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- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

Chap. 11 Static Equilibrium

Equilibrium conditions

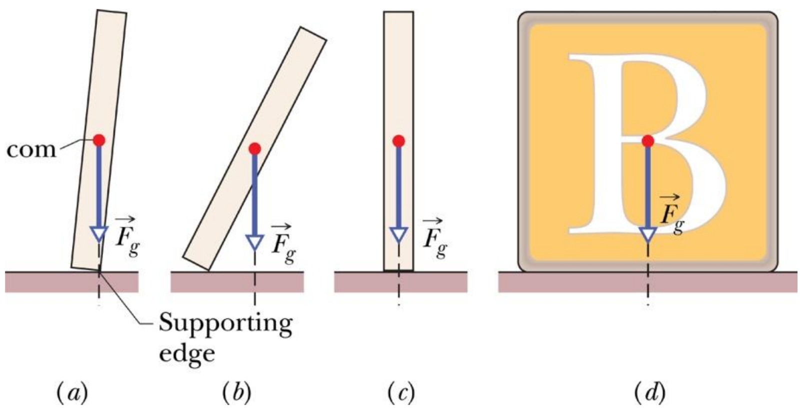
$$\sum_i \vec{F}_{\text{net}} = 0$$

$$\sum_i \vec{\tau}_{\text{net}} = 0$$

What is equilibrium?

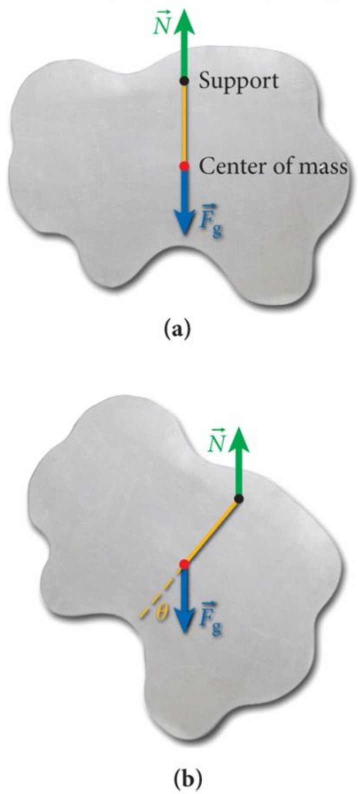
1. C.o.m의 linear momentum $\vec{P}$가 일정.
2. 임의의 점에 대한 angular momentum $\vec{L}$이 일정

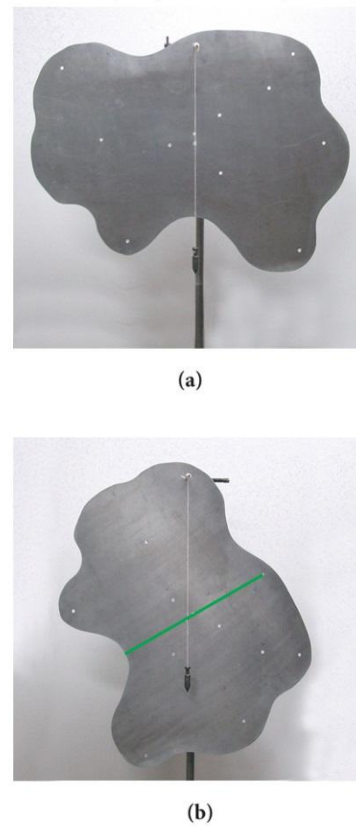
만일 $\vec{P} = 0, \ \vec{L} = 0$ 이라면 static equilibrium 이라고 한다.



Unstable equilibrium Stable equilibrium

Finding COM





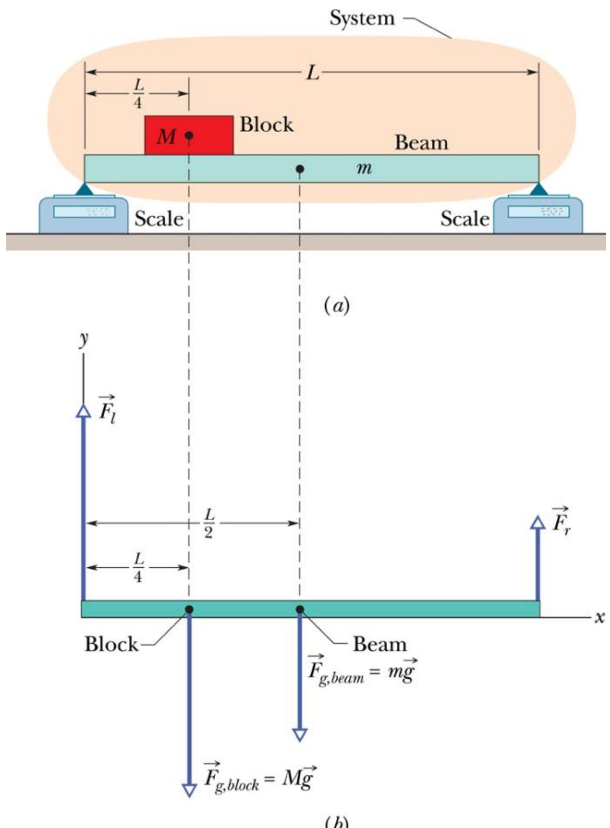
Equilibrium conditions

$$\vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt}$$

$$\vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt}$$

$$\vec{F}_{\text{ext}} = 0$$ (힘의 균형)

$$\vec{\tau}_{\text{ext}} = 0$$ (torque의 균형)



힘 $$F_l + F_r - Mg - mg = 0$$

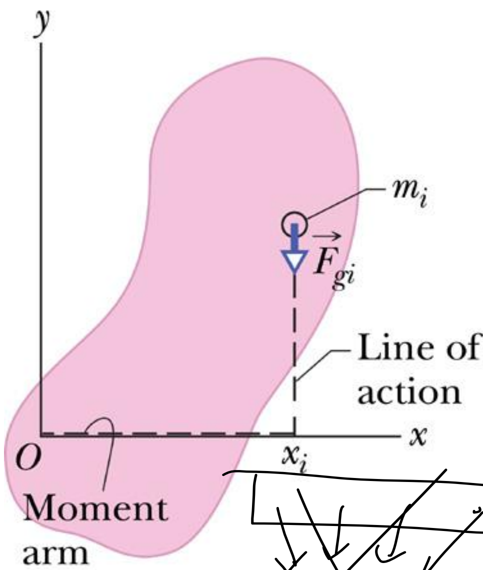
$$F_r = \frac{Mg}{4} + \frac{mg}{2}$$

왼쪽 끝에 대한 torque

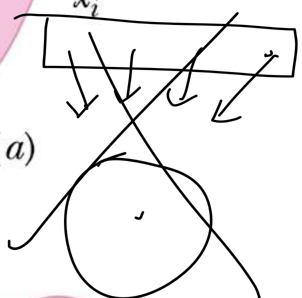
$$0 \cdot F_l - \frac{L}{4}Mg - \frac{L}{2}mg + LF_r = 0$$

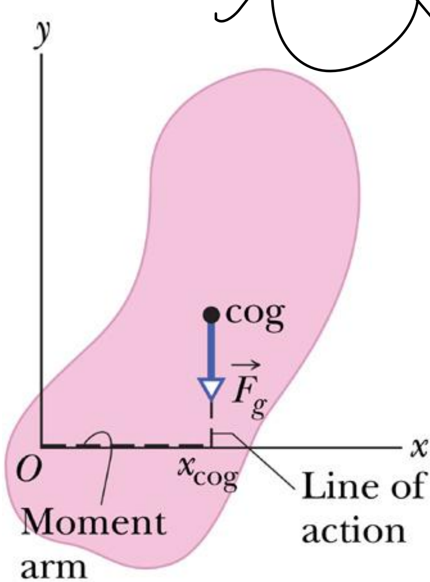
$$F_r = Mg/4 + mg/2$$

$$F_l = 3Mg/4 + mg/2$$



(a)





(b)

Center of gravity

1. 물체에 작용하는 중력은 사실상 **center of gravity**라는 한 점에 작용하는 것으로 볼 수 있다.
2. 중력이 물체의 모든 부분에 대해 같으면 com과 cog는 같다.

$$\tau_i = x_i F_{gi} \longrightarrow \tau_{\text{net}} = \sum \tau_i = \sum x_i F_{gi}$$

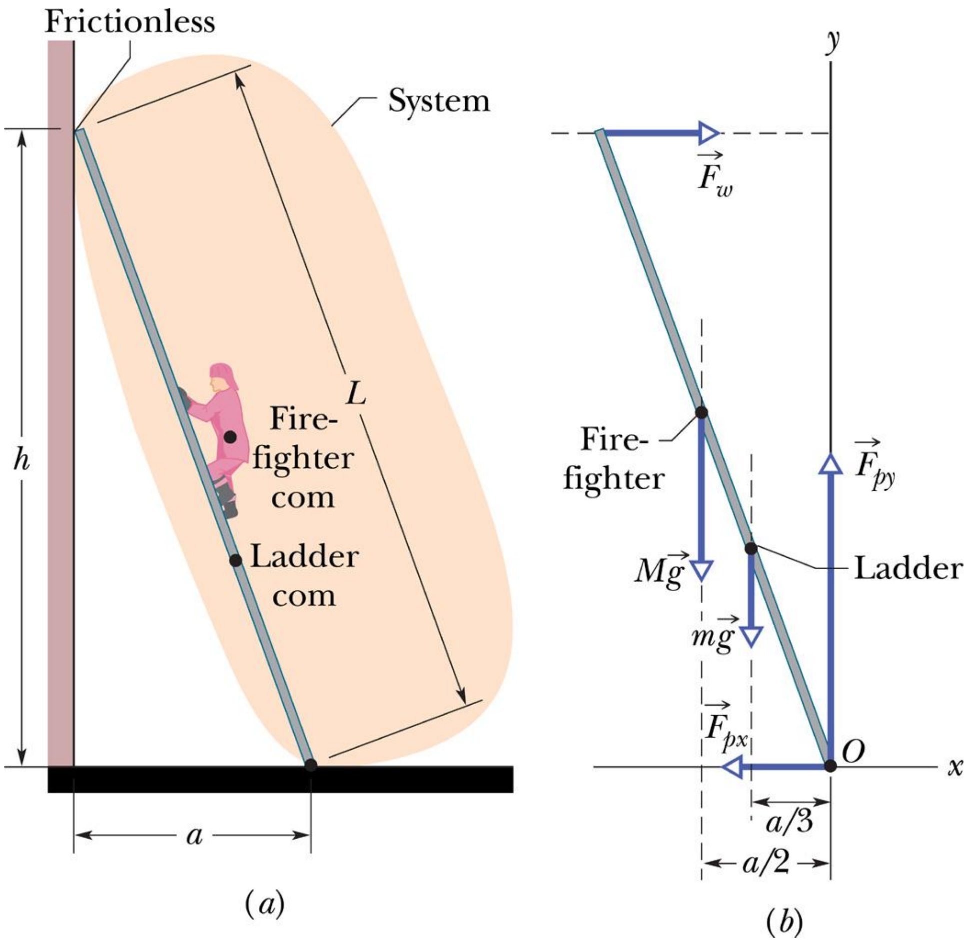
$$\tau = x_{\text{cog}} \sum F_{gi}$$

$$x_{\text{cog}} \sum F_{gi} = \sum x_i F_{gi} = \sum x_i m_i g_i$$

만일 g_i가 모든 점에 대해 같다면 $x_{\text{cog}} \sum m_i = \sum x_i m_i$

$$x_{\text{cog}} = \frac{1}{M} \sum m_i x_i \longrightarrow x_{\text{cog}} = x_{\text{com}}$$

Sample prob.



$F_w = F_{px}$

$F_{py} = (M+m)g$

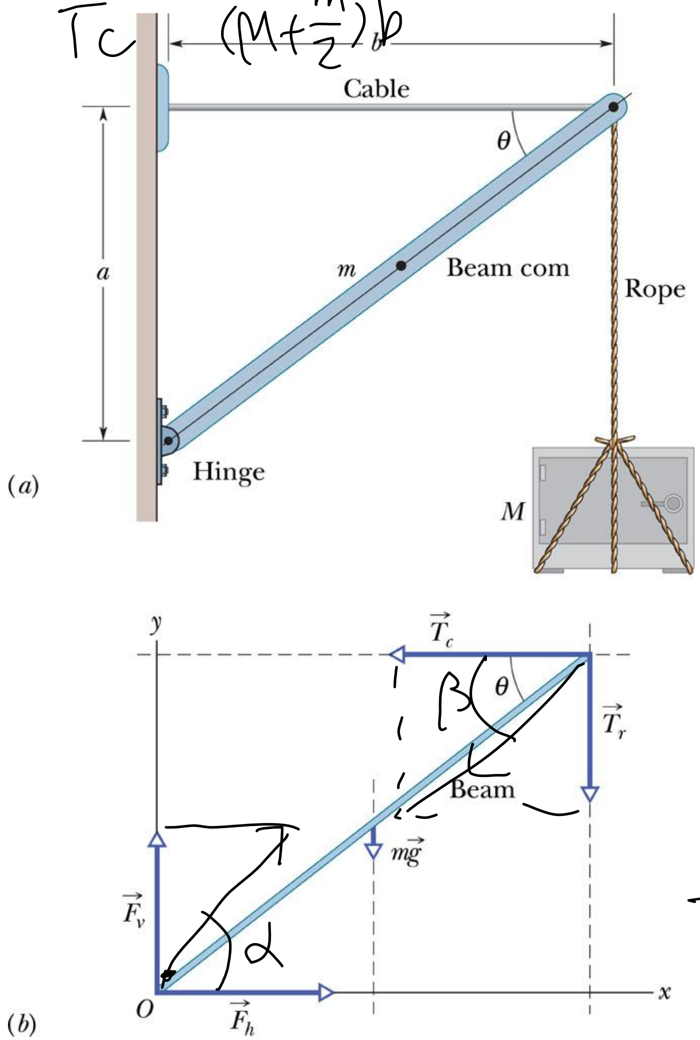
$F_w h = Mg\frac{a}{2} + mg\frac{a}{3}$

$F_w = F_{px} = ga\left(\frac{M}{2} + \frac{m}{3}\right)$

$$\tan\alpha = \frac{F_v}{F_h} = \frac{(M+m)g}{(M+\frac{m}{2})gb/a} = \frac{a(M+m)}{b(M+\frac{m}{2})}$$

Sample prob.

$$\tan\beta = \frac{T_r}{T_c} = \frac{Ma}{(M+\frac{m}{2})b}$$



~~$T_r = F_h$~~

~~$F_v = T_r$~~

$T_c = F_h$

$F_v = T_r + mg$

$$T_c a = T_r b + mg\frac{b}{2}$$

$$F_v b = mg\frac{b}{2} + F_h a$$

$$F_h a = (F_v - mg)b + \frac{mgb}{2}$$

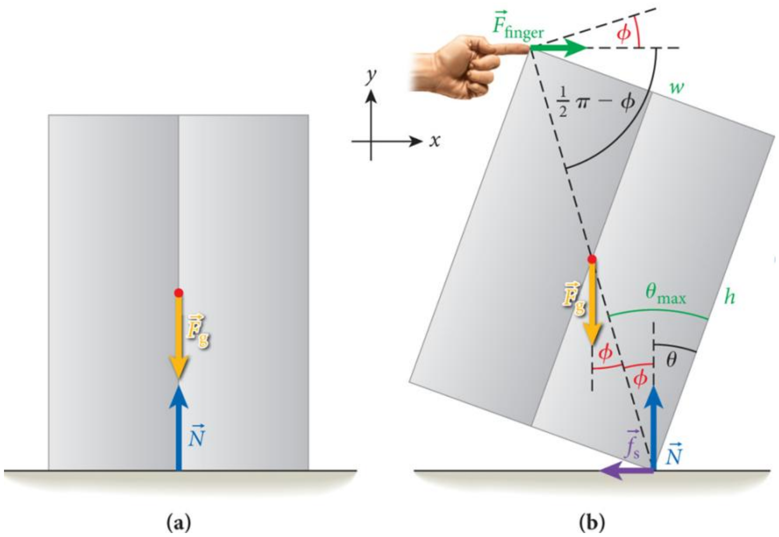
$$= \left(F_v - \frac{mg}{2}\right)b \rightarrow \left(M+\frac{m}{2}\right)gb$$

∴

$T_r = Mg$, $F_v = (M+m)g$

$$F_h = T_c = \left(M+\frac{m}{2}\right)g\frac{b}{a}$$

Ex. 11.5 Pushing a box



$N = mg$

$F_{\text{finger}} = f_s$

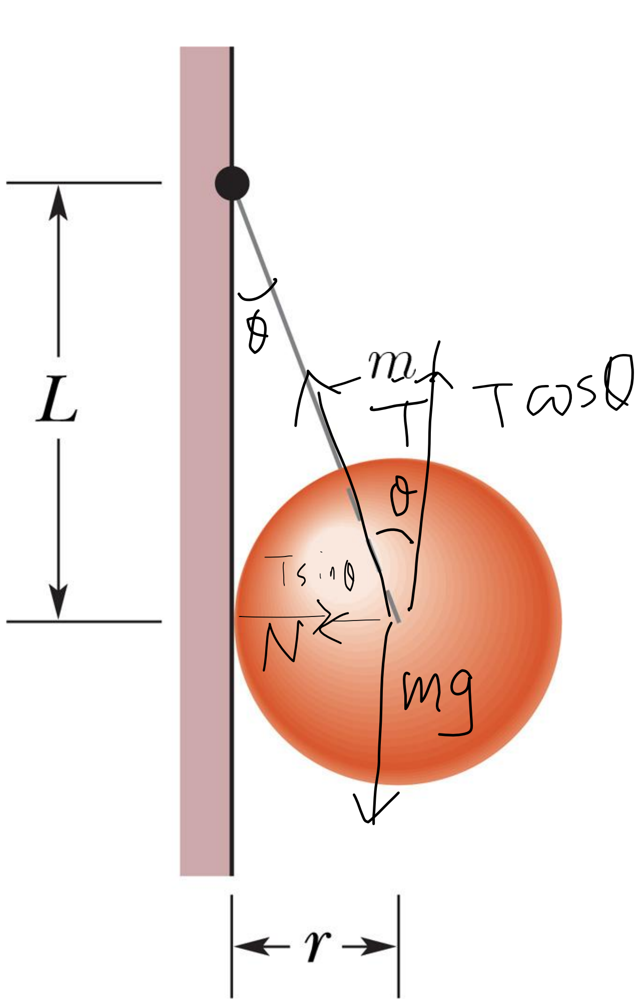
$$\theta_{\text{max}} = \tan^{-1}\frac{w}{h}$$

$$F_{\text{finger}} = \frac{mg}{2}\tan\phi$$

$$F_g(l/2)\sin\phi - F_{\text{finger}}\, l \sin(\pi/2 - \phi) = 0$$

$$F_{\text{finger}} = \frac{1}{2}mg\tan\left[\tan^{-1}\frac{w}{h} - \theta\right]$$

Prob. 1



(a) 줄의 tension

$$N = T\sin\theta$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{N}{mg} \qquad T = \frac{mg\,L}{\sqrt{L^2 + r^2}}$$

$$N = mg\,\frac{r}{L}$$

(b) 공이 벽에 가하는 힘

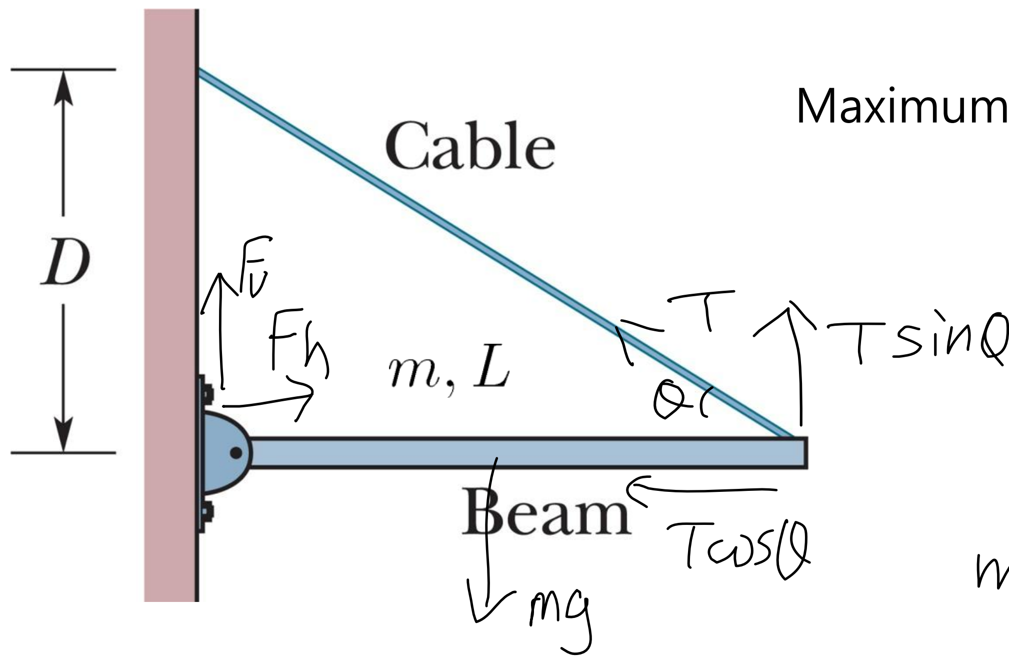
$$F_{bw} = mg\,\frac{r}{L}$$

$$D = \frac{L}{\sqrt{\frac{4T^2}{m^2g^2} - 1}}$$

Prob. 2

$$\left(-1 + \left(\frac{2T}{mg}\right)^2\right) D^2 = L^2$$

Maximum tension T, 끊어지지 않을 거리 D?



$$T\cos\theta = F_h$$

$$T\sin\theta + F_v = mg$$

$$mg\frac{L}{2} = LT\sin\theta$$

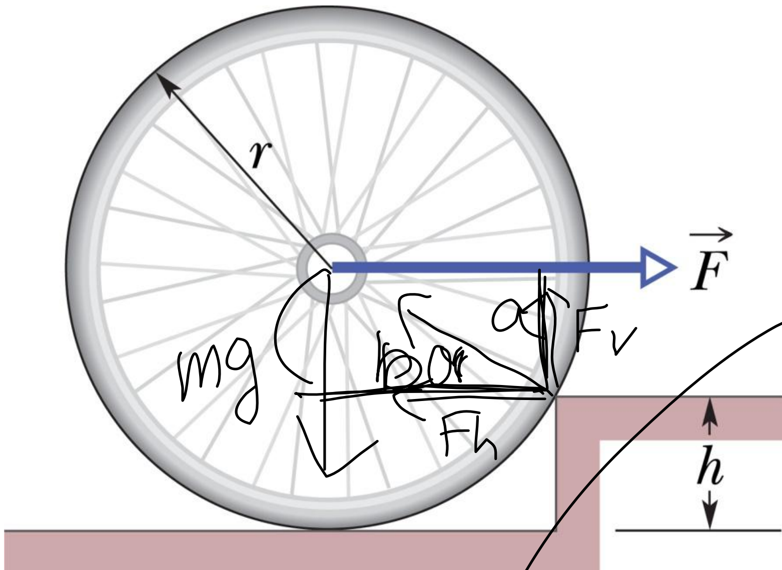
$$T = \frac{mg}{2\sin\theta}$$

$$\sin\theta = \frac{mg}{2T} = \frac{D}{\sqrt{L^2+D^2}}$$

$$L^2 + D^2 = \left(\frac{mg}{2T}\right)^{-2} D^2$$

Prob. 3

$$\frac{F_v}{F_h} = \frac{mg}{F} = \frac{r-h}{\sqrt{2rh-h^2}} = \frac{a}{b}$$



$F = F_h$

$F_v = mg$

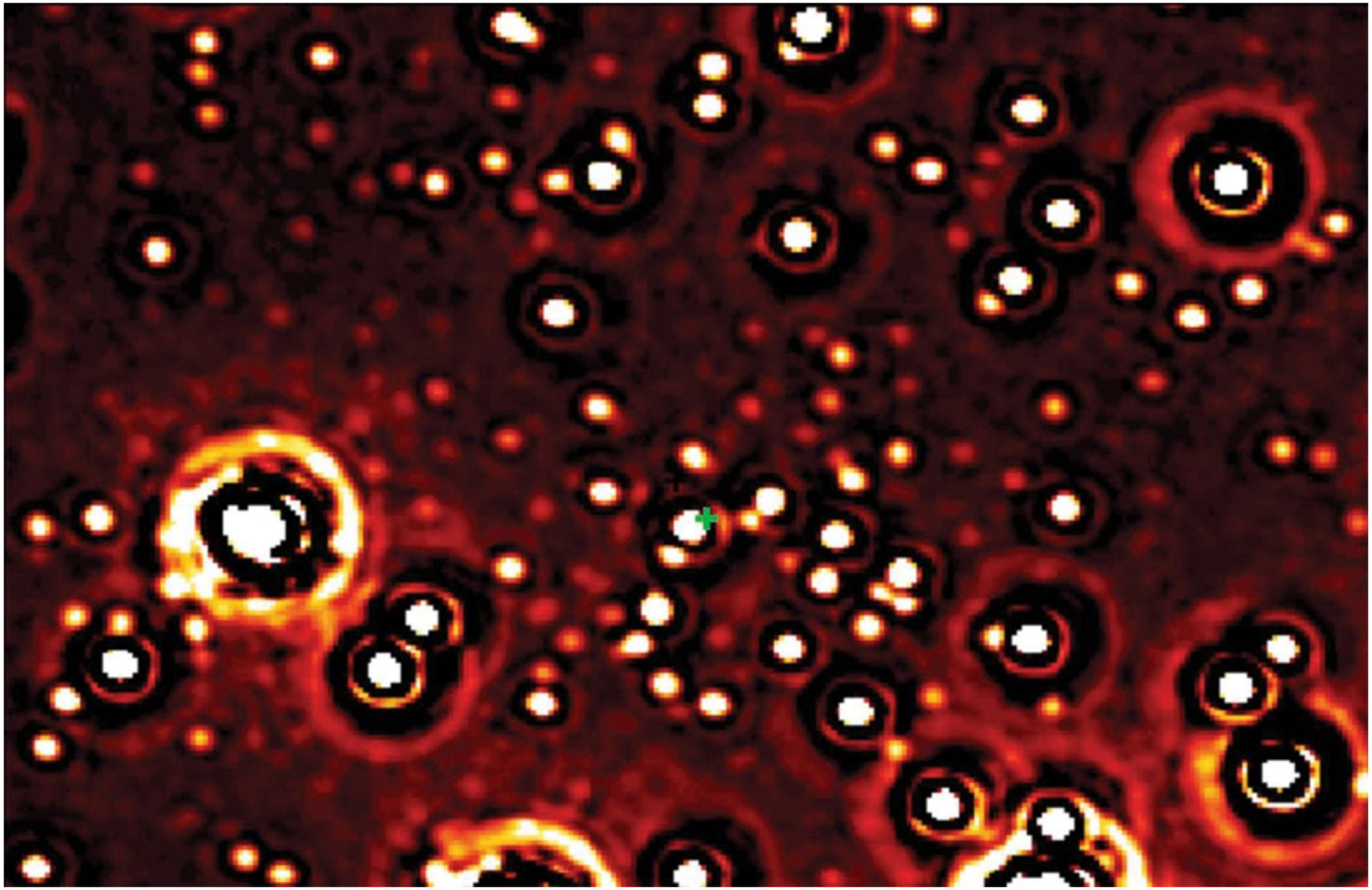
$$Fa = mg\,b$$

$a = r-h$

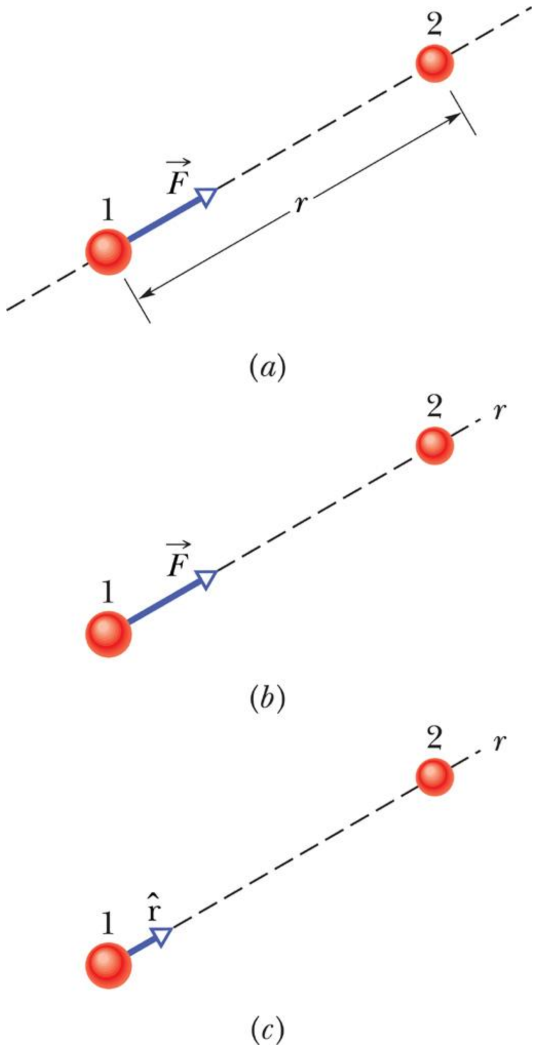
$b^2 = r^2 - (r-h)^2 = 2rh - h^2$

$$F = mg\frac{\sqrt{2rh-h^2}}{r-h}$$

Chap. 12 Gravitation



Newton's law of gravitation



$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r}$$

Gravitation constant G

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

$$= 6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2}$$

$[G] = L^3 M^{-1} T^{-2}$

Newton's shell theorem

Shell theorem

(1) 공 모양의 균일한 껍질은 마치 모든 질량이 중심에 모여있는 것처럼 외부의 입자를 잡아당긴다.

(2) 공 모양의 균일한 껍질 내부에 있는 입자는 이 껍질에 의한 중력이 상쇄되어 없어진다.

Gravitation and superposition principle

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}$$

$$= \sum_{i=2}^{n} \vec{F}_{1i}$$

연속적인 물체의 경우

$$\vec{F}_1 = \int d\vec{F}$$