GEST 011, Newton's Clock & Heisenberg's Dice, Fall 2013

Anatomy of The Uncertainty Principle

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The Uncertainty Principle

(the dictum)

$x \leftrightarrow p$	:	$\Delta x \Delta p \geq \frac{1}{2}\hbar$
$L_x \leftrightarrow L_y$	:	$\Delta L_x \Delta L_y \geq \frac{1}{2}\hbar$
$L_y \leftrightarrow L_z$	:	$\Delta L_y \Delta L_z \geq \frac{1}{2}\hbar$
$L_z \leftrightarrow L_x$	:	$\Delta L_z \Delta L_x \geq \frac{1}{2}\hbar$
$S_x \leftrightarrow S_y$	:	$\Delta S_x \Delta S_y \geq \frac{1}{2}\hbar$
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The uncertainty relation

$$\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$$

One cannot measure precisely both x and p at the same time.

The uncertainty relation

$$\Delta t \cdot \Delta E \geq \frac{1}{2}\hbar$$

The states with broader energy distribution decays faster.

Do we understand its meaning?

Original, But Incorrect, Idea

(Heisenberg, Zeitschrift für Physik 1927)

"Measurement-Disturbance Relation"

At the instant of time when the position is determined, that is, at the instant when the photon is scattered by the electron, the electron undergoes a discontinuous change in momentum. This change is the greater the smaller the wavelength of the light employed, i.e., the more exact the determination of the position.

Modern Version

(Robertson, Phys. Rev. 1929; Kennard, Z. Physik 1927; Weyl 1928)

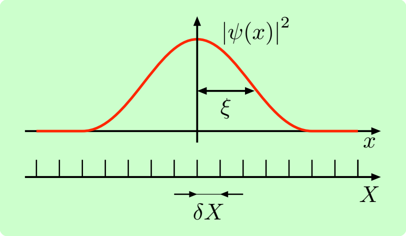
"Uncertainty Relation"

- The uncertainty principle prevails intrinsically for any quantum state of the particle.
- It can be manifested differently depending on specific measurements.

Example

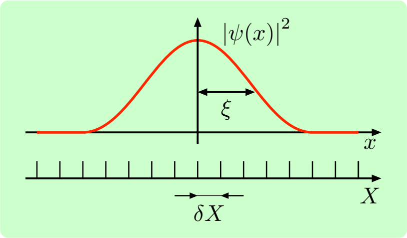
Initial State

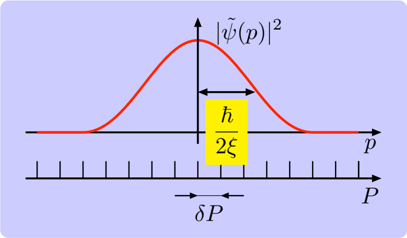
(right before the measurement)



Initial State

(right before the measurement)

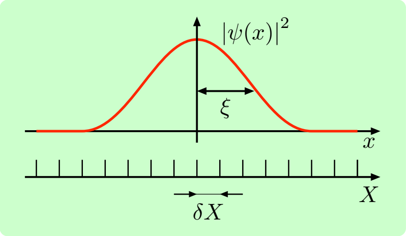




$$\psi(x) = \int \frac{dk}{2\pi}\, \tilde{\psi}(k)\, \exp(ikx)$$

$$\delta X \ll \xi\,, \quad \delta P \ll \frac{\hbar}{2\xi}$$

Measuring Only Position or Momentum



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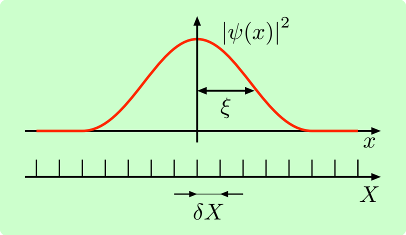
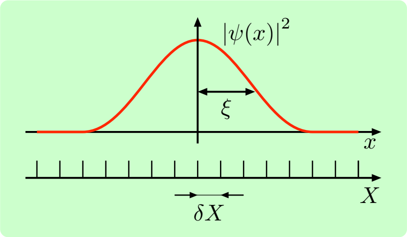
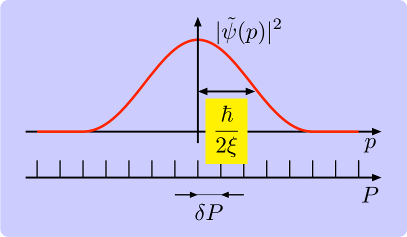


$$\Delta X = \xi + \frac{1}{2}\delta X \approx \xi$$

Measuring Only Position or Momentum





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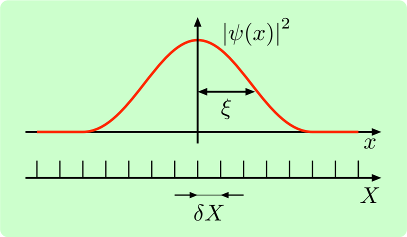

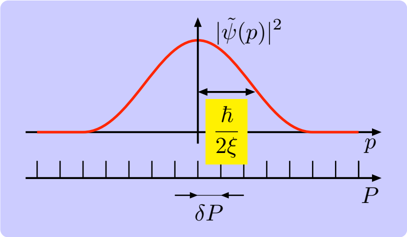

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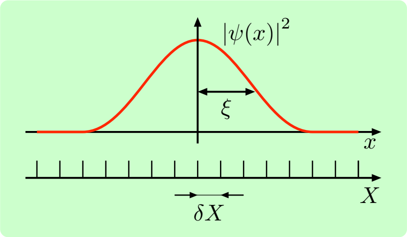

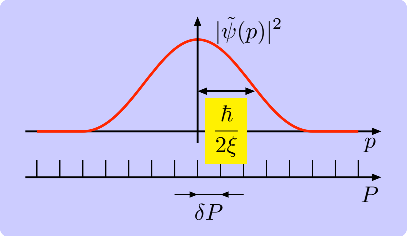

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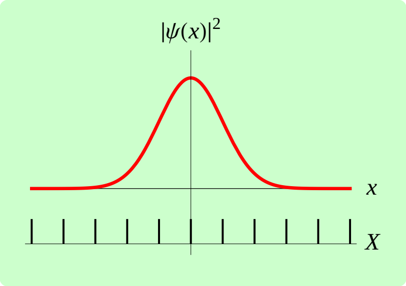
$$\Delta X \Delta P \gtrsim \frac{\hbar}{2}$$

Measuring Position and Then Momentum 1



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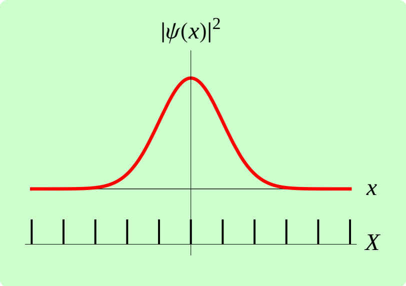

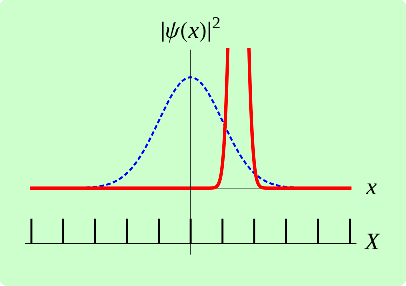

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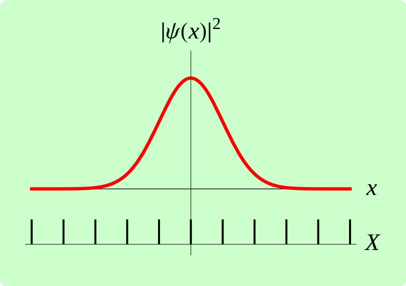

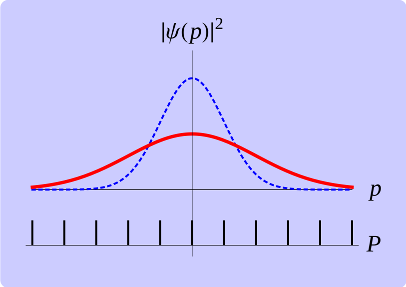

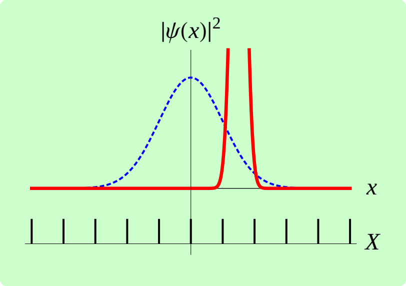

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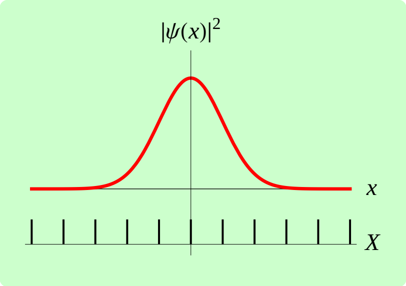
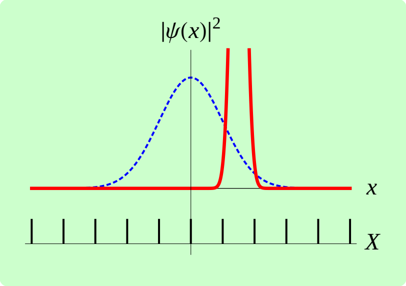
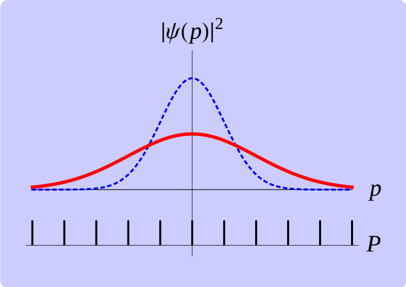




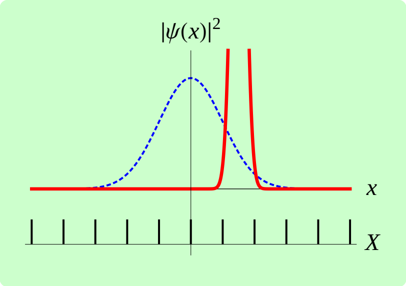


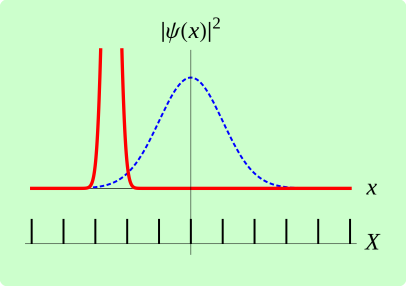
$$\Delta X \approx \xi$$

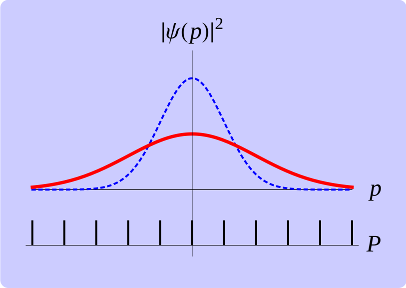
$$\Delta P \geq \frac{\hbar}{2\,\delta X}$$

$$\boxed{\Delta X \Delta P \gtrsim \frac{\hbar}{2}\frac{\xi}{\delta X} \gg \frac{\hbar}{2}}$$

Measuring Position and Then Momentum 2







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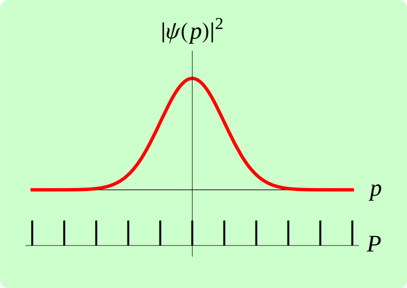

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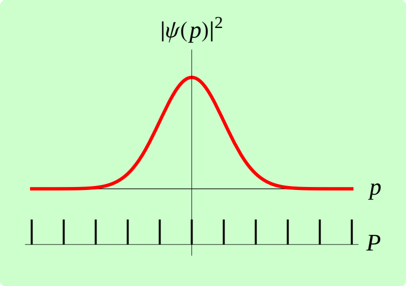

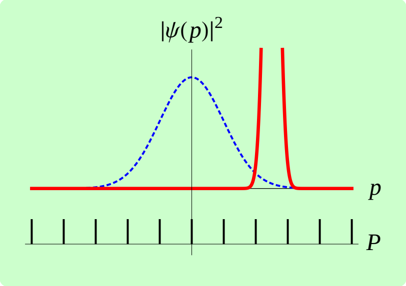

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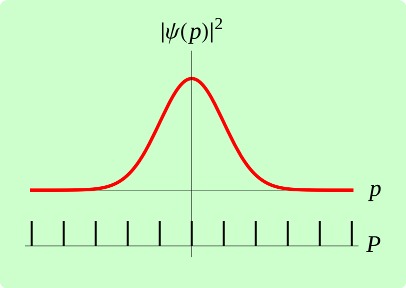

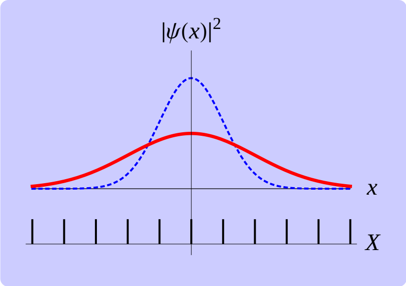

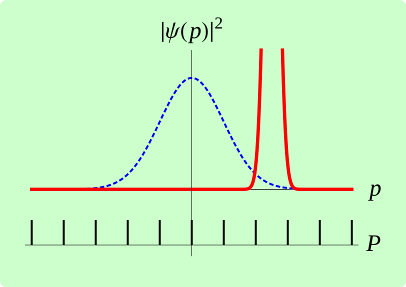

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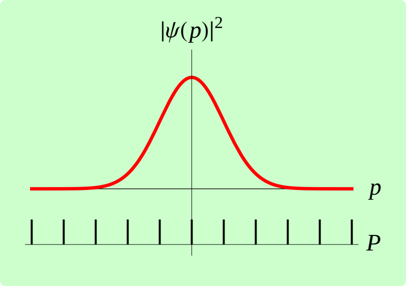

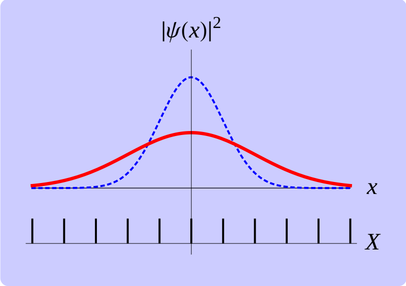

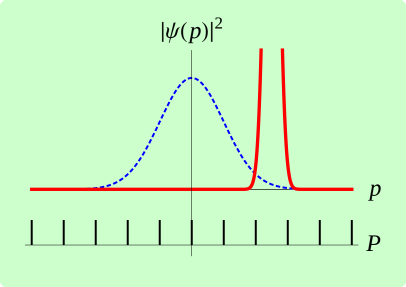

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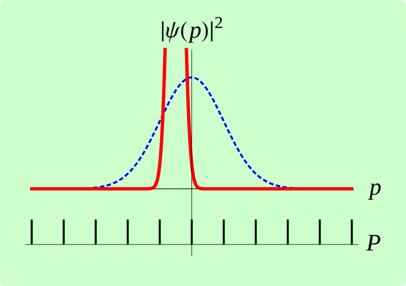

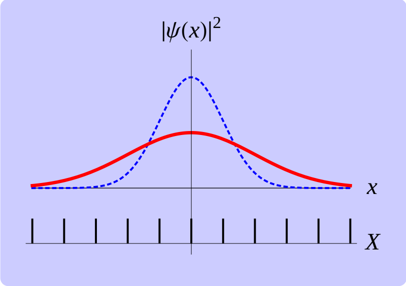

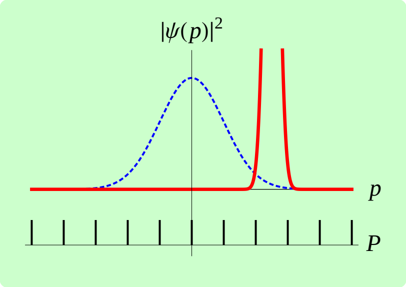

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Position & Momentum "Simultaneously"

At best, equivalent to the case of "position-then-momentum" or "momentum-then-position".

$$\boxed{\Delta X \Delta P \gg \frac{\hbar}{2}}$$

References

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