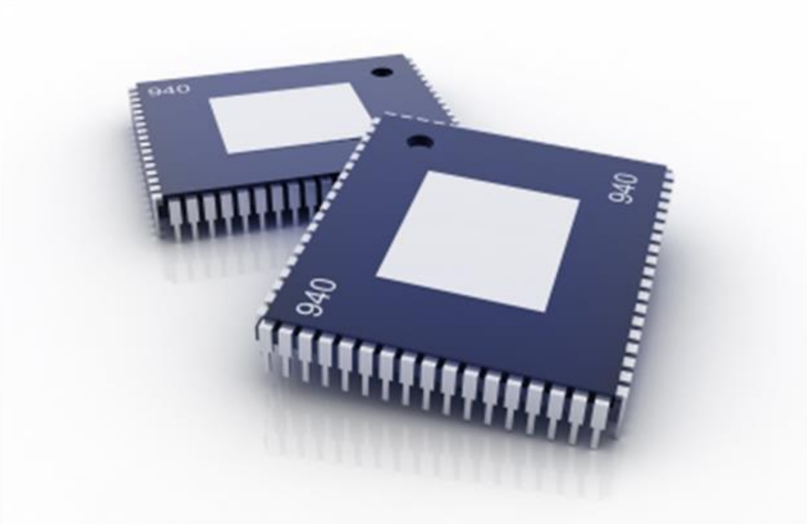

LECTURE 3

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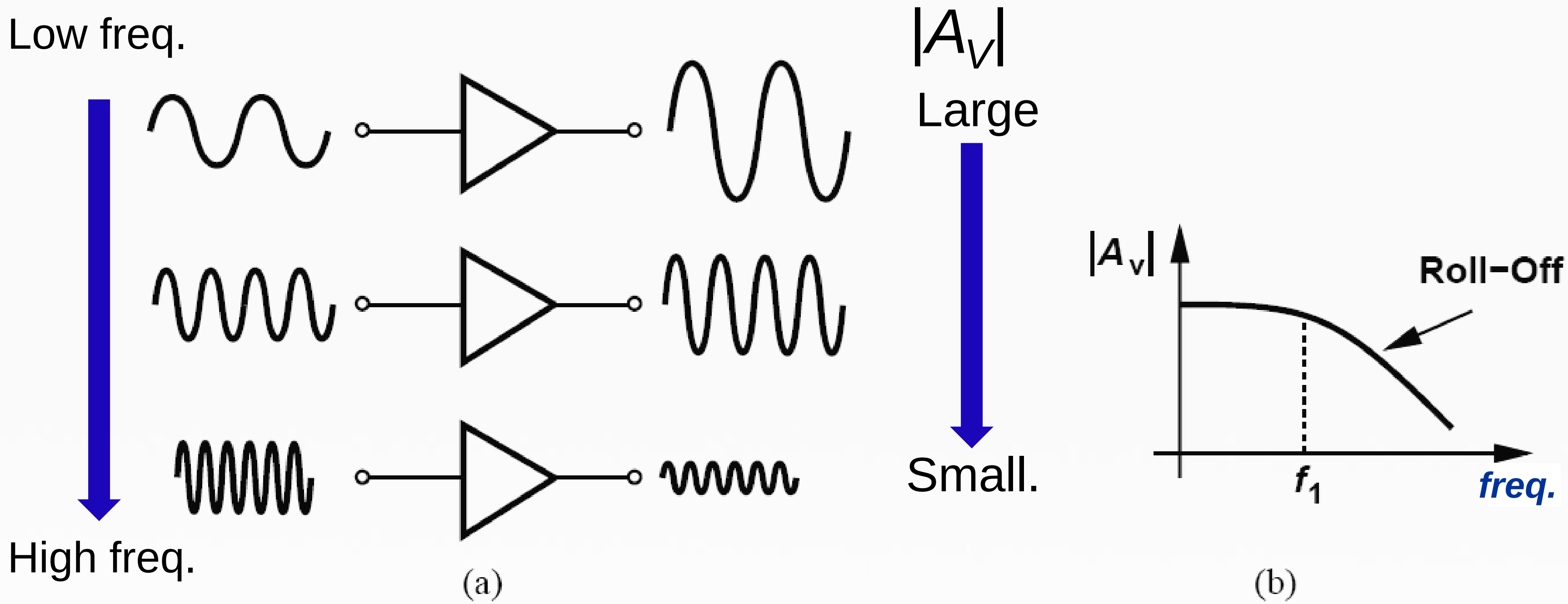
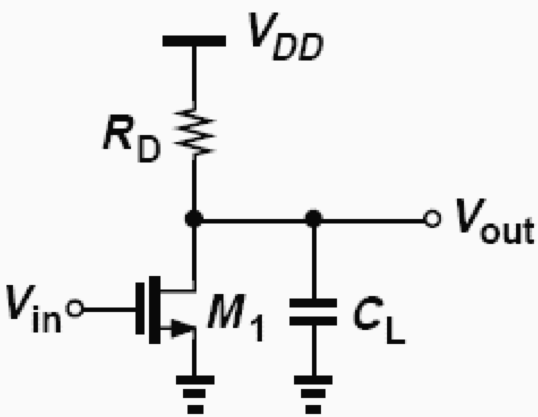
4. Frequency Response of Electronic Circuits

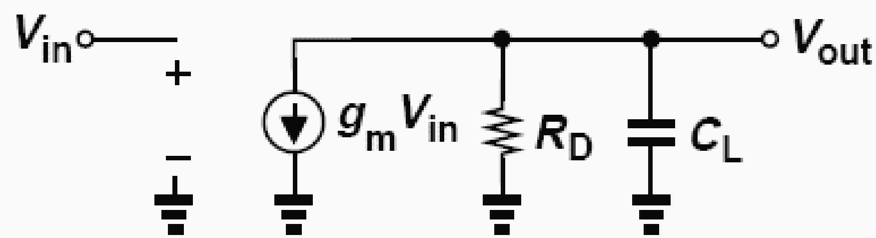
Gain Roll-off of Amplifier



$$freq.\uparrow \quad \Rightarrow \quad |A_V| \rightarrow 0$$

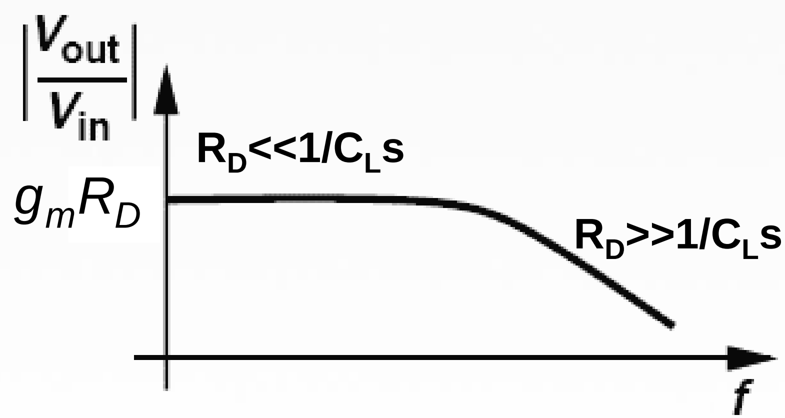
Gain Roll-off: Common Source





$$V_{out} = -g_m V_{in}\left(R_D \parallel \frac{1}{C_L s}\right)$$

$$f\uparrow \quad \Rightarrow \quad \frac{1}{C_L s} = 0 \quad \Rightarrow \quad A_v \rightarrow 0$$



Bode Plot

Recall the electrical circuit theorem

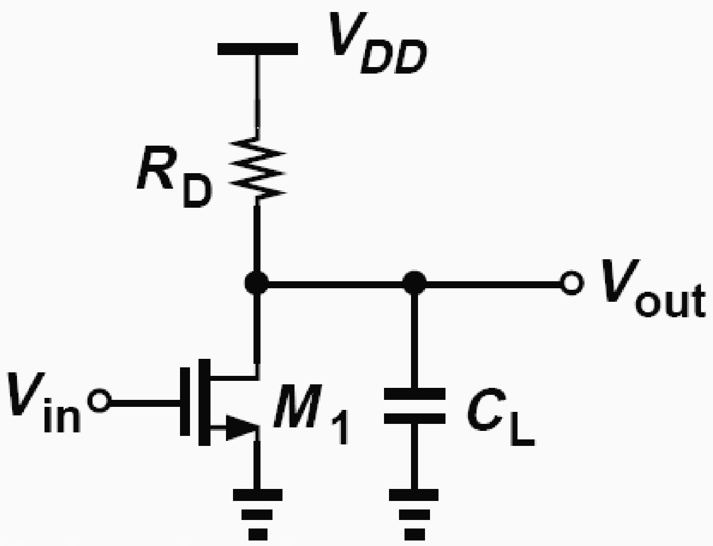
$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

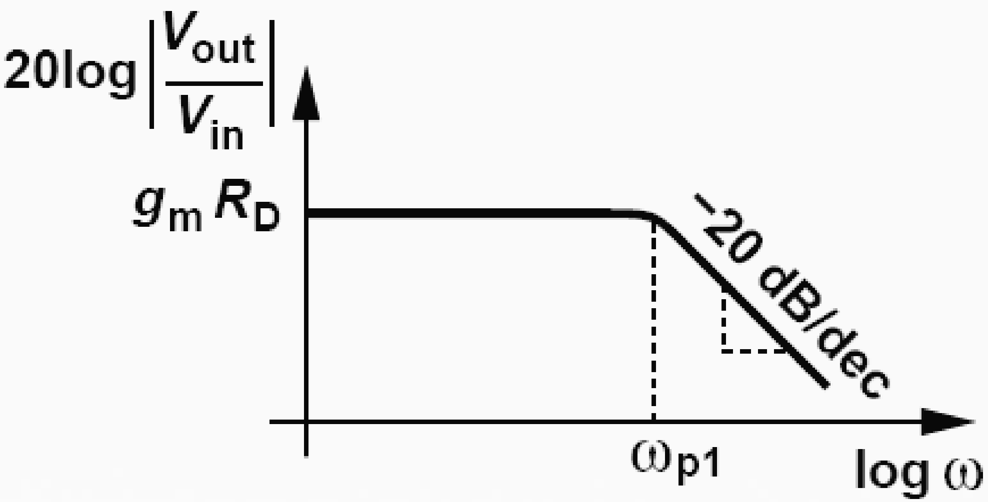
ω_z : zero frequency

ω_p : pole frequency

- **Increase by 20dB/decade**
- **Decrease by 20dB/decade**

Example: Bode Plot



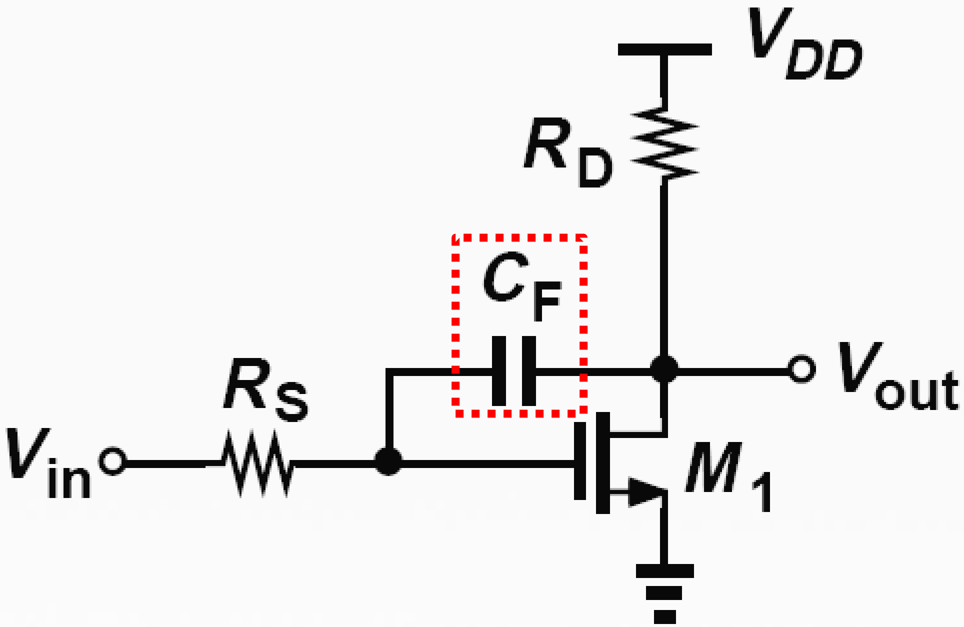


$$\frac{V_{out}}{V_{in}} = \frac{-g_m R_D}{1 + \dfrac{s}{1/R_D C_L}}$$

$$|\omega_{p1}| = \frac{1}{R_D C_L}$$

The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as ω pass ω_{p1}.

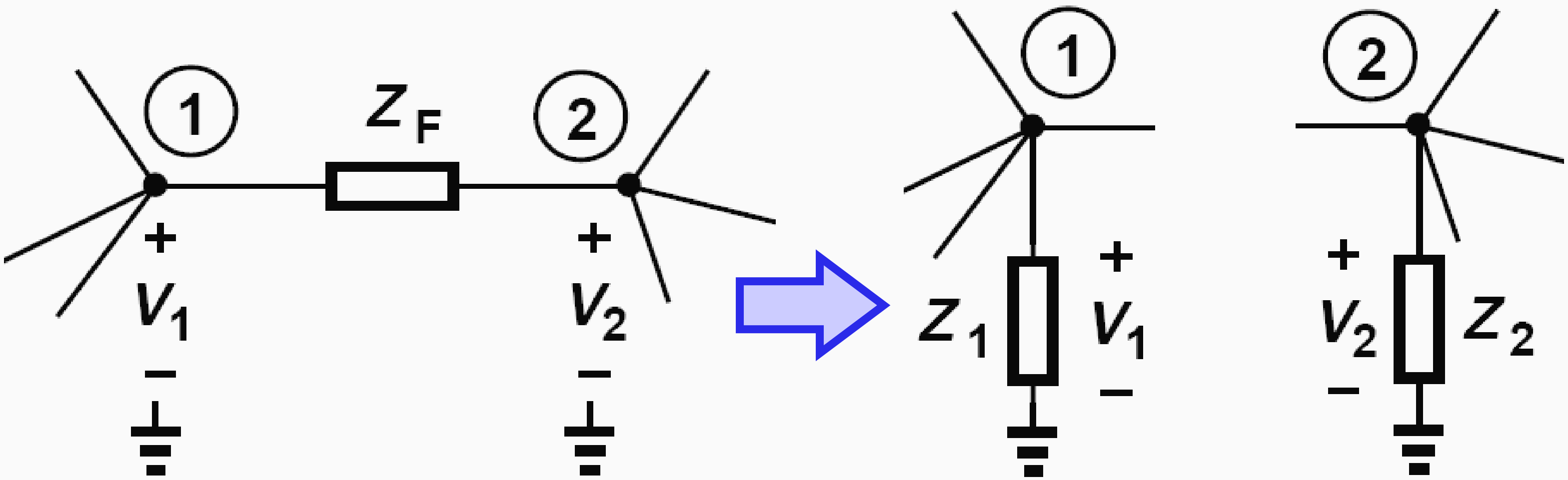
Circuit with Floating Capacitor



The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.

⇒ **Miller's theorem**

Miller's Theorem



$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}$$

$$\frac{V_2 - V_1}{Z_F} = \frac{V_2}{Z_2}$$

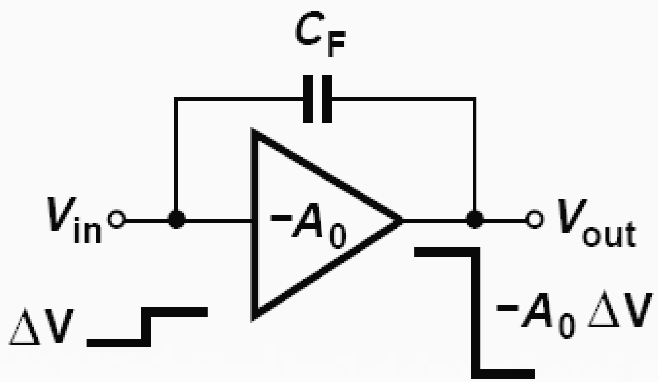
$$A_v = \frac{V_2}{V_1}$$

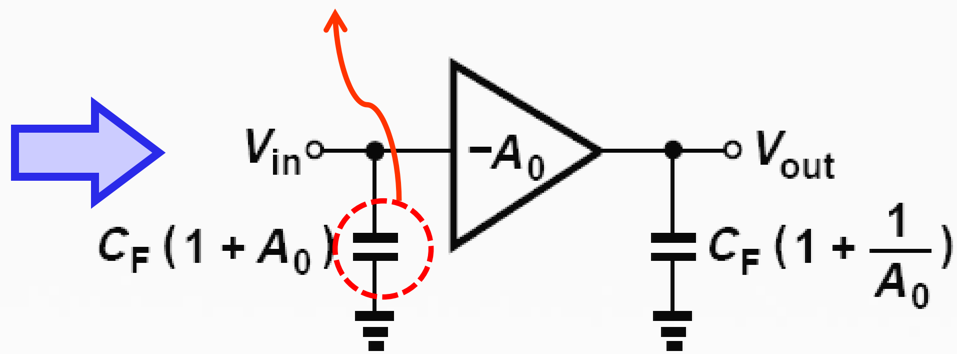
$$Z_1 = \frac{Z_F}{1 - A_v}$$

$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

Miller's Multiplication

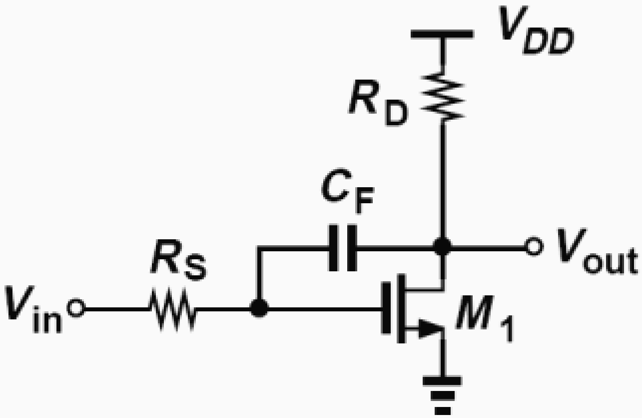
$$Z_1 = \frac{Z_F}{1 - A_v} = \frac{1}{(1 + A_0)C_F s}$$

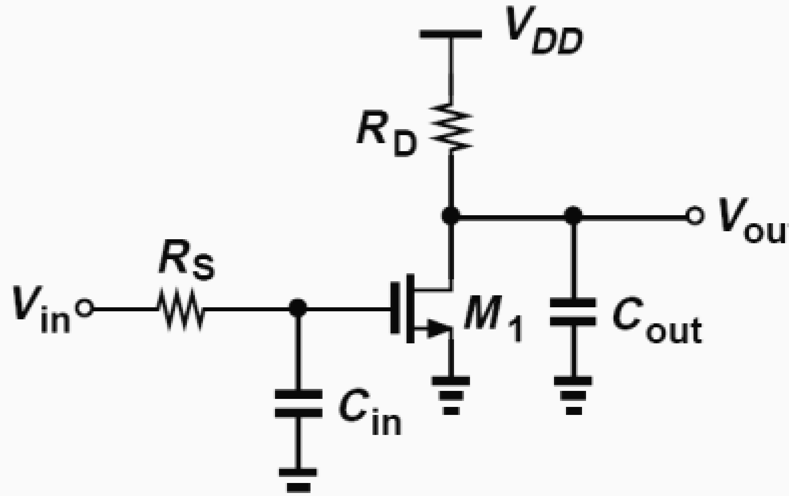




The input capacitor is larger than the original floating capacitor (Miller multiplication)

Example: Miller Theorem





$$A_0 = -g_m R_D$$

$$C_{in} = (1 - A_0)C_F = (1 + g_m R_D)C_F$$

$$C_{out} = (1 - 1/A_0)C_F = \left(1 + \frac{1}{g_m R_D}\right)C_F$$

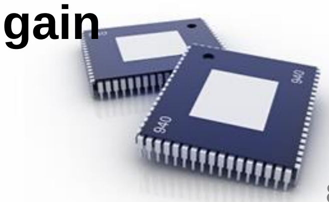
$$\omega_{p,in} = \frac{1}{R_S \cdot C_{in}} = \frac{1}{R_S(1 + g_m R_D)C_F}$$

$$\omega_{p,out} = \frac{1}{R_D \cdot C_{out}} = \frac{1}{R_D\left(1 + \frac{1}{g_m R_D}\right)C_F}$$

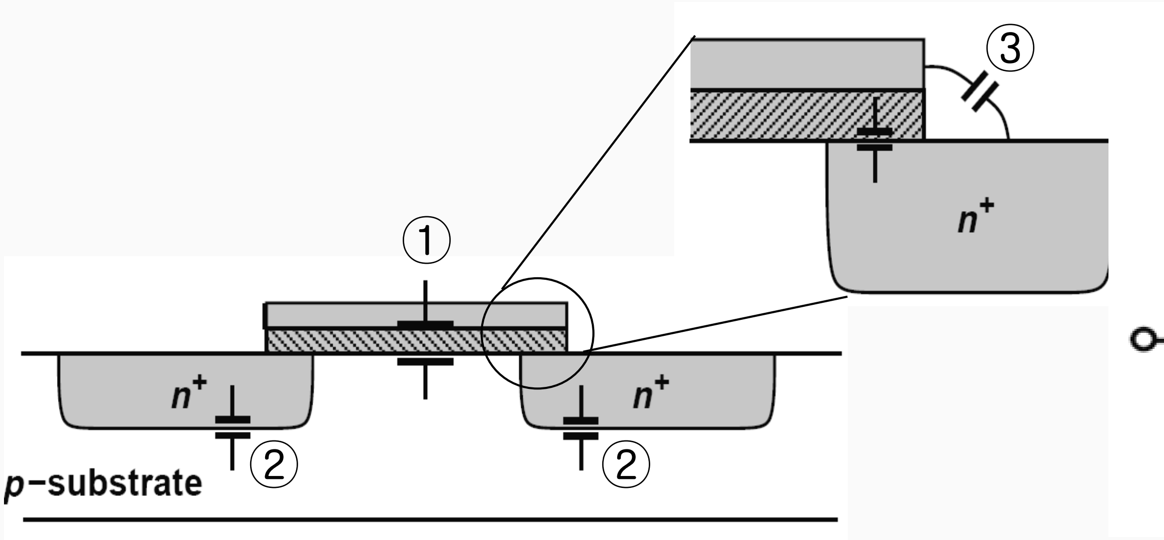
$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{p,in}}\right)\left(1 + \frac{s}{\omega_{p,out}}\right)}$$

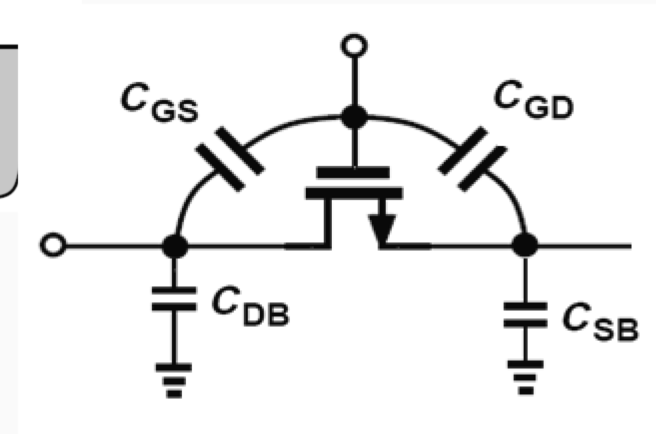
Demerits

1. **Discard zeros**
2. **Approximating gain**



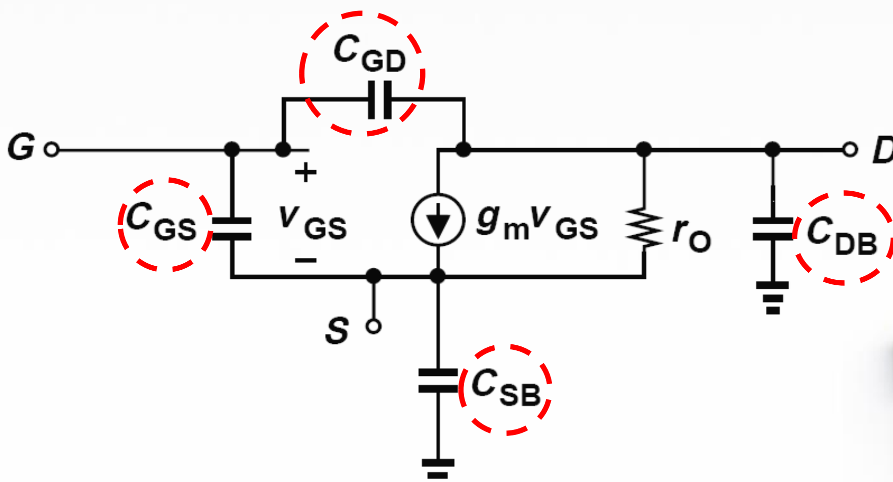
MOS Intrinsic Capacitances ?





①Oxide capacitance ② Junction capacitances ③ Overlap capacitance

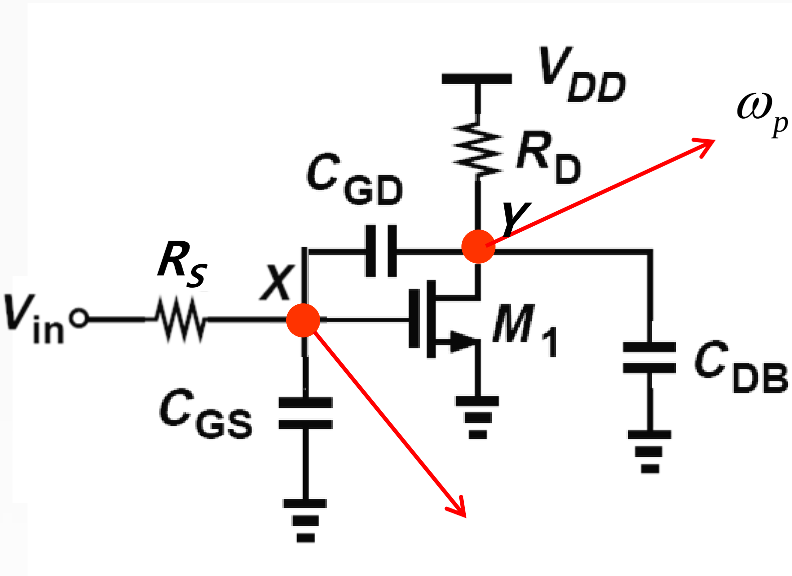
Small-signal equivalent circuit



Frequency Response of CS Stage

Use the Miller theorem

$r_O = \infty$



$$\omega_{p,Y} = \frac{1}{R_D\left[C_{DB} + \left(1 + \frac{1}{g_m R_D}\right)C_{GD}\right]s}$$

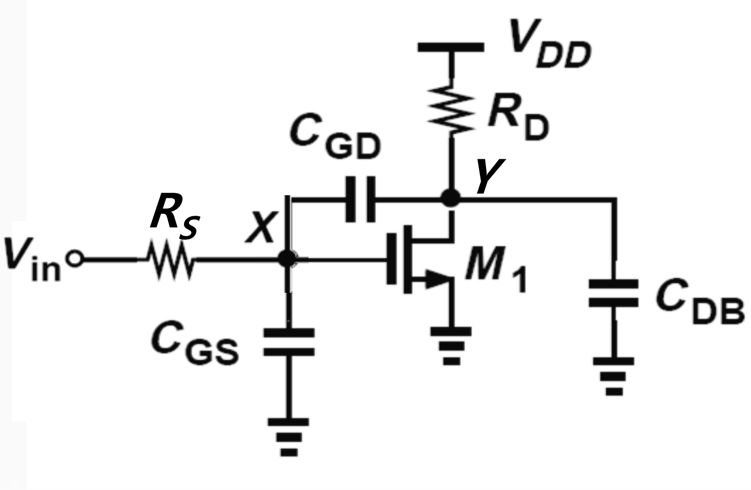
Miller theorem

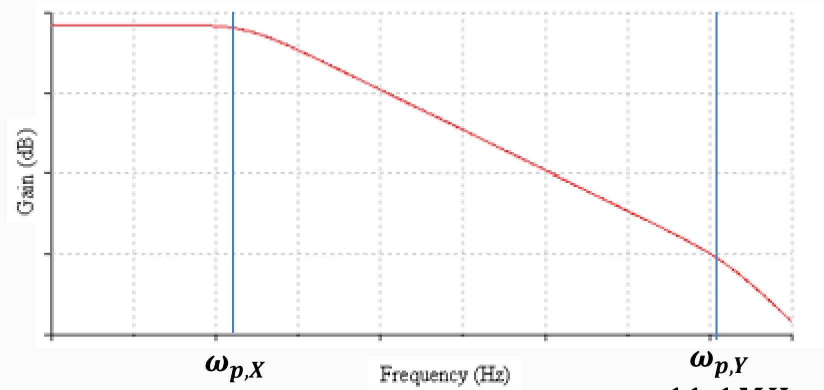
$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS} + \left(1 + g_m R_D\right)C_{GD}\right]s}$$

Miller theorem

Example 1

Calculate the $\omega_{p,X}, \omega_{p,Y}$ ***(if,*** $g_m = 150\mu A/V$, $R_s = 100k\Omega$, $R_D = 40k\Omega$, $C_{GS} = 23fF, C_{GD} = 2fF$, $C_{DB} = 35fF$)





$\omega_{p,X} = 43.4MHz$ $\omega_{p,Y} = 11.1MHz$

$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS} + (1+g_m R_D)C_{GD}\right]s}$$

$$= \frac{1}{100k \cdot \left[23f + (1+150\mu \cdot 40k) \cdot 2f\right] \cdot 2\pi}$$

$$= \frac{1}{100k \cdot \left[23f + (1+150\mu \cdot 40k) \cdot 2f\right] \cdot 2\pi}$$

$$= 43.4MHz$$

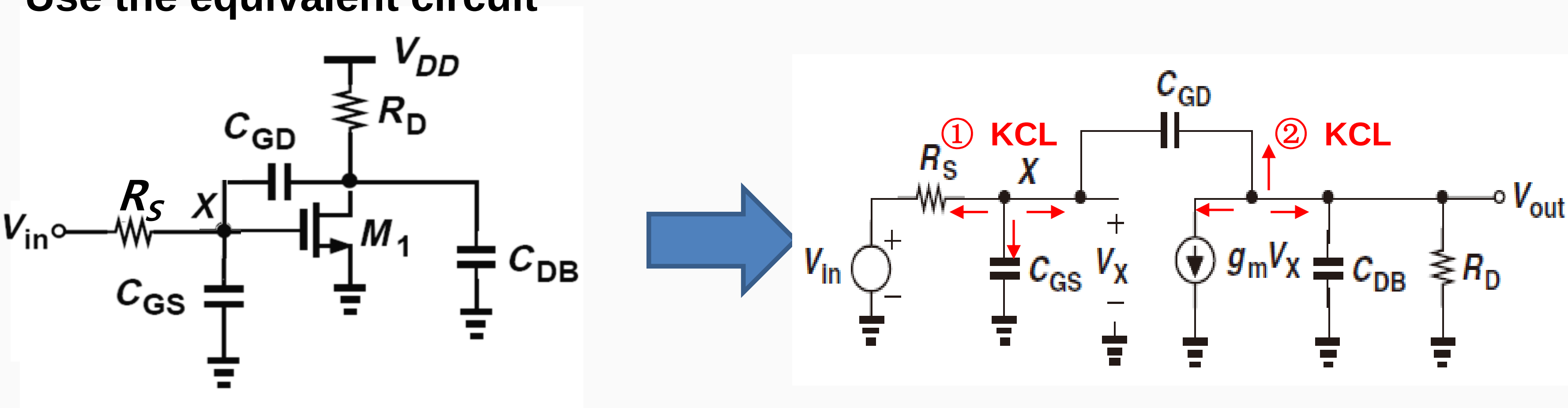
$$\omega_{p,Y} = \frac{1}{R_D\left[C_{DB} + \left(1+\frac{1}{g_m R_D}\right)C_{GD}\right]s}$$

$$= \frac{1}{40k \cdot \left[35f + \left(1+\frac{1}{150\mu \cdot 40k}\right)2f\right] \cdot 2\pi}$$

$$= 111.1MHz$$

Frequency Response of CS Stage

Use the equivalent circuit



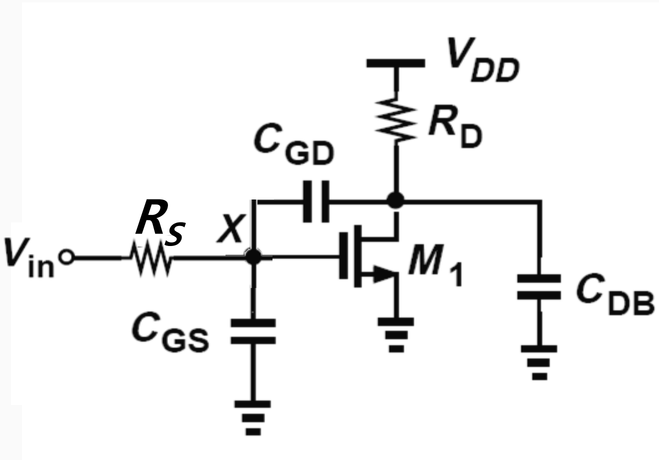
① $$\frac{V_X - V_{in}}{R_S} + V_X C_{GS} s + (V_X - V_{out}) C_{GD} s = 0$$

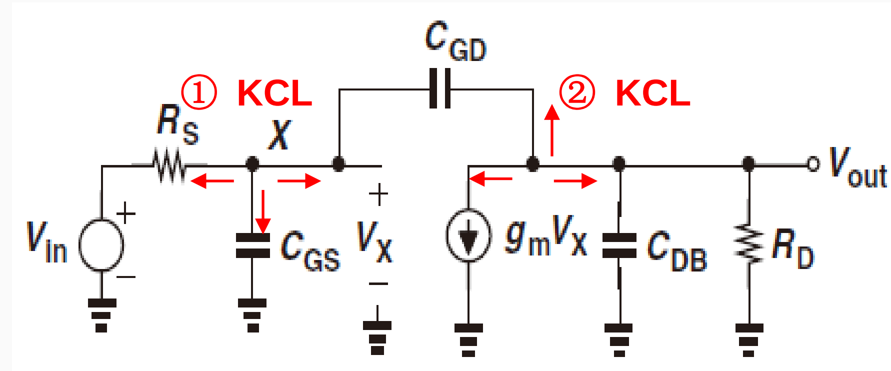
② $$(V_{out} - V_X) C_{GD} s + g_m V_X + V_{out}\left(\frac{1}{R_D} + C_{DB} s\right) = 0$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m) R_D}{R_S R_D \xi s^2 + \left[R_S(1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})\right] s + 1}$$

$$(\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})$$

Frequency Response of CS Stage





Use the equivalent circuit

Use the Miller theorem

$$\omega_{p1} = \frac{1}{R_S(1+g_mR_D)C_{GD} + R_SC_{GS} + R_D(C_{GD}+C_{DB})} \iff \omega_{p1} = \frac{1}{R_S\left[C_{GS}+(1+g_mR_D)C_{GD}\right]s}$$

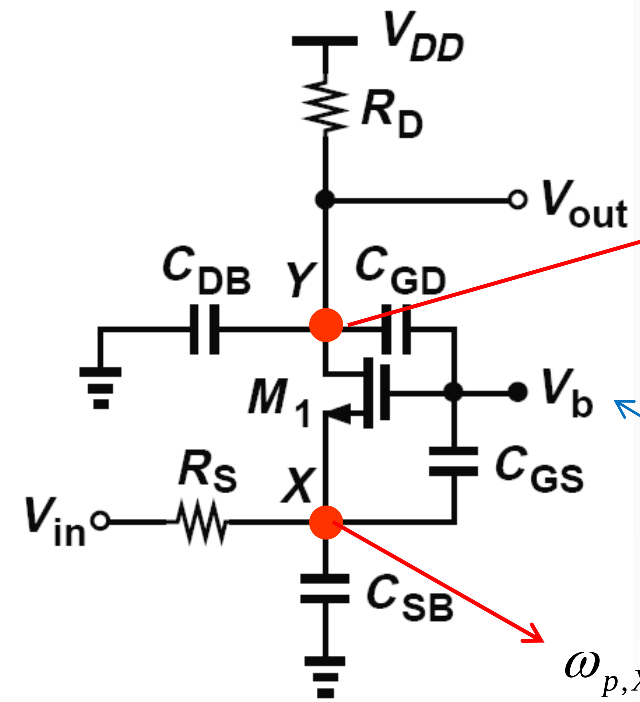
$$\omega_{p2} = \frac{R_S(1+g_mR_D)C_{GD} + R_SC_{GS} + R_D(C_{GD}+C_{DB})}{R_SR_D(C_{GS}C_{GD}+C_{GS}C_{DB}+C_{GD}C_{DB})} \iff \omega_{p2} = \frac{1}{R_D\left[C_{DB}+\left(1+\frac{1}{g_mR_D}\right)C_{GD}\right]s}$$

If, $C_{GS} >> \frac{(1+g_mR_D)C_{GD}+R_D(C_{GD}+C_{DB})}{R_S}$

$$= \frac{1}{R_D(C_{GD}+C_{DB})}$$

Frequency Response of CG Stage

$r_O = \infty$



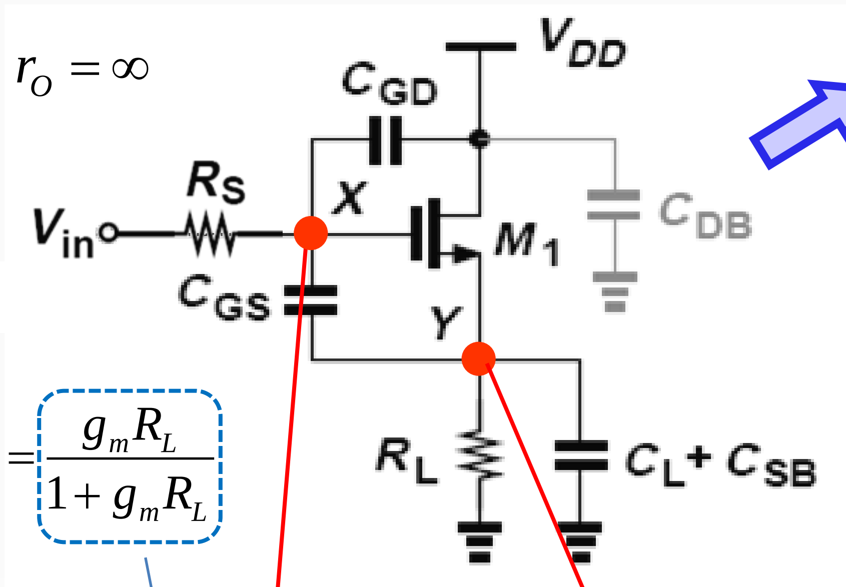
$$\omega_{p,Y} = \frac{1}{R_D(C_{GD} + C_{DB})}$$

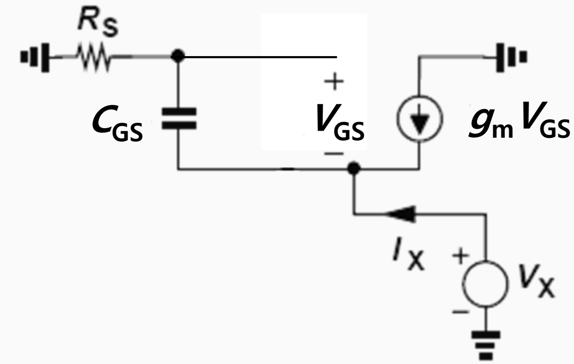
ac ground

$$\omega_{p,X} = \frac{1}{\left(R_S \,\|\, \frac{1}{g_m}\right)(C_{GS} + C_{SB})}$$

High speed(No miller effect)

Frequency Response of SF Stage





$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = \frac{g_m R_L}{1 + g_m R_L}$$

$$\omega_{p,X} = \frac{1}{R_S \cdot \left(C_{GD} + (1 - A_v) C_{GS}\right)}$$

$$= \frac{1}{R_S \cdot \left(C_{GD} + \frac{C_{GS}}{1 + g_m R_L}\right)}$$

$$\left(I_X + g_m V_{GS}\right)\left(\frac{1}{C_{GS} s}\right) = -V_{GS}$$

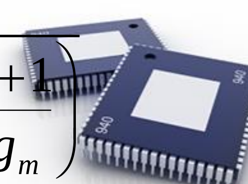
$$V_{GS} = -I_X \frac{1}{C_{GS} s + g_m}$$

Using KVL

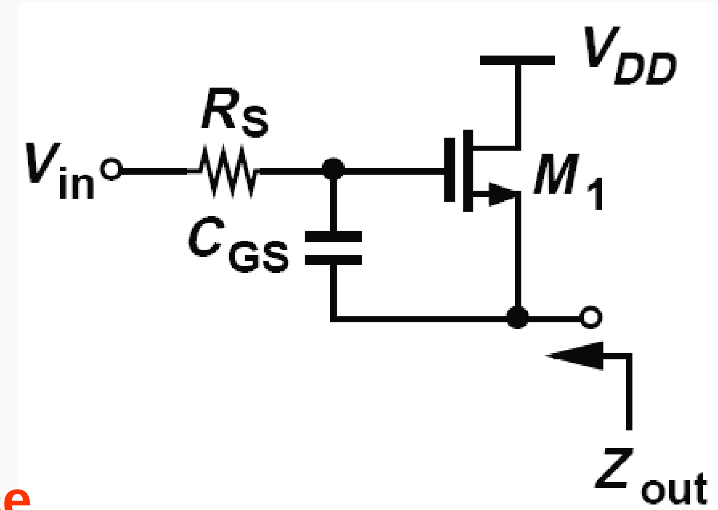
$$\left(I_X + g_m V_{GS}\right) R_S - V_{GS} = V_X$$

$$\therefore Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

$$\omega_{p,Y} = \frac{1}{R_L \cdot \left(C_L + C_{SB} + \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}\right)}$$



Output Impedance of Source Follower

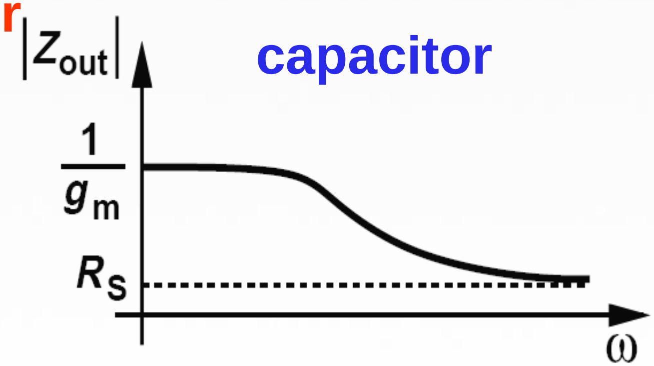


$$Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

$$\begin{cases} s = 0 \Rightarrow Z_{out} = \dfrac{1}{g_m} \\ s = \infty \Rightarrow Z_{out} = R_S \end{cases}$$

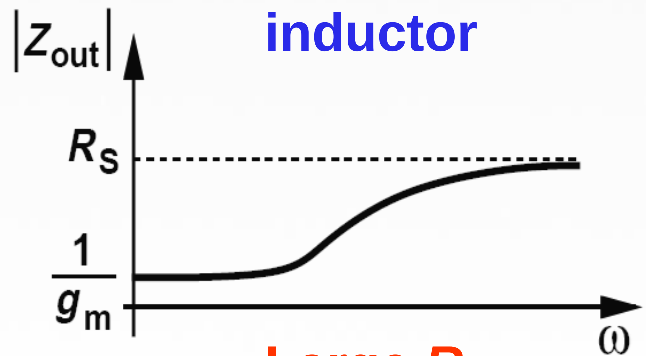
Source Follower

capacitor



Small R_S

inductor



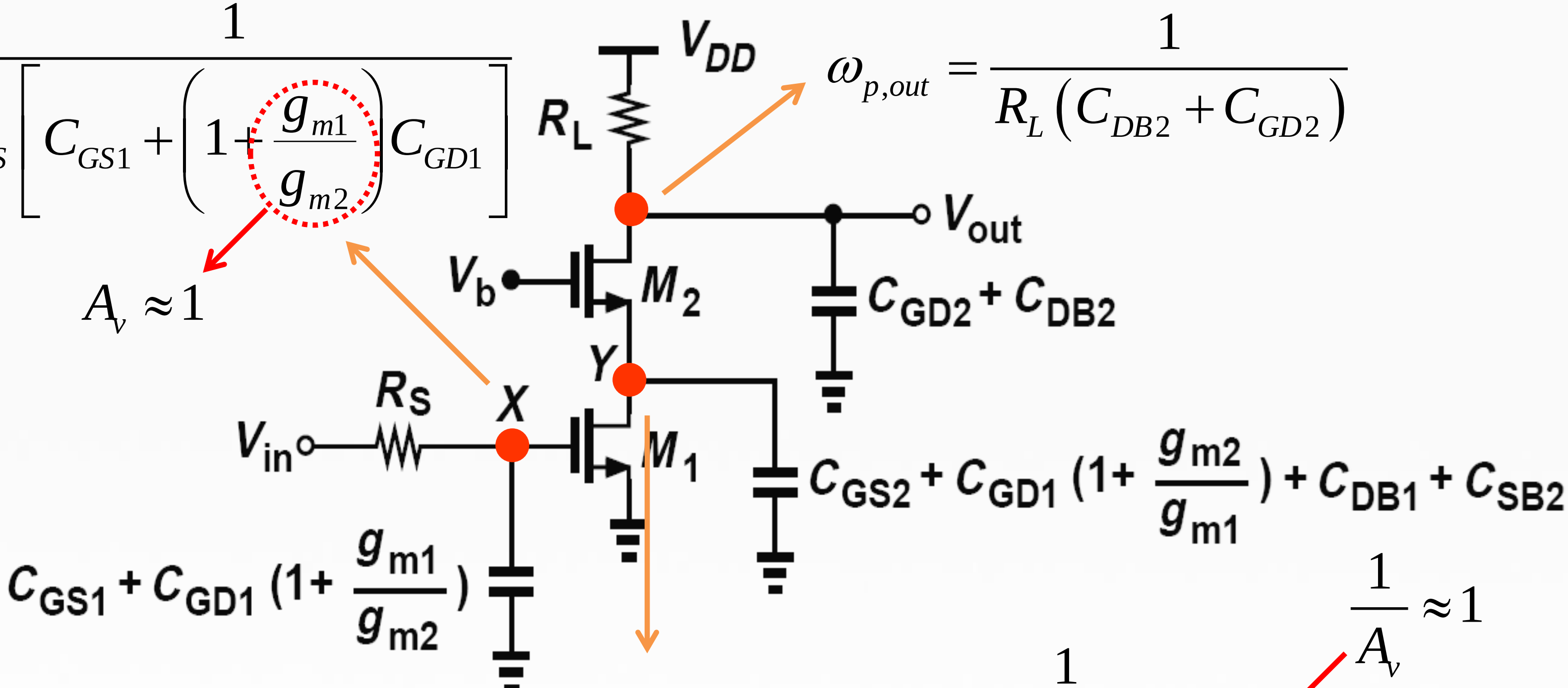
Large R_S

Poles of MOS Cascode

$$\omega_{p,X} = \frac{1}{R_S\left[C_{GS1} + \left(1+\frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

$A_v \approx 1$

$$\omega_{p,out} = \frac{1}{R_L\left(C_{DB2} + C_{GD2}\right)}$$

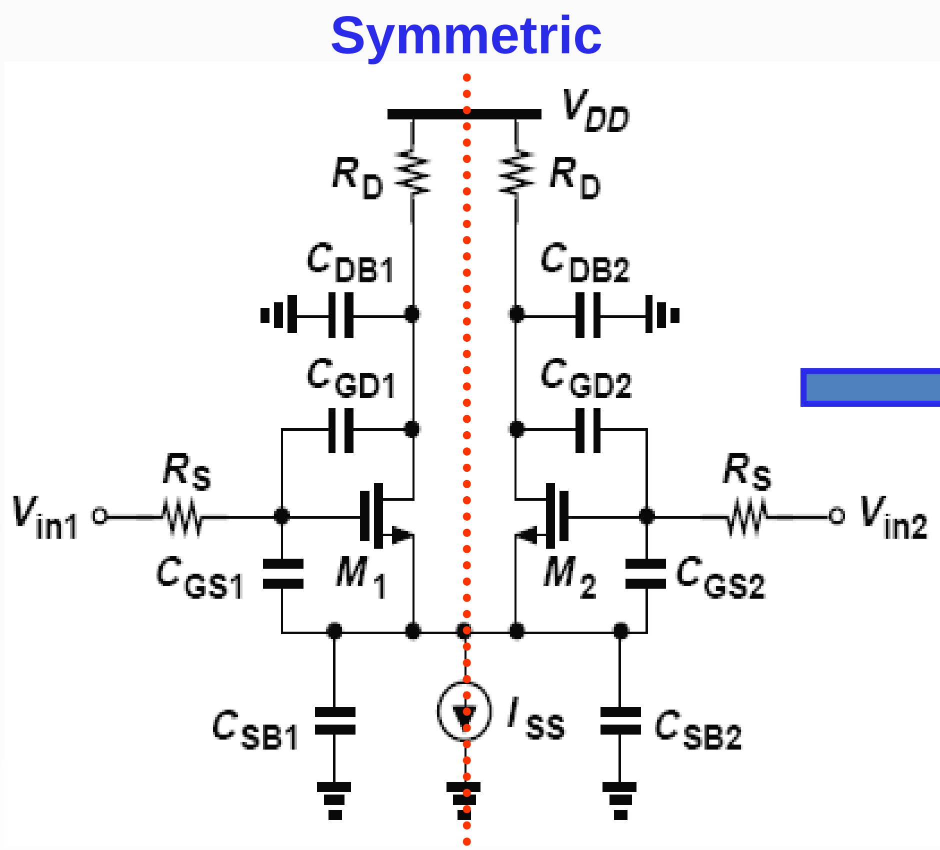


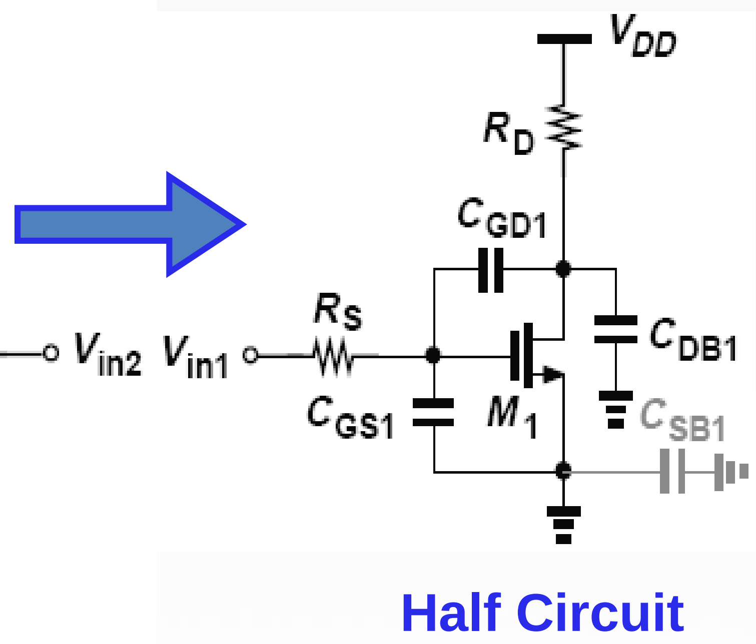
$\frac{1}{A_v} \approx 1$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1+\frac{g_{m2}}{g_{m1}}\right)C_{GD1}\right]}$$

High speed(reject the miller effect)

Differential Pair Frequency Response





its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.