Communication Systems II

[KECE322_01]

<2012-2nd Semester>

Lecture #23

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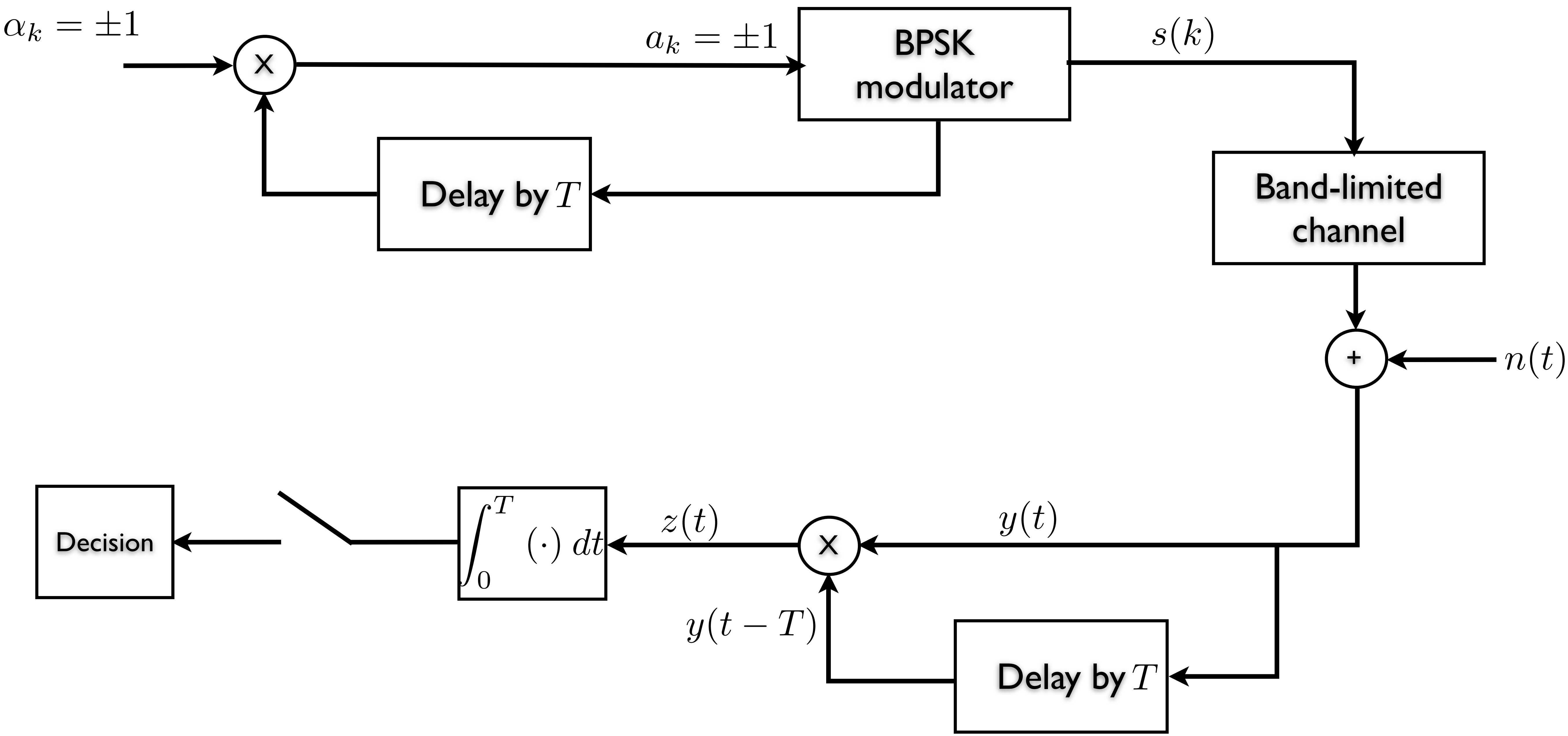
Outline

- Binary differential PSK (BDPSK)
- Quadrature Amplitude Modulation (QAM)

Binary Differential Phase-Shift-Keying (BDPSK)

- Differential coherent modulations are used when tracking the phase is difficult or costly.
- These techniques allow the extraction/recovery of the information bits without knowledge of phase.
- Let us treat only binary differential phase-shift-keying (BDPSK).

- Block diagram



- Received signal for $t \in [kT,\ (k+1)T]$

$$y(t) = a_k \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t + \theta) + n(t)$$

- After delay by

$$y(t-T) = a_{k-1} \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c (t-T) + \theta) + n(t)$$

$$= a_{k-1} \sqrt{\frac{2E_b}{T}} \cos(2\pi f_c t + \theta) + n(t)$$

← Assume $f_c T = \text{integer}$

- Key assumption

  - Assume that $\theta(t)$ is unknown to the receiver but slowly varying such as it is roughly constant over 2 successive bits.

$$\frac{1+\cos(4\pi f_c t + 2\theta)}{2}$$

$$\begin{aligned} z(t) &= y(t)y(t-T) \\ &= a_k a_{k-1} \frac{2\mathcal{E}_b}{T} \cos(2\pi f_c t + \theta)\cos(2\pi f_c t + \theta) + a_k \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_c t + \theta) n(t) \\ &\quad + a_{k-1} \sqrt{\frac{2\mathcal{E}_b}{T}} \cos(2\pi f_c t + \theta) n(t) + n^2(t) \end{aligned}$$

- After integration

$$z = a_k a_{k-1} \frac{2\mathcal{E}_b}{T} \int_{kT}^{(k+1)T} \frac{1+\cos(4\pi f_c t + 2\theta)}{2}\, dt + \text{ noise}$$

$$= a_k a_{k-1} \frac{2\mathcal{E}_b}{T} \frac{T}{2} + \text{ noise}$$

$$= a_k a_{k-1} \mathcal{E}_b + \text{noise}$$

$$\text{noise} = \begin{cases} 2n_1 + n_2 \\ n_2 \end{cases}$$

$$n_1 = \sqrt{\frac{2\mathcal{E}_b}{N_0}} \int_0^T \cos(2\pi f_c t + \theta) n(t)\, dt$$

$$n_2 = \int_0^T n^2(t)\, dt$$

Differential encoding and decoding

- If the symbol $a_k \in \{-1,\ 1\}$ are obtained from the information symbols $\alpha_k \in \{-1,\ 1\}$. From the differential encoder we have:

$$a_k = a_{k-1} \cdot \alpha_k$$

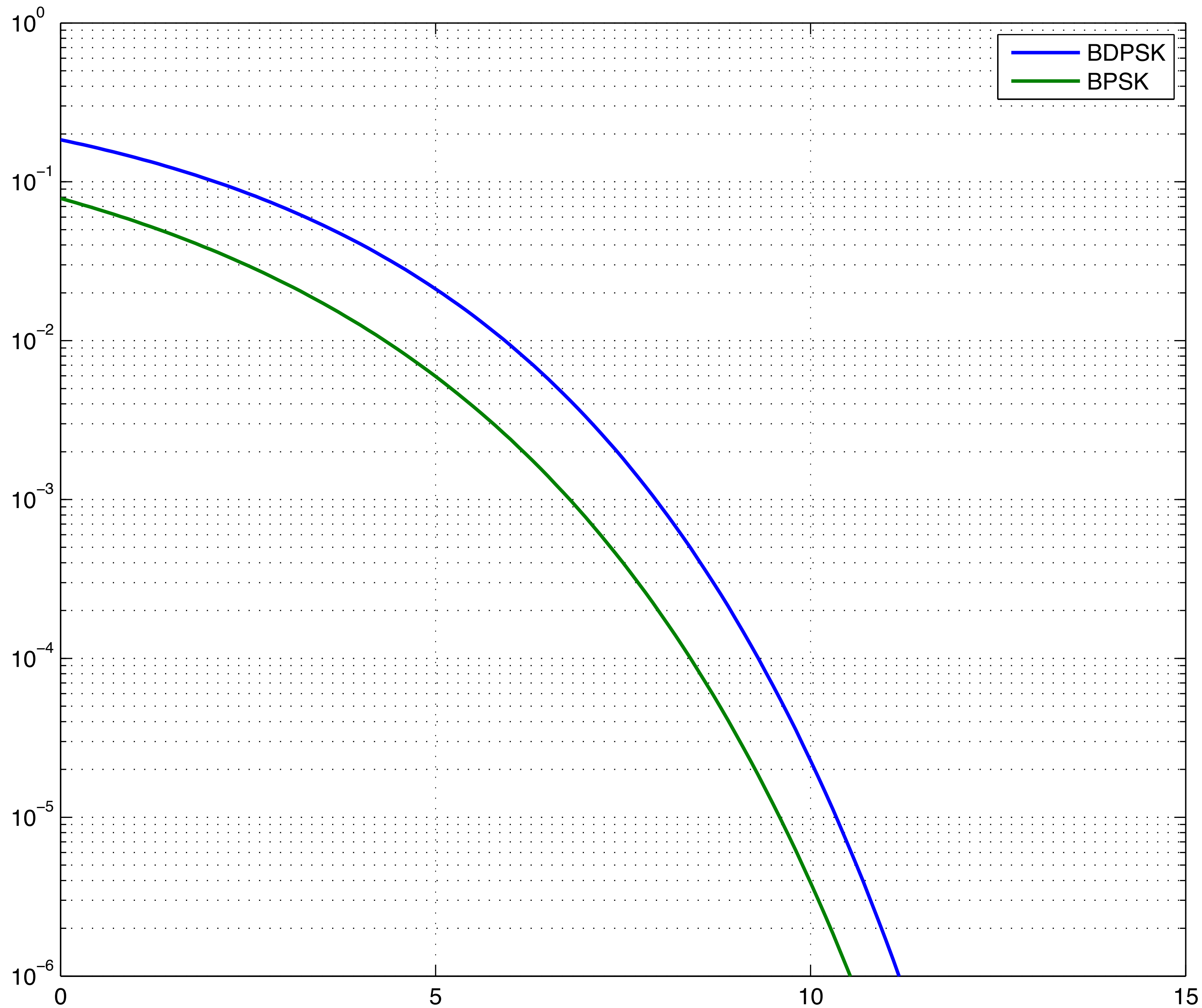
a_k	a_{k-1}	α_k
1	1	1
1	-1	-1
-1	1	-1
-1	-1	1

$$z = a_k a_{k-1} \mathcal{E}_b + \text{ noise } = (a_{k-1})^2 \alpha_k \mathcal{E}_b + \text{ noise}$$

$$= \alpha_k \mathcal{E}_b + \text{ noise}$$

Performance of BDPSK

$$P_b = \frac{1}{2} e^{-\mathcal{E}_b/N_0}$$



Quadrature Amplitude Modulation (QAM)

- Transmit signal waveforms

$$u_m(t) = A_{mc} g_T(t) \cos 2\pi f_c t + A_{ms} g_T(t) \sin 2\pi f_c t, \qquad m = 1, 2, \ldots, M$$

$$= \sqrt{A_{mc}^2 + A_{ms}^2} g_T(t) \cos(2\pi f_c t + \theta_m)$$

$$= \Re\{\sqrt{A_{mc}^2 + A_{ms}^2} g_T(t) e^{j(2\pi f_c t + \theta_m)}\}$$

$$= \Re\{s_{lm}(t) e^{2j\pi f_c t}\}$$

where $s_{lm}(t) = \sqrt{A_{mc}^2 + A_{ms}^2} g_T(t) e^{j\theta_m}$

$$= s_{m1}\phi_1(t) + s_{m2}\phi_2(t)$$

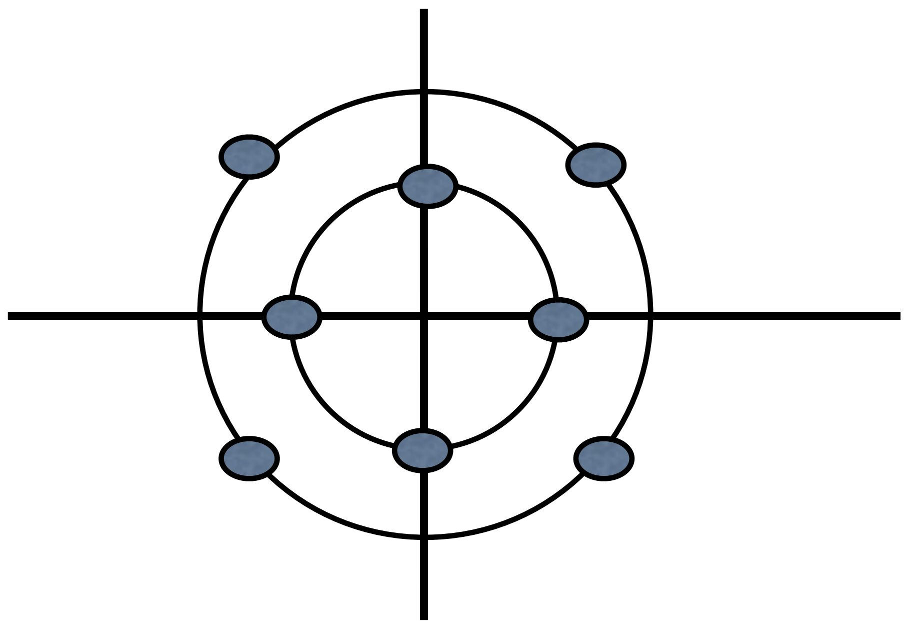
$$\theta_m = \tan^{-1} \frac{A_{ms}}{A_{mc}}$$

$$\phi_1(t) = \sqrt{\frac{1}{\mathcal{E}_s}} g_T(t) \cos(2\pi f_c t)$$

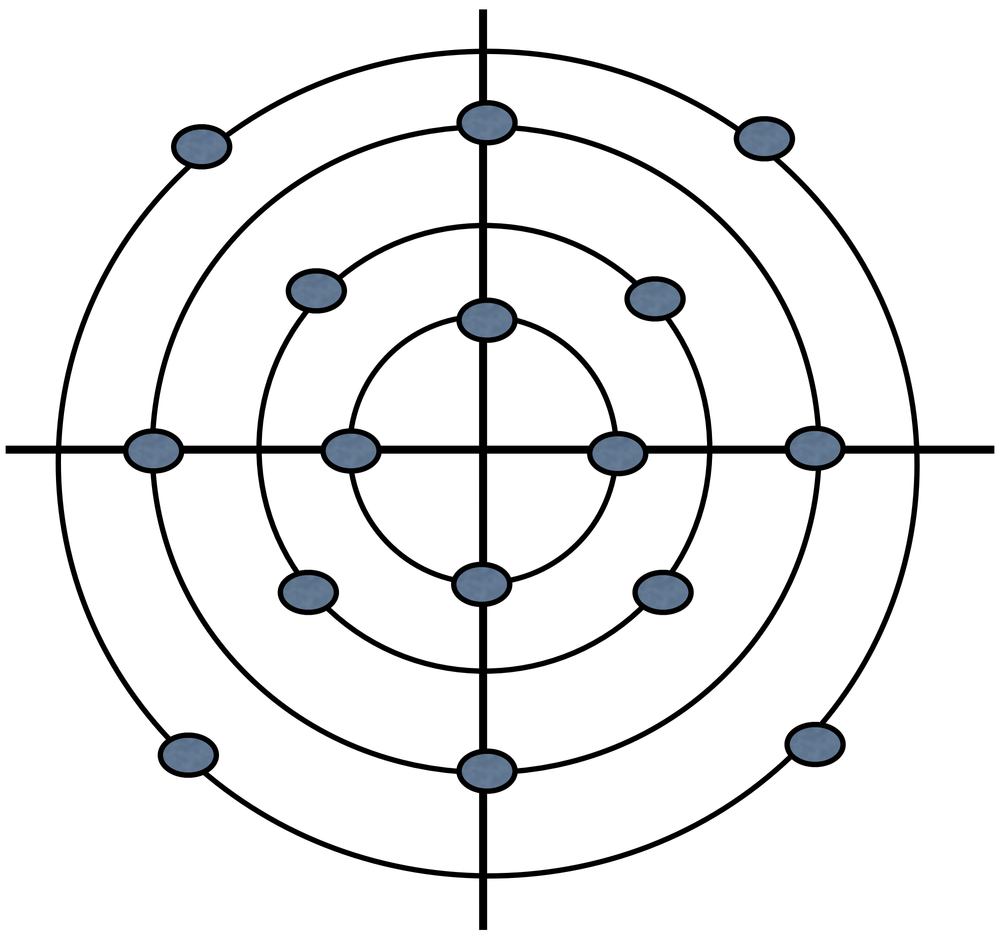
$$\mathbf{s}_m = \left(\sqrt{\mathcal{E}_s} A_{mc}, \quad \sqrt{\mathcal{E}_s} A_{ms}\right)$$

$$\phi_2(t) = \sqrt{\frac{1}{\mathcal{E}_s}} g_T(t) \sin(2\pi f_c t)$$

Example



$M = 8$



$M = 16$

Spectral efficiency

$$u_m(t) = \sqrt{A_{mc}^2 + A_{ms}^2} g_T(t) \cos(2\pi f_c t + \theta_m) = A_m g_T \cos(2\pi f_c t + \theta_n),$$

$$m = 1, 2, \ldots, M_1$$
$$n = 1, 2, ..., M_2$$

- Number of bits per symbol

Let $M_1 = 2^{k_1}$ and $M_2 = 2^{k_2}$

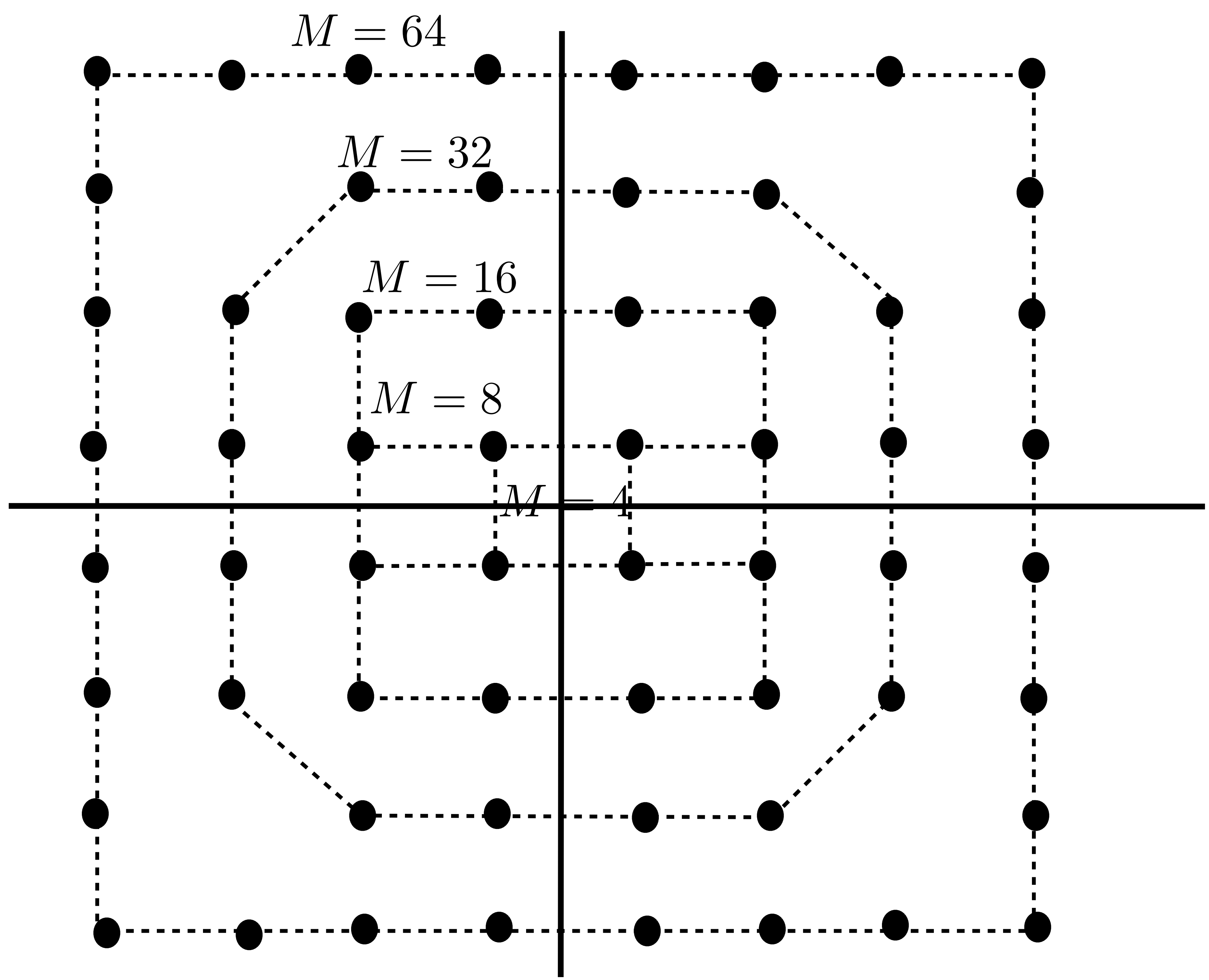
$$k_1 + k_2 = \log_2 M_1 + \log_2 M_2$$ bits/symbol

- Symbol rate

$$R_s = \frac{R_b}{k_1 + k_2}$$

Rectangular QAM

- Signal amplitudes take the set of values $\{(2m-1-M)d,\ \ m=1,2,\ldots,M\}$
- Signal space diagram



Rectangular QAM

- Average energy per symbol

$$\mathcal{E}_{av} = \frac{1}{M}\sum_{i=1}^{M} ||\mathbf{s}_i||^2.$$

- Distance between two symbols

$$d_{mn} = \sqrt{||\mathbf{s}_m - \mathbf{s}_n||^2}.$$

Probability of Error for QAM

- For rectangular signal constellations in which $M = 2^k$ where k is even, the QAM signal constellation is equivalent to two PAM signals on quadrature carriers, each having $\sqrt{M} = 2^{k/2}$ signal points.
  - Since the signals in the phase-quadrature components can be perfectly separated at the demodulator, the probability of error for QAM is easily determined from the probability of error for PAM.
  - Specifically, the probability of a correct decision for the M-ary QAM system is

$$P_c = (1 - P_{\sqrt{M}})^2$$

    - where $P_{\sqrt{M}}$ is the probability of error of an $\sqrt{M}$ -ary PAM with one-half the average power in each quadrature signal of the equivalent QAM system.
  - By appropriately modifying the probability of error for M-ary QAM, we obtain

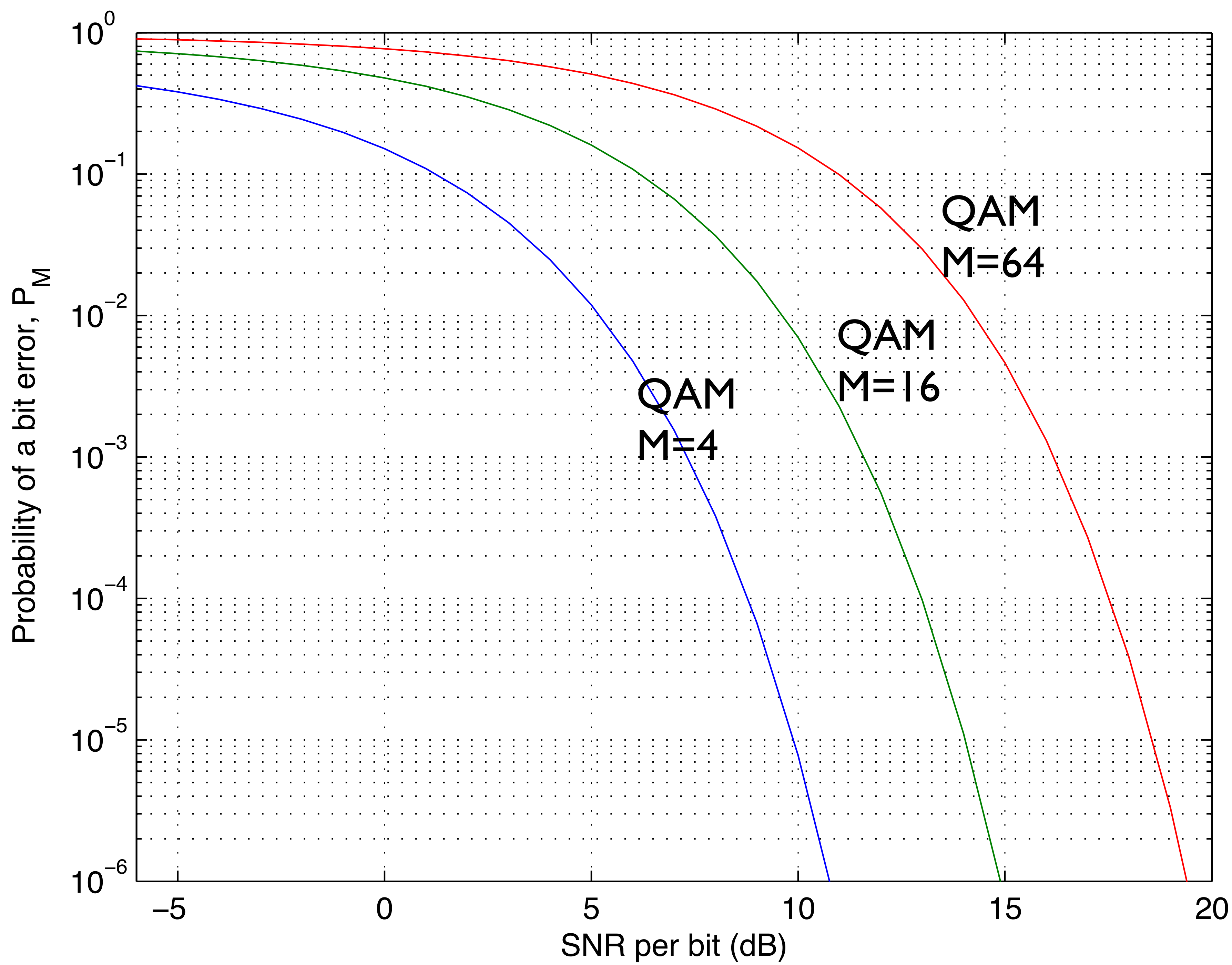
$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_{av}}{N_0}}\right)$$

~ where E_{av}/N_0 is the SNR per symbol.

- Therefore, the probability of a symbol error for the M-ary QAM is

$$P_M = 1 - (1 - P_{\sqrt{M}})^2$$

  - Note that this result is exact for $M = 2^k$ when k is even.



- SER of M-QAM

$$P_M = 1 - (1 - P_{\sqrt{M}})^2 = 1 - (1 - 2P_{\sqrt{M}} + P_{\sqrt{M}}^2)$$

$$\leq 2P_{\sqrt{M}} \quad = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{\mathcal{E}_{av}}{N_0}}\right)$$

$$\leq 4Q\left(\sqrt{\frac{3}{M-1}\frac{\mathcal{E}_{av}}{N_0}}\right)$$

- Comparison with M-PSK

$$P_M \approx 2Q\left(\sqrt{2\rho_s}\sin\frac{\pi}{M}\right)$$

- Define the ratio of the arguments of Q function for the two signal format:

$$\mathcal{R}_M = \frac{\frac{3\mathcal{E}_{av}}{(M-1)N_0}}{2\rho_s \sin^2 \frac{\pi}{M}} \quad = \frac{3/(M-1)}{2\sin^2 \frac{\pi}{M}}$$

- M=4,

  $$\mathcal{R}_4 = 1$$

  - which means the SER performances of QAM and PSK are the same.

- M>4,

  $$\mathcal{R}_M > 1$$

  - which means the SER of QAM is better than the one of PSK.

Advantage of M-ary QAM over M-ary PSK

M	$10\log_{10}\mathcal{R}_M$
8	1.65
16	4.20
32	7.02
64	9.95