Mobile Communications (KECE425)

Lecture Note 21

5-21-2014

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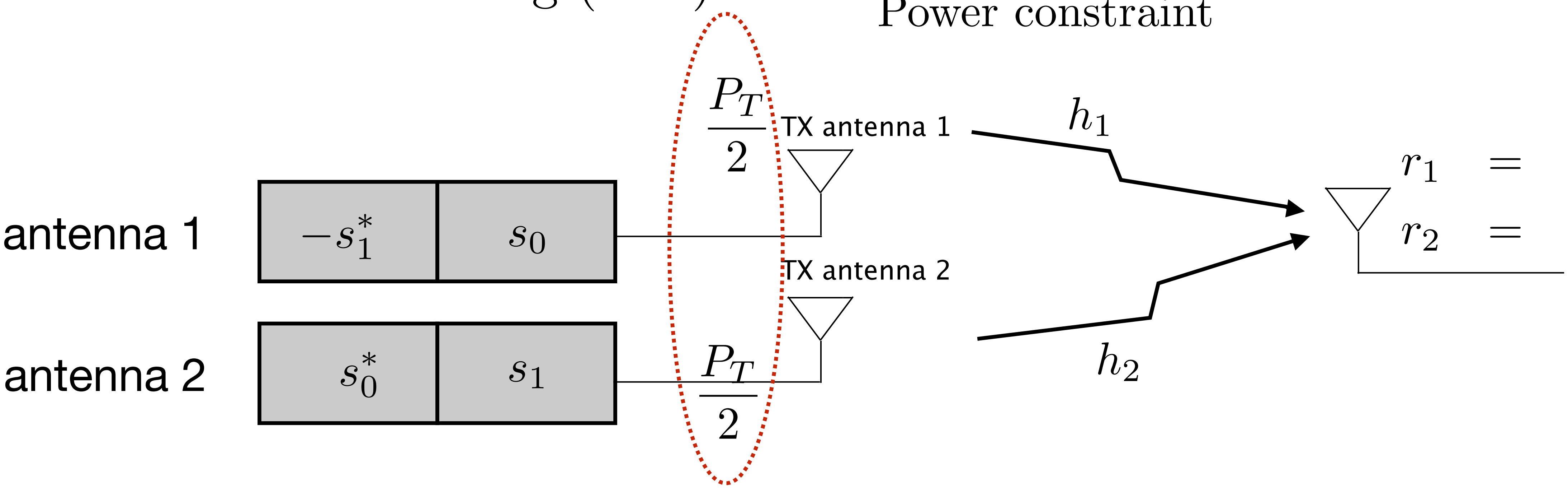
Summary

- Complexity issues of diversity systems
    - ADC and Nyquist sampling theorem
- Transmit diversity
    - Channel is known at the transmitter (Closed-loop transmit diversity: CLTD)
    - Channel is unknown at the transmitter (Space-time block coding: STBC)
- Transmit-Receive diversity (Maximal ratio transmission)
- Multi-user opportunistic diversity

Open-Loop Transmit Diversity

- There are many open loop transmit diversity schemes.
- Out of them, we only study the space-time block coding (STBC) with dual transmit antennas.
- Alamouti devised the STBC with two antennas in 1998 and it is often called as Alamouti coding.

- Alamouti coding (2×1)



$$
\begin{aligned}
r_1 &= s_0 h_1 + s_1 h_2 + n_1 \\
r_2 &= -s_1^* h_1 + s_0^* h_2 + n_2
\end{aligned}
$$

[Space-time block code (STBC)

	antenna1	antenna2
time t	s_0	s_1
time t+T	$-s_1^*$	s_0^*

- QPSK example

$$
\begin{aligned}
00 \rightarrow s^1 &= 1+j \\
01 \rightarrow s^2 &= 1-j \\
11 \rightarrow s^3 &= -1+j \\
10 \rightarrow s^4 &= -1-j
\end{aligned}
$$

[Space-time block code (STBC)

	antenna1	antenna2
time t	s_0	s_1
time t+T	$-s_1^*$	s_0^*

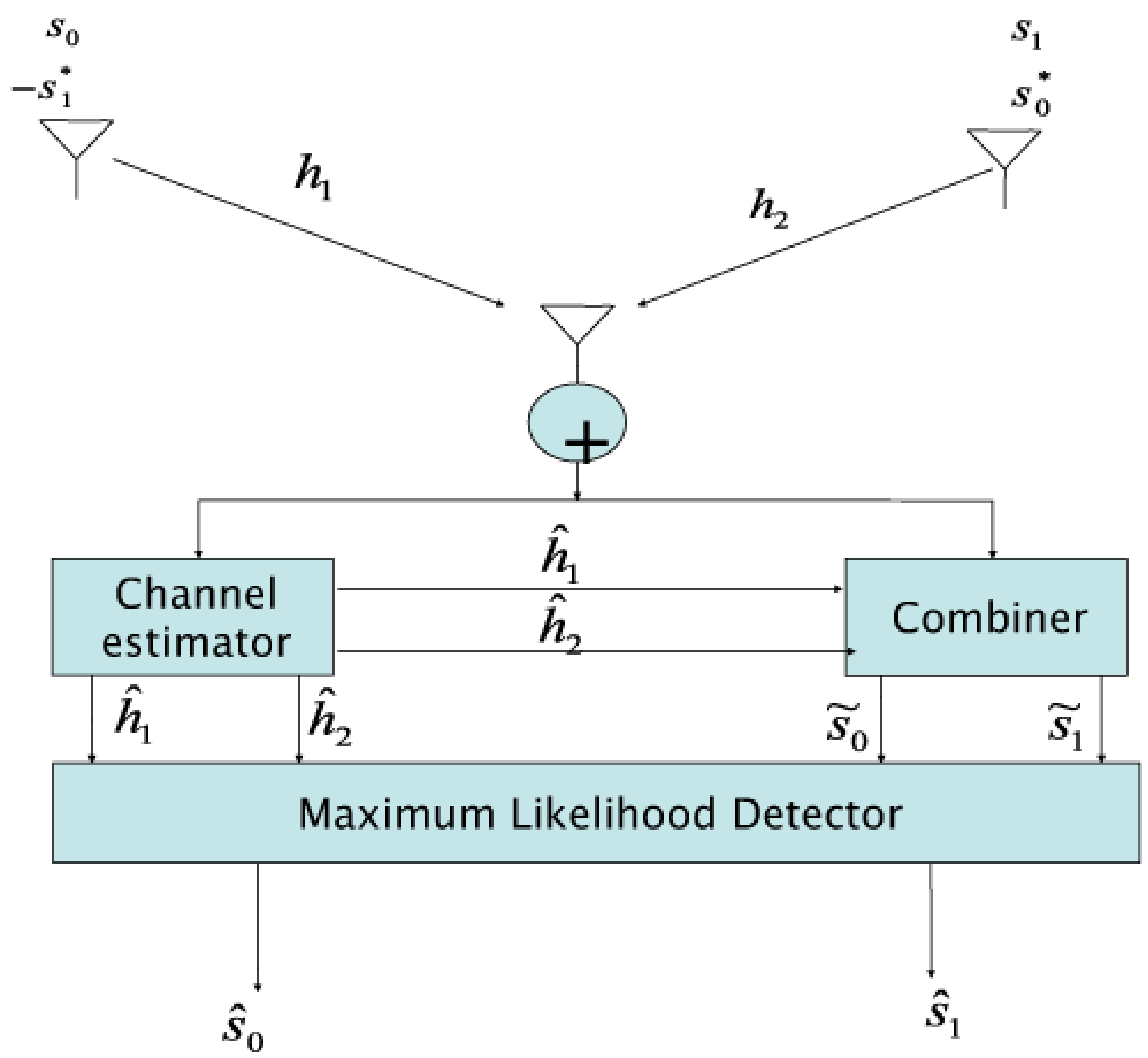
data: $01001011\cdots \quad \Longrightarrow \quad s_0 s_1 s_2 s_4 \cdots = s^2 s^1 s^4 s^3 \cdots$

time	ant1	ant2
T	$1-j$	$1+j$
$2T$	$-1+j$	$1+j$
$3T$	$-1-j$	$-1+j$
$4T$	$1+j$	$-1+j$

$\vdots$

$$
\begin{aligned}
r_1 &= s_0 h_1 + s_1 h_2 + n_1 \\
r_2 &= -s_1^* h_1 + s_0^* h_2 + n_2 \\
r_3 &= s_2 h_1 + s_3 h_2 + n_1 \\
r_4 &= = -s_3^* h_1 + s_2^* h_2 + n_2
\end{aligned}
$$

- Detection of space-time block coding signal



$$\begin{aligned} r_1 &= s_0 h_1 + s_1 h_2 + n_1 \\ r_2 &= -s_1^* h_1 + s_0^* h_2 + n_2 \end{aligned}$$

$$\begin{aligned} v_1 &= h_1^* r_1 + h_2 r_2^* \\ v_2 &= h_2^* r_1 - h_1 r_2^* \end{aligned}$$

$$\begin{aligned} v_1 &= (|h_1|^2 + |h_2|^2) s_0 + h_1^* n_1 + h_2 n_2^* \\ v_2 &= (|h_1|^2 + |h_2|^2) s_1 + h_2^* n_1 - h_2 n_2^* \end{aligned}$$

- Received signal vector
  - At odd time

$$v_{2k-1} = \left(|h_1|^2 + |h_2|^2\right) s_0 + h_1^* n_1 + h_2 n_2^*$$

* SNR

$$\gamma_t = \frac{\left(|h_1|^2 + |h_2|^2\right)^2 E_s/2}{N_0 \left(|h_1|^2 + |h_2|^2\right)} = \frac{\left(|h_1|^2 + |h_2|^2\right) E_s/2}{N_0} = \frac{\gamma_1 + \gamma_2}{2}$$

  - At even time

$$v_{2k} = \left(|h_1|^2 + |h_2|^2\right) s_1 + h_2^* n_1 - h_2 n_2^*$$

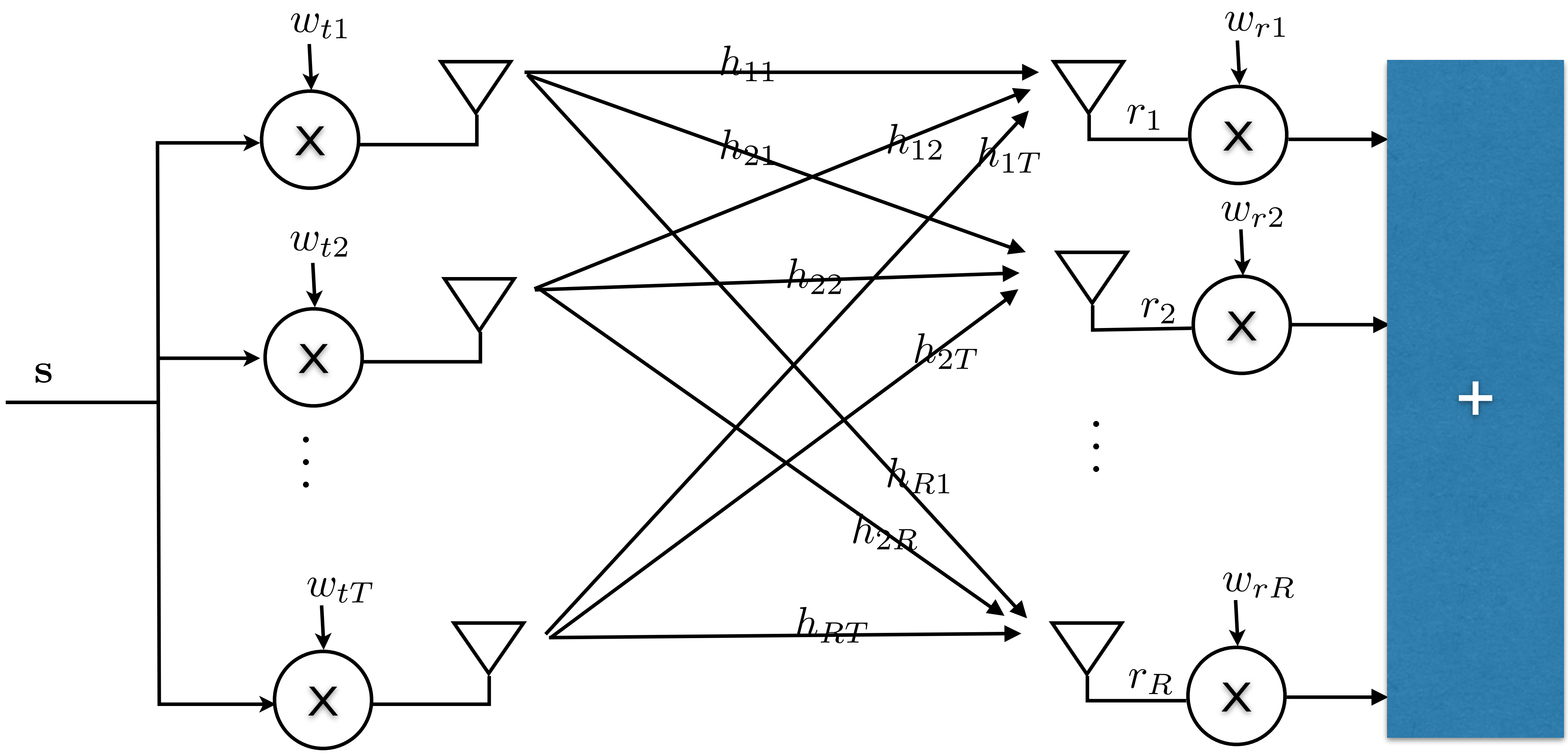
* SNR

$$\gamma_t = \frac{\gamma_1 + \gamma_2}{2}$$

- Performance of STBC scheme
  - It gives the same performance as CLTD with two antennas only when each of diversity branch has the same average SNR, $\bar{\gamma}_l = \bar{\gamma}$ for all l (that is, i.i.d.).

Maximal Ratio Transmission (MRT)

- MRT is also called multiple input multiple output (MIMO)-MRC.



- MIMO channel can be represented in matrix form:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1T} \\ h_{21} & h_{22} & \cdots & h_{2T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{R1} & h_{R2} & \cdots & h_{RT} \end{bmatrix}$$

- Vector representation

$$\mathbf{w}_t = [w_{t1} \; w_{t2} \; \cdots \; w_{tT}]^T$$

$$\mathbf{w}_r = [w_{r1} \; w_{r2} \; \cdots \; w_{rR}]^T$$

$$\mathbf{n} = [n_1 \; n2 \; \cdots \; n_R]^T$$

- Received signal:

$$
\begin{aligned}
r_1 &= (w_{t,1}h_{11} + w_{t,2}h_{12} + \cdots + w_{t,T}h_{1T})s + n_1 \\
r_2 &= (w_{t,1}h_{21} + w_{t,2}h_{22} + \cdots + w_{t,T}h_{2T})s + n_2 \\
&\vdots \\
r_R &= (w_{t,1}h_{R1} + w_{t,2}h_{R2} + \cdots + w_{t,T}h_{RT})s + n_R
\end{aligned}
$$

  - Received signal in vector form:

$$\mathbf{r} = \mathbf{H}\mathbf{w}_t s + \mathbf{n}$$

- Combined signal:

$$r_t = \mathbf{w}_r \mathbf{r}$$

- Optimal receive weight vector $\mathbf{w}_r$ can be easily shown to be given as

$$\mathbf{w}_r = c\,(\mathbf{H}\mathbf{w}_t)^H = c\mathbf{w}_t^H \mathbf{H}^H$$

where $(\cdot)^H$ denote the Hermitian operation.

  - In this case, the received signal can be written as

$$\begin{aligned} r_t &= \mathbf{w}_r \mathbf{r} \\ &= \mathbf{w}_r(\mathbf{H}\mathbf{w}_t s + \mathbf{n}) \\ &= c\mathbf{w}_t^H \mathbf{H}^H \mathbf{H}\mathbf{w}_t s + c\mathbf{w}_t^H \mathbf{H}^H \mathbf{n} \end{aligned}$$

- SNR of the received signal
  - Received signal can be written as

$$r_t = c\mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t s + c\mathbf{w}_t^H \mathbf{H}^H \mathbf{n}$$

  - SNR of r_t

$$\gamma_t = \frac{1}{\sigma_n^2} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t$$

- Optimal transmit weight vector, $\mathbf{w}_t^{\text{opt}}$

$$\begin{aligned} \mathbf{w}_t^{\text{opt}} &= \max_{\mathbf{w}_t} \gamma_t \\ &= \max_{\mathbf{w}_t} \frac{1}{\sigma_n^2} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t \\ &= \max_{\mathbf{w}_t} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t \end{aligned}$$

- Find the optimal weight vector $\mathbf{w}_t$ to maximize the SNR γ_t.

$$\mathbf{w}_t^{\text{opt}} = \max_{\mathbf{w}_t} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t$$

  - We can solve this problem by making use of Rayleigh-Ritz theorem.

- Rayleigh-Ritz theorem

$$\mathbf{x}^H \mathbf{A} \mathbf{x} \leq ||\mathbf{x}|| \lambda_{\max}$$

where $\mathbf{A}$ is the Hermitian matrix, $\mathbf{x}$ is an y non-zero complex vector and $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{A}$.

- Equality holds if and only if $\mathbf{x}$ is the eigenvector corresponding to $\lambda_{\max}$.

- Based on Rayleigh-Ritz theorem, we can find the optimal weight vector $\mathbf{w}_t^{\text{opt}}$, we can find the optimal weight vector as

$$\mathbf{w}_t^{\text{opt}} = \sqrt{\Omega} \mathbf{U}_{\max}$$

where $\mathbf{U}_{\max}$ is the eigenvector corresponding to the largest eigenvalue of the quadratic form $\mathbf{F} = \mathbf{H}^H \mathbf{H}$ and $\mathbf{U}_{\max}^H \mathbf{U}_{\max} = \mathbf{I}$

- Combined SNR with the optimum weight vector

$$\gamma_t = \frac{\Omega \lambda_{\max}}{\sigma_n^2}$$

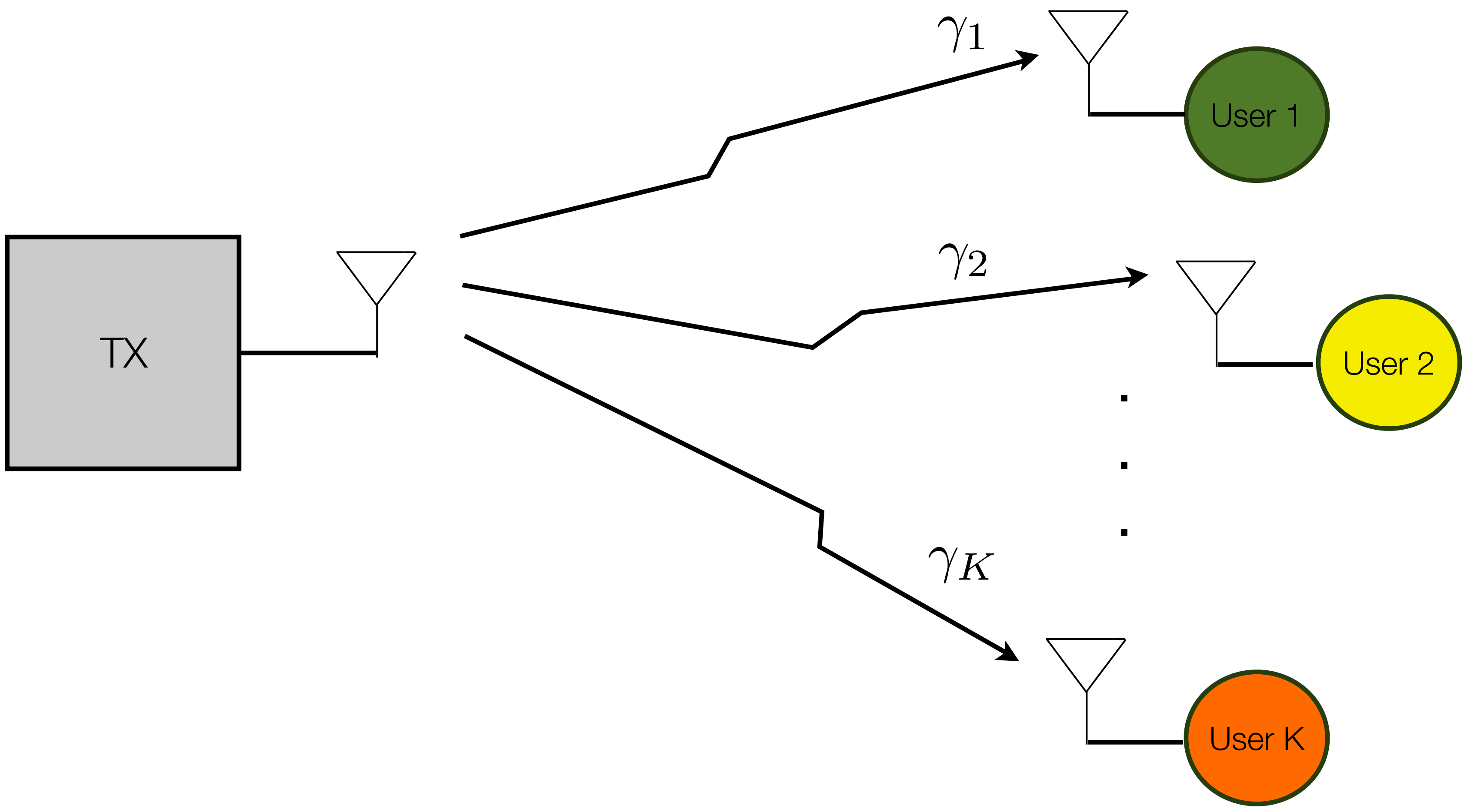
Multi-User Opportunistic Diversity

- We often need to select users if there are more than users to support the service, for a certain limited frequency (or/and time) resource.

- Example:
  - There are 50 MHz bandwidth for the service and each user takes 5 MHz bandwidth. In this case, we can support 10 users for a given time.
  - However, more than 50 users, saying 100 users, are willing to communicate at the same time, what is the best way to select users among 100 users?

- Multi-user opportunistic diversity scheme is simply to select the users with the strongest SNRs.

- Schematic concept of multi-user diversity (MUD).



- Choose the user which has the largest SNR among K users.

- If one user is selected out of K users at every selection period, the selected user k^* can be written as

$$k^* = \max_k(\gamma_1, \gamma_2, \cdots, \gamma_K)$$

- By doing this, we can improve the channel capacity such as

$$\begin{aligned} C &= E\left[\log_2(1+\gamma_k^*)\right] \\ &= \int_0^\infty \log_2\left(1+\gamma_k^*\right) p_{\gamma_{k^*}}\left(\gamma_{k^*}\right) d\gamma_{k^*} \end{aligned}$$

- Multi-user diversity gain