- Average BER of BPSK using SC

$$P_b(e) = \int_0^\infty Q(\sqrt{2\gamma}) p_{\gamma_{\rm sc}}(\gamma)\, d\gamma$$

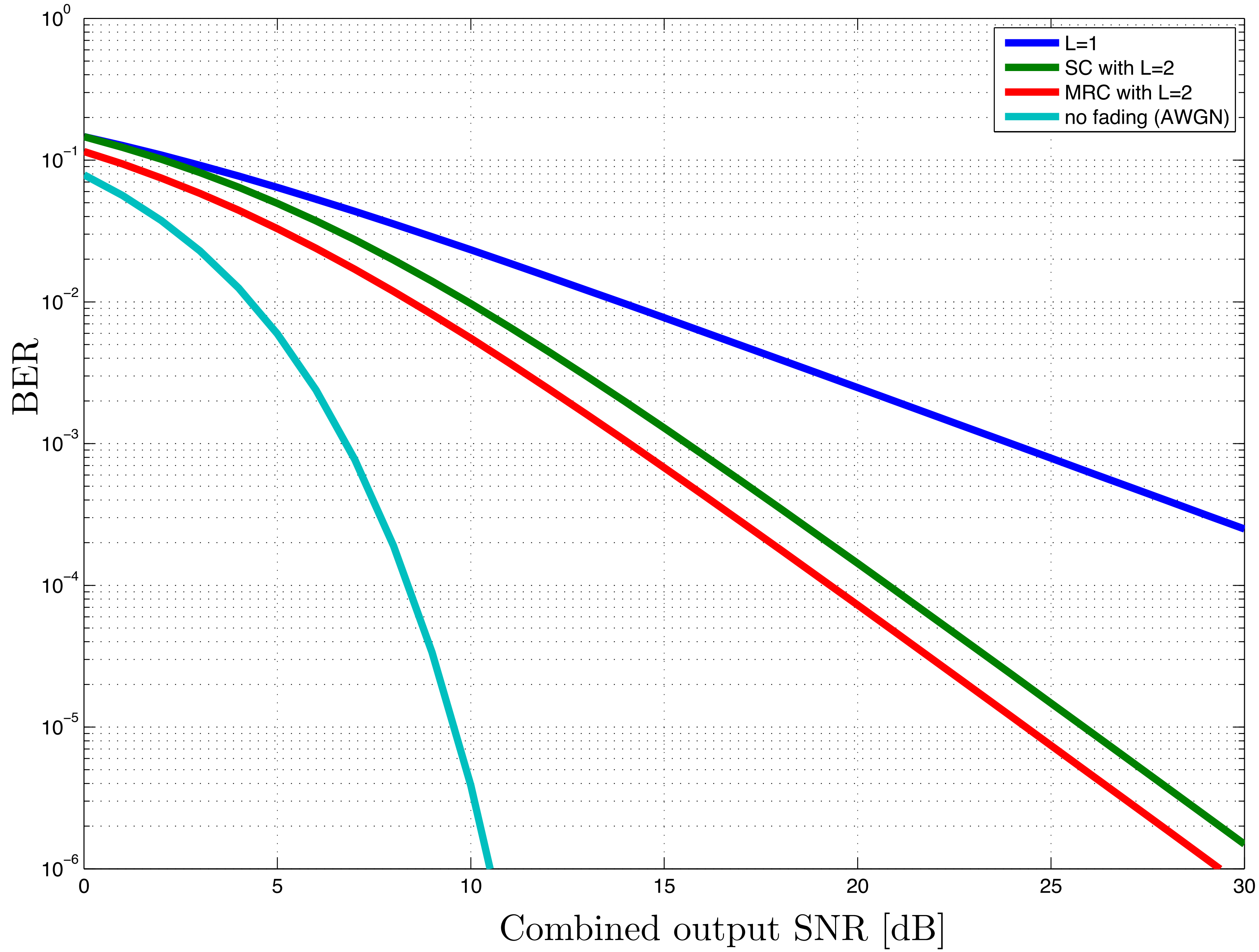
- Average BER of BPSK using SC for $L = 2$

$$P_b(e) = \frac{2}{\bar{\gamma}} \int_0^\infty Q(\sqrt{2\gamma}) e^{-\gamma/\bar{\gamma}}\, d\gamma - \frac{2}{\bar{\gamma}} \int_0^\infty Q(\sqrt{2\gamma}) e^{-2\gamma/\bar{\gamma}}\, d\gamma$$

$$= 2 \times \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right) - \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}/2}{1+\bar{\gamma}/2}}\right)$$

$$= \frac{1}{2} - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}} + \frac{1}{2}\sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}}$$

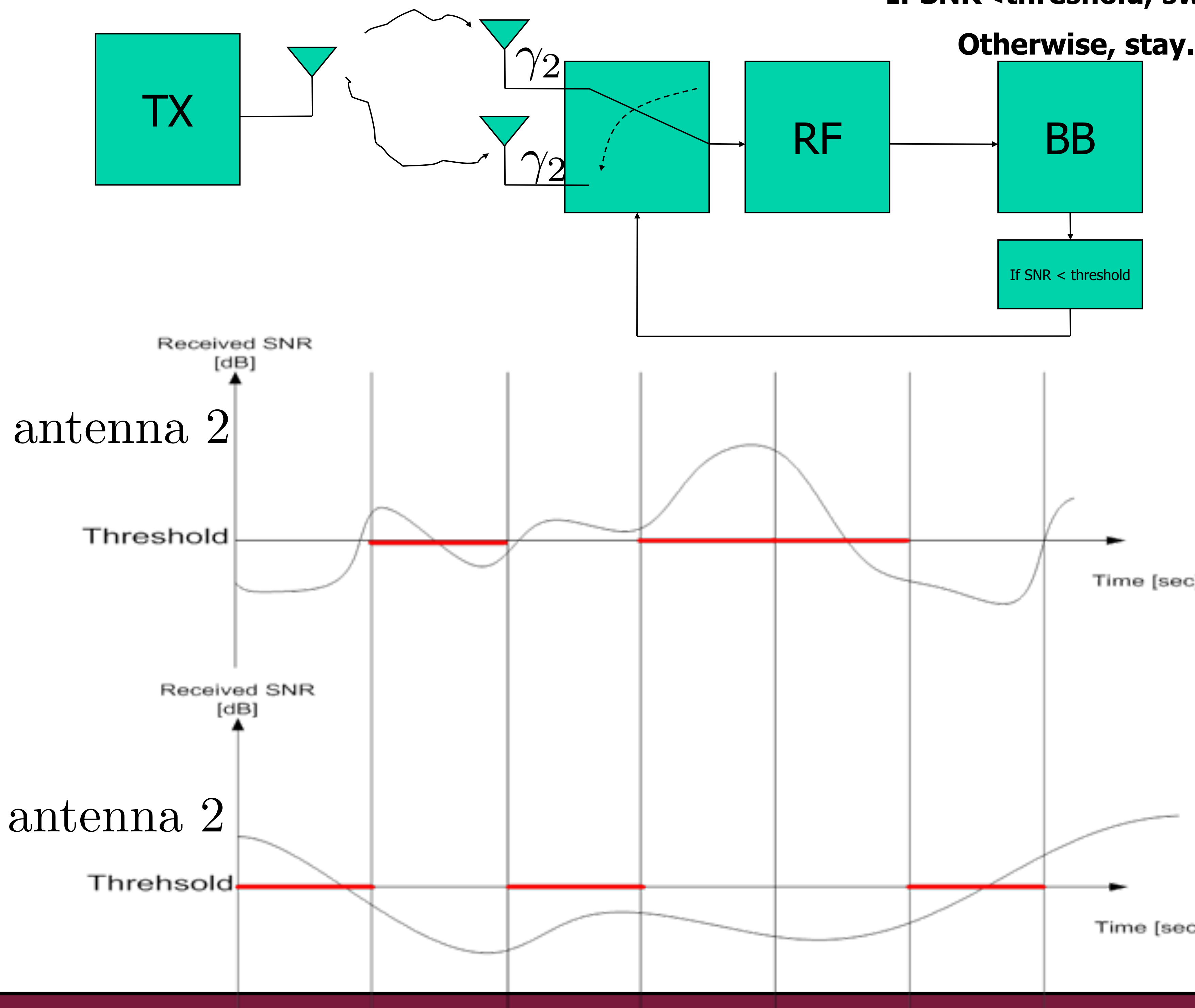
- Comparison of BER of BPSK over Rayleigh channel between MRC and SC for $L = 2$

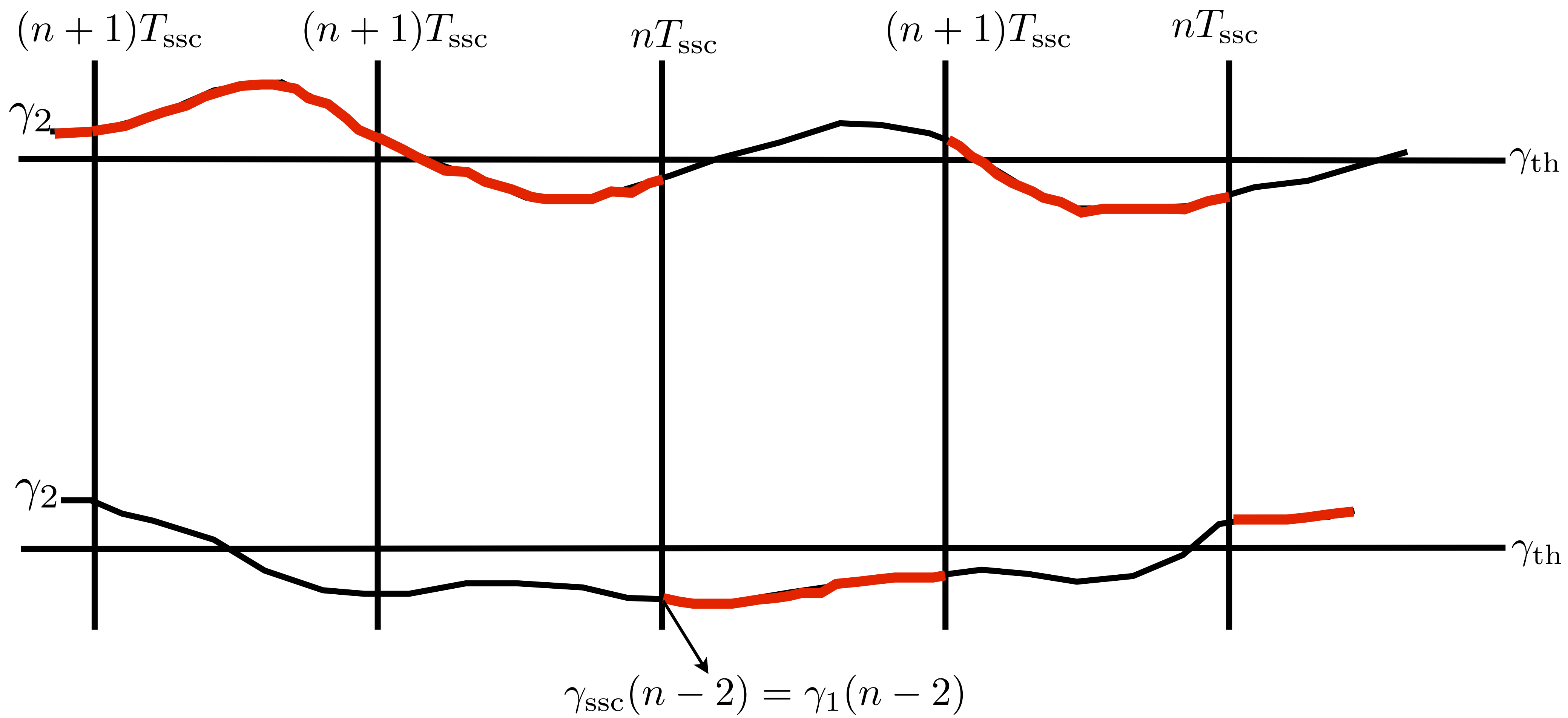


Switched Combining

If SNR<threshold, switch.

Otherwise, stay.





$$\gamma_{\text{ssc}}(n) = \gamma_1(n) \quad \text{iff} \quad \begin{cases} \gamma_{\text{ssc}}(n-1) = \gamma_1(n-1) \quad \text{and} \quad \gamma_1(n) \geq \gamma_{\text{th}} \\ \text{or} \\ \gamma_{\text{ssc}}(n-1) = \gamma_2(n-1) \quad \text{and} \quad \gamma_2(n) < \gamma_{\text{th}} \end{cases}$$

$\gamma_{\text{ssc}}(n) = \gamma_2(n)$ as above with interchanging γ_1 and γ_2.