

Wireless Communications (ITC731)

Lecture Note 6

9-April-2013

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Summary

- Receive diversity
 - Maximal ratio combining
 - Equal gain combining
 - Selection combining
 - Switched combining

- Transmit diversity
 - Channel known at the transmitter
 - Channel unknown at the transmitter

- Transmit-Receive diversity

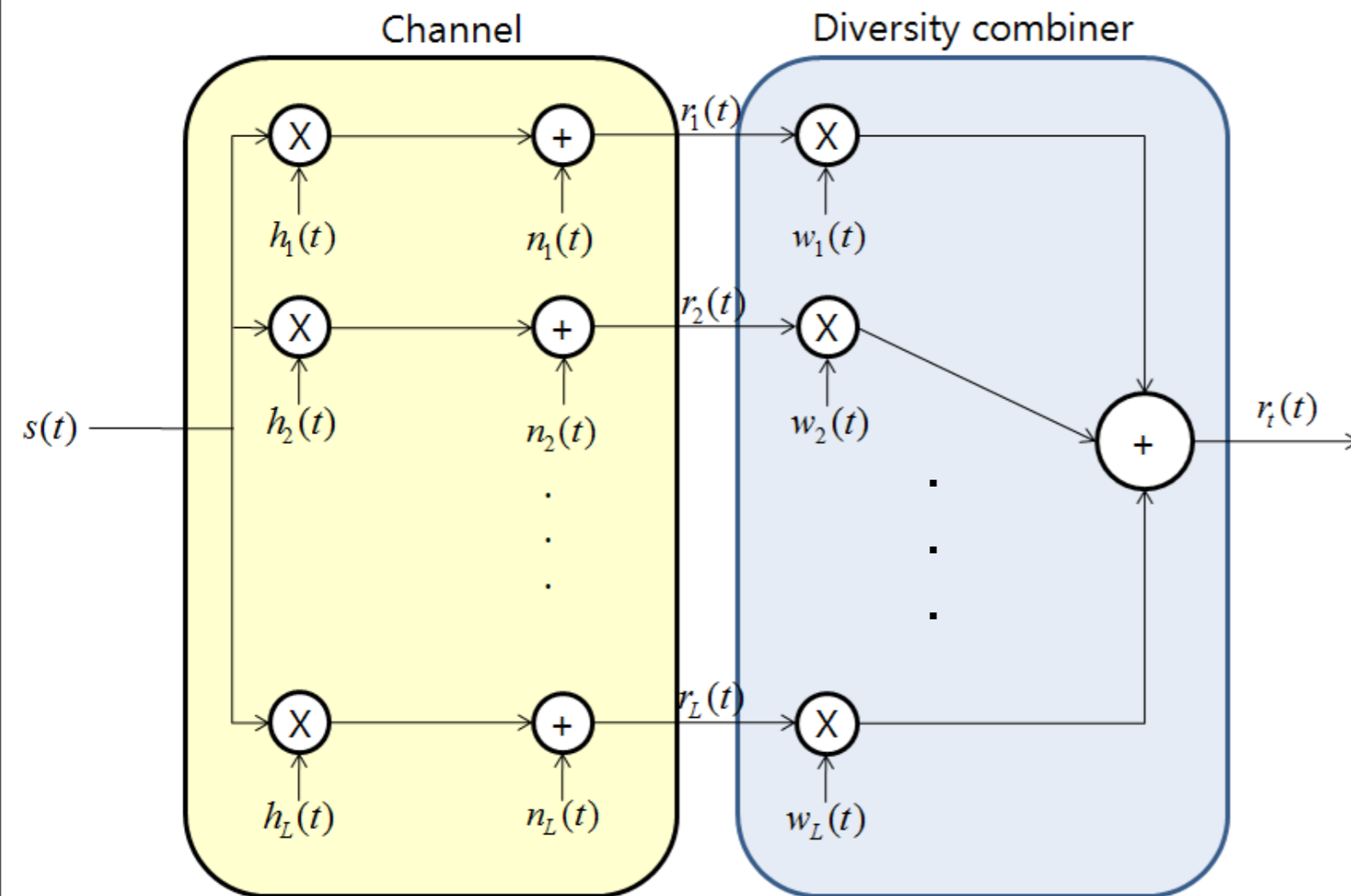
- Multi-user diversity

Receiver Diversity

- Maximal ratio combining
- Equal gain combining
- Selection combining
- Switched combining

Linear Receiver Antenna Diversity Combining

■ Block diagram



Combined received signal

$$\begin{aligned} r_t(t) &= \sum_{l=1}^L w_l(t) r_l(t) \\ &= \sum_{l=1}^L w_l(t) (h_l(t) s(t) + n_l(t)) \end{aligned}$$

Maximal Ratio Combining

- MRC maximizes the received signal-to-noise ratio.
 - ~ What is the optimum weight vector to maximize the SNR of the combined signal $r_t(t)$?
- Optimum weight vector

$$r_t(t) = \sum_{l=1}^L w_l r_l(t) = \sum_{l=1}^L w_l h_l(t) s(t) + \sum_{l=1}^L w_l n_l(t)$$

- ~ Combined output SNR

$$\gamma_t = \frac{|\sum_{l=1}^L w_l h_l|^2 E_s}{\sum_{l=1}^L |w_l|^2 N_0} = \frac{E_s}{N_0} \frac{|\sum_{l=1}^L w_l h_l|^2}{\sum_{l=1}^L |w_l|^2}$$

- ~ Cauchy-Schwartz inequality

$$\left| \sum_{l=1}^L w_l h_l \right|^2 \leq \left| \sum_{l=1}^L w_l \right|^2 \left| \sum_{l=1}^L h_l \right|^2$$

Equality hold iff $w_l = c h_l^*(t)$ with an arbitrary constant value of c .

- Using the optimal weight vector, we have

$$\gamma_t \leq \frac{E_s}{N_0} \sum_{l=1}^L |h_l|^2 = \frac{E_s}{N_0} \sum_{l=1}^L \alpha_l^2 = \sum_{l=1}^L \gamma_l$$

where $\gamma_l = \frac{\Omega_l E_s}{N_0}$, that is, SNR at each branch.

- Maximal ratio combining

$$\gamma_t = \sum_{l=1}^L \gamma_l$$

$$w_l = c h_l^*(t) \text{ for an arbitrary constant value of } c$$

Received Output SNR of MRC

Received output SNR of MRC

$$\gamma_t = \sum_{l=1}^L \gamma_l$$

~ Average received output SNR

$$\bar{\gamma}_t = \sum_{l=1}^L \bar{\gamma}_l$$

- If $\Omega_l = \Omega$ for $l = 1, \dots, L$ (identical channels) and hence, $\bar{\gamma}_l = \bar{\gamma}$

$$\bar{\gamma}_t = L\bar{\gamma}$$

Average BER/SER of MRC

■ Average BER/SER

$$P(e) = \int_0^{\infty} P(e|\gamma_t) p_{\gamma_t}(\gamma_t) d\gamma_t$$

~ For example for BPSK,

$$P(e|\gamma_t) = Q\left(\sqrt{2\gamma_t}\right) = Q\left(\sqrt{2 \sum_{l=1}^L \gamma_l}\right)$$

~ or it can be written as

$$P(e) = \underbrace{\int_0^{\infty} \cdots \int_0^{\infty}}_{L\text{-fold}} P(e|\gamma_1, \cdots, \gamma_L) p_{\gamma_1, \cdots, \gamma_L}(\gamma_1, \cdots, \gamma_L) d\gamma_1 \cdots d\gamma_L$$

~ For independent channels,

$$p_{\gamma_1, \cdots, \gamma_L}(\gamma_1, \cdots, \gamma_L) = \prod_{l=1}^L p_{\gamma_l}(\gamma_l)$$

■ Note that

$$\begin{aligned} Q(\sqrt{2\gamma_t}) &= Q\left(\sqrt{2\sum_{l=1}^L \gamma_l}\right) \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left[-\frac{\sum_{l=1}^L \gamma_l}{\sin^2 \phi}\right] d\phi \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{l=1}^L \exp\left[-\frac{\gamma_l}{\sin^2 \phi}\right] d\phi \end{aligned}$$

■ Average BER of BPSK using MRC

$$\begin{aligned} P(e) &= \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{l=1}^L \exp\left[-\frac{\gamma_l}{\sin^2 \phi}\right] \prod_{l=1}^L p_{\gamma_l}(\gamma_l) d\gamma_1 \cdots d\gamma_L \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left\{ \int_0^\infty e^{-\frac{\gamma_1}{\sin^2 \phi}} p_{\gamma_1}(\gamma_1) d\gamma_1 \cdots \int_0^\infty e^{-\frac{\gamma_L}{\sin^2 \phi}} p_{\gamma_L}(\gamma_L) d\gamma_L \right\} \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{l=1}^L M_{\gamma_l} \left(-\frac{1}{\sin^2 \phi} \right) d\phi \end{aligned}$$

- For i.i.d. Rayleigh channels, MGF of SNR at each branch is given as

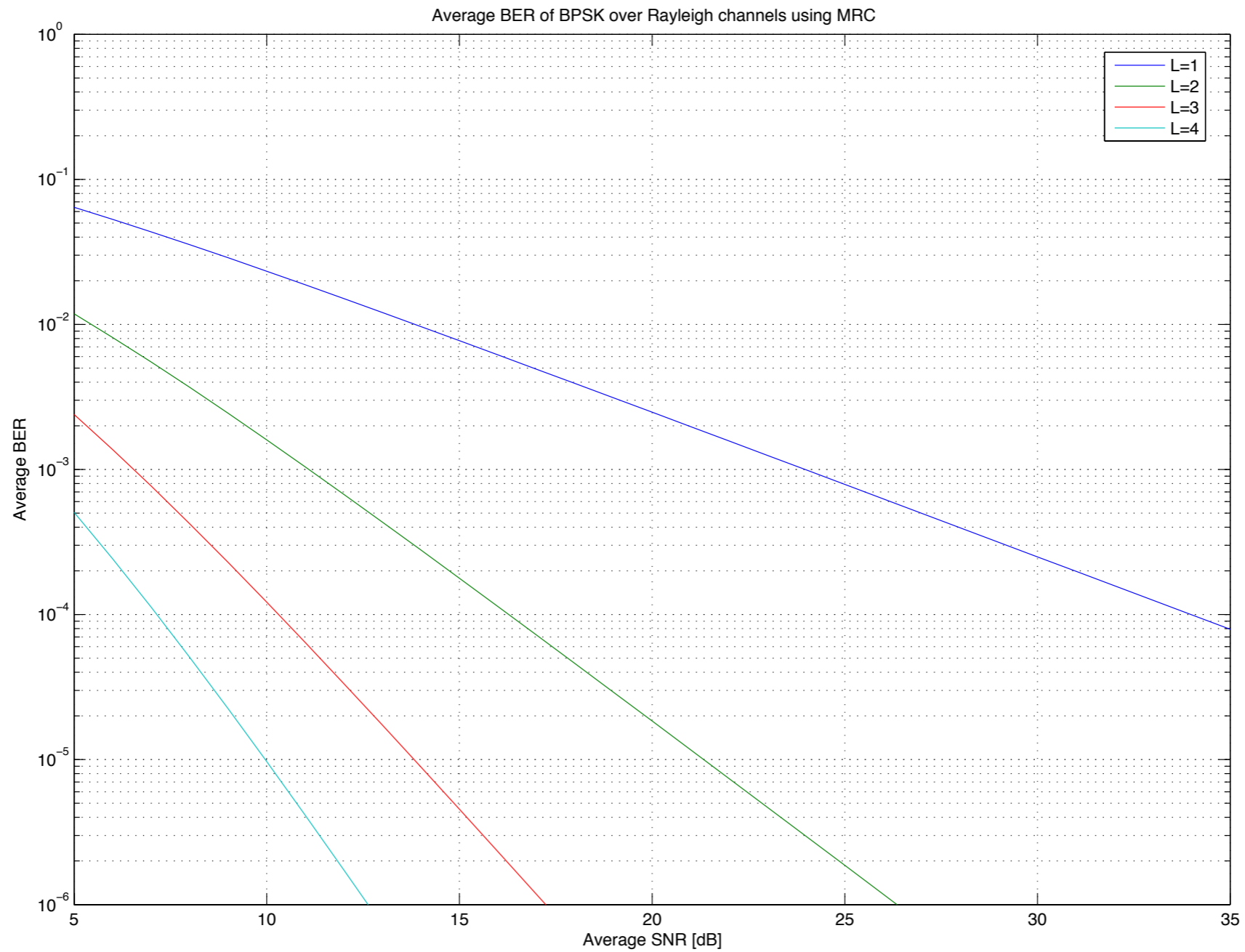
$$M_{\gamma_l}(s) = (1 - \bar{\gamma}s)^{-1}$$

- ~ Then BER of MRC over Rayleigh channels is

$$\begin{aligned} P(e) &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}}{\sin^2 \phi}\right)^{-L} d\phi \\ &= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2 \phi}{\sin^2 \phi + \bar{\gamma}}\right)^L d\phi \\ &= I_L(\bar{\gamma}) \\ &= \frac{1}{2} - \left(\frac{1}{2} - A(\bar{\gamma})\right) \sum_{l=0}^L \binom{2l}{l} \cdot (A(\bar{\gamma}))^l (1 - A(\bar{\gamma}))^l \end{aligned}$$

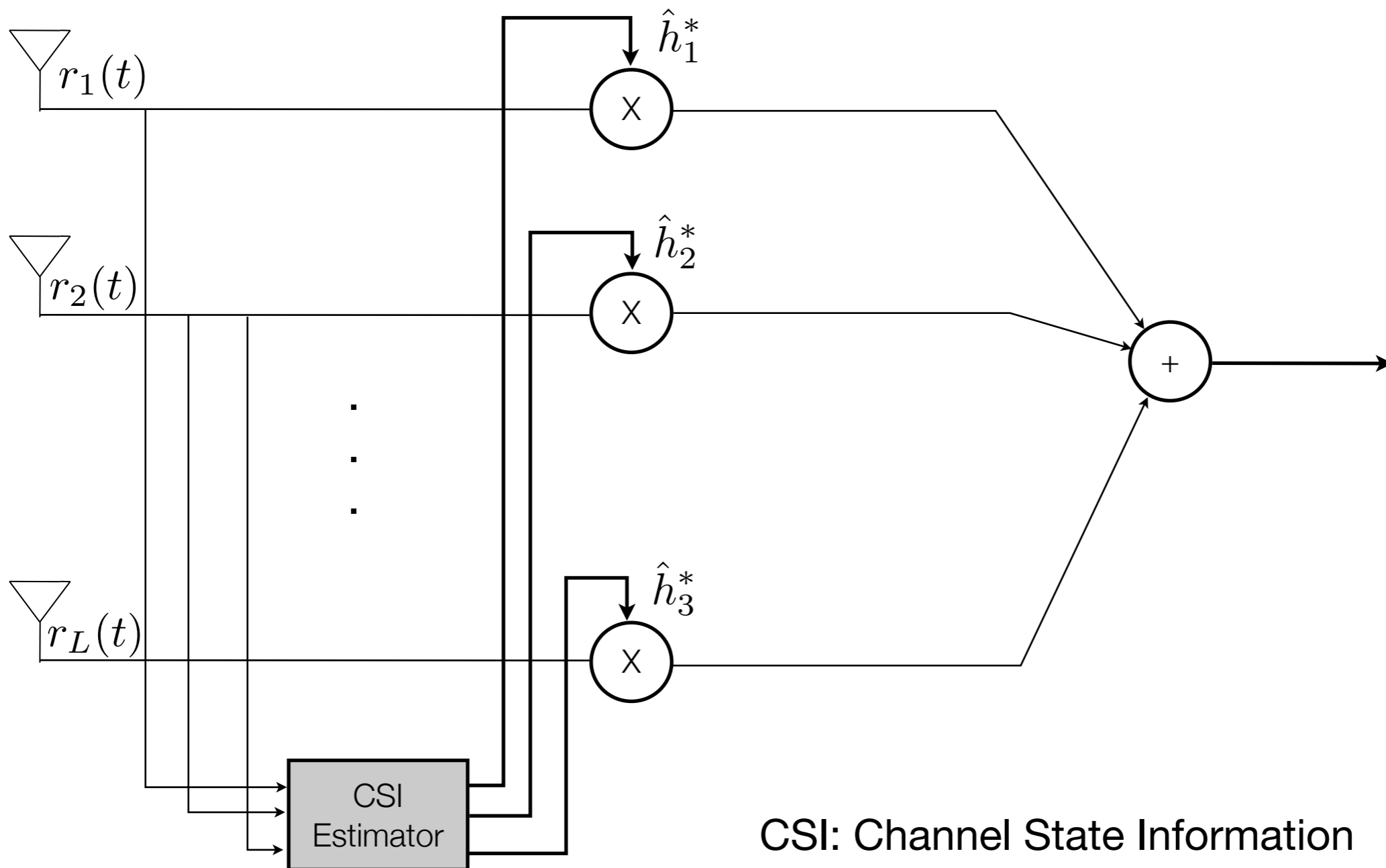
where $A(\bar{\gamma}) = \frac{1}{2} \left[1 - \sqrt{\frac{\bar{\gamma}}{1 + \bar{\gamma}}}\right]$

BER of BPSK over Rayleigh Channels using MRC



Channel Estimation

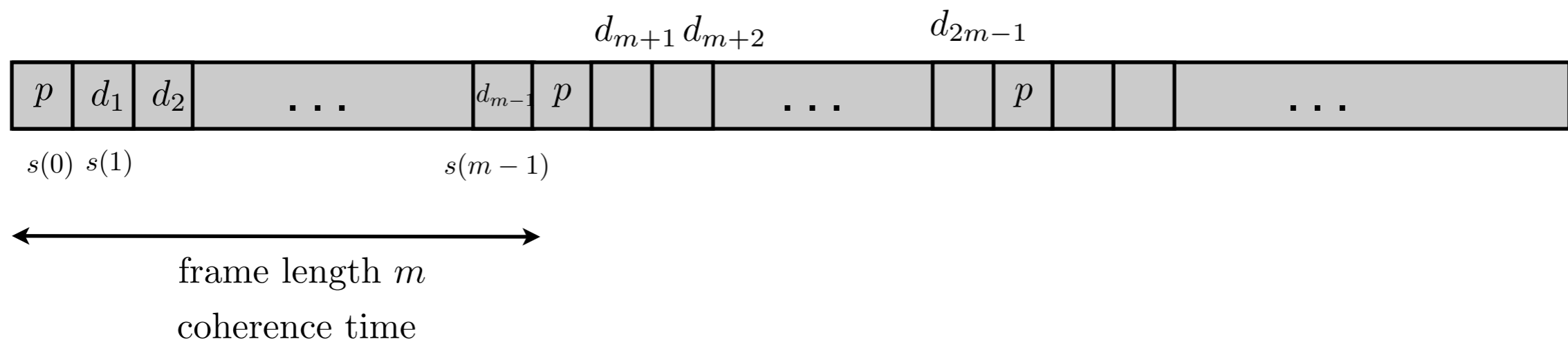
- MRC requires channel estimation at every diversity branch.



■ How does CSI estimator work?

~ Popular channel estimation

- Pilot symbol assisted modulation (PSAM)
- Idea of PSAM: Insert known symbols (pilots) into the stream of useful information symbols.



■ Discrete-time received signal

$$r[k] = h[k]s[k] + n[k]$$

~ Without loss of generality, we can write

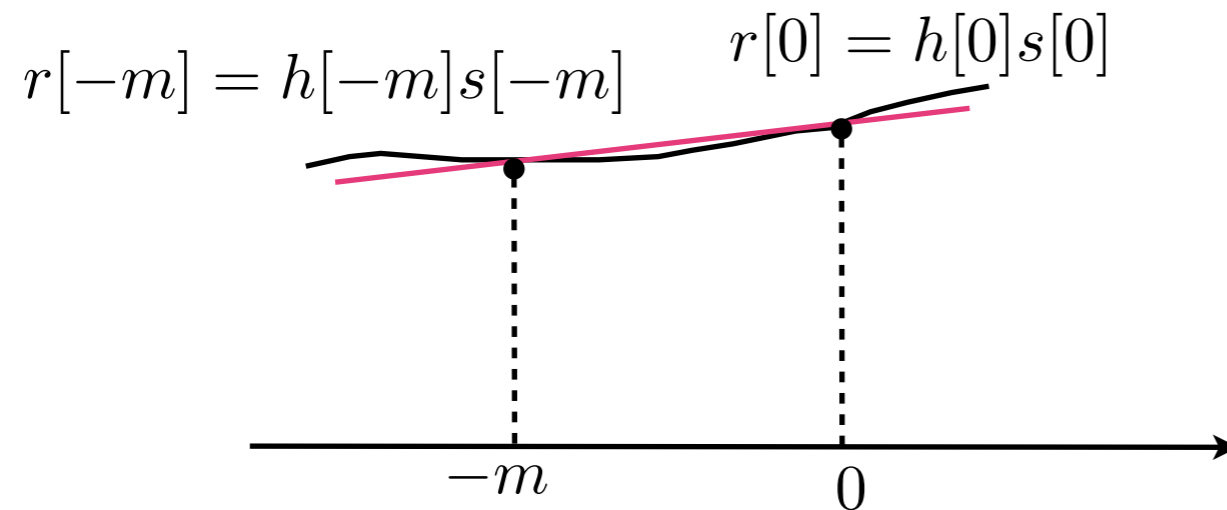
- $s[km]$: pilot signal for k integer.
- Surrounded by information data $s[k]$ for $-\lfloor \frac{M}{2} \rfloor \leq k \leq \lfloor \frac{M}{2} \rfloor$

■ Estimation approach

- ~ Simple approach (Linear regression)
- ~ Optimal approach

Channel Estimation using Linear Regression

Let us ignore the effect of noise.



$$\hat{h}[k] = \left(\frac{ph[0] - ph[-m]}{0 - (-m)} (k - 0) + ph[0] \right) \cdot \frac{1}{p}$$

➔
$$\hat{h}[k] = \frac{k}{m} (h[0] - h[-m]) + h[0], \quad \text{for } -m + 1 \leq k \leq -1$$

$$\hat{h}[k] = \frac{k}{m} (h[1] - h[0]) + h[1], \quad \text{for } -1 \leq k \leq m - 1$$

⋮
⋮
⋮

Channel Estimation using Optimal Approach

- $\hat{h}[k]$: weighted sum of the surrounding $K+1$ received pilots

$$\hat{h}[k] = \sum_{i=-\lfloor \frac{K}{2} \rfloor}^{\lfloor \frac{K}{2} \rfloor} \lambda(i, k) r(im)$$

$\lambda(i, k)$: weighted depends on the symbol portion k within the frame length m .

- Optimal approach: find $\lambda(i, k)$ to minimize the variance of channel estimation error such that

$$\begin{aligned} \sigma_{\epsilon}^2 &= E[\epsilon^2[k]] \\ &= E[|h[k] - \hat{h}[k]|^2] \\ &= E\left[\left| h[k] - \sum_{i=-\lfloor \frac{K}{2} \rfloor}^{\lfloor \frac{K}{2} \rfloor} \lambda(i, k) r(im) \right|^2 \right] \end{aligned} \quad \Rightarrow \quad \lambda_{\text{opt}}(i, k) = \min_{\lambda(i, k)} \sigma_{\epsilon}^2$$

■ Solution

$$\frac{\partial \sigma_\epsilon^2}{\partial \lambda(i, k)} = 0 \quad \text{for } \lfloor -\frac{K}{2} \rfloor \leq i \leq \lfloor \frac{K}{2} \rfloor$$

That is,

$$\begin{aligned} \frac{\partial \sigma_\epsilon^2}{\partial \lambda(\lfloor -\frac{K}{2} \rfloor, k)} &= 0 \\ \frac{\partial \sigma_\epsilon^2}{\partial \lambda(\lfloor -\frac{K}{2} \rfloor + 1, k)} &= 0 \\ &\vdots \\ \frac{\partial \sigma_\epsilon^2}{\partial \lambda(\lfloor \frac{K}{2} \rfloor, k)} &= 0 \end{aligned} \quad \Rightarrow \quad K + 1 \text{ equations}$$

and note that

$$\frac{\partial \sigma_\epsilon^2}{\partial \lambda(i, k)} = 2E \left[(h^*[k] - \sum_{j=-\lfloor \frac{K}{2} \rfloor}^{\lfloor \frac{K}{2} \rfloor} \lambda(i, k) r[jm]) \right] = 0 \quad \text{for } \lfloor -\frac{K}{2} \rfloor \leq i \leq \lfloor \frac{K}{2} \rfloor$$

~ We can simply write the above $K+1$ equations in matrix form such as

$$\begin{bmatrix} R_{rr}(0) & R_{rr}(1) & \cdots & R_{rr}(k) \\ R_{rr}(1) & R_{rr}(2) & \cdots & R_{rr}(k-1) \\ \vdots & \vdots & \ddots & \vdots \\ R_{rr}(k) & R_{rr}(k-1) & \cdots & R_{rr}(0) \end{bmatrix} \begin{bmatrix} \lambda \left(\left\lfloor -\frac{K}{2} \right\rfloor, k \right) \\ \lambda \left(\left\lfloor -\frac{K}{2} \right\rfloor + 1, k \right) \\ \vdots \\ \lambda \left(\left\lfloor \frac{K}{2} \right\rfloor, k \right) \end{bmatrix} = \begin{bmatrix} R_{h^*r} \left(-\left\lfloor \frac{K}{2} \right\rfloor, k \right) \\ R_{h^*r} \left(-\left\lfloor \frac{K}{2} \right\rfloor + 1, k \right) \\ \vdots \\ R_{h^*r} \left(\left\lfloor \frac{K}{2} \right\rfloor, k \right) \end{bmatrix}$$

where $R_{rr}(i, j) = E[r[im] \cdot r[jm]] = R(|i - j|)$

$$R_{h^*r} \left(\left\lfloor \frac{K}{2} + n \right\rfloor, k \right) = \sum_{j=-\left\lfloor \frac{K}{2} \right\rfloor}^{\left\lfloor \frac{K}{2} \right\rfloor} \lambda(j, k) R_{rr} \left(\left\lfloor \frac{K}{2} + n - j \right\rfloor \right)$$

$$\begin{aligned} R_{h^*r} &= E[h^*[k]r[im]] = p \\ &= E[h^*[k](h[im]s[jm] + n[jm])] \\ &= E[h^*[k]h[jm]]p \\ &= pJ_0(2\pi f_m |k - jm|) \end{aligned}$$

Now straightforward to solve for $\lambda(i, k)$.

Other Linear Diversity Combining

■ Combining Schemes

- ~ Equal gain combining (EGC)
- ~ Selection combining (SC)
- ~ Switched and stay combining (SSC)

- Most of performance evaluation can be computed by MGF approach while PDF based approach is not often feasible.

Equal Gain Combining

- Let us rewrite the channel gain as

$$h_l(t) = \alpha_l(t)e^{j\theta_l(t)}$$

- Then, EGC compensates only the phase distortion of the channel so that the weight is given as

$$w_l = e^{-j\theta_l(t)}$$

$$\begin{aligned} r_c(t) &= [r_1(t) \ r_2(t) \ \cdots \ r_L(t)] \cdot [w_1 \ w_2 \ w_L] \\ &= s(t) \sum_{l=1}^L \alpha_l + \sum_{n=1}^L e^{-j\theta_l(t)} n_l(t) \end{aligned}$$

Combined output SNR of EGC

$$\gamma_{\text{EGC}} = \frac{\left(\sum_{l=1}^L \alpha_l\right)^2 E_s}{\sum_{l=1}^L N_l}$$

N_l : AWGN power spectral density on the l th path

E_s : Energy (in joules) per symbol

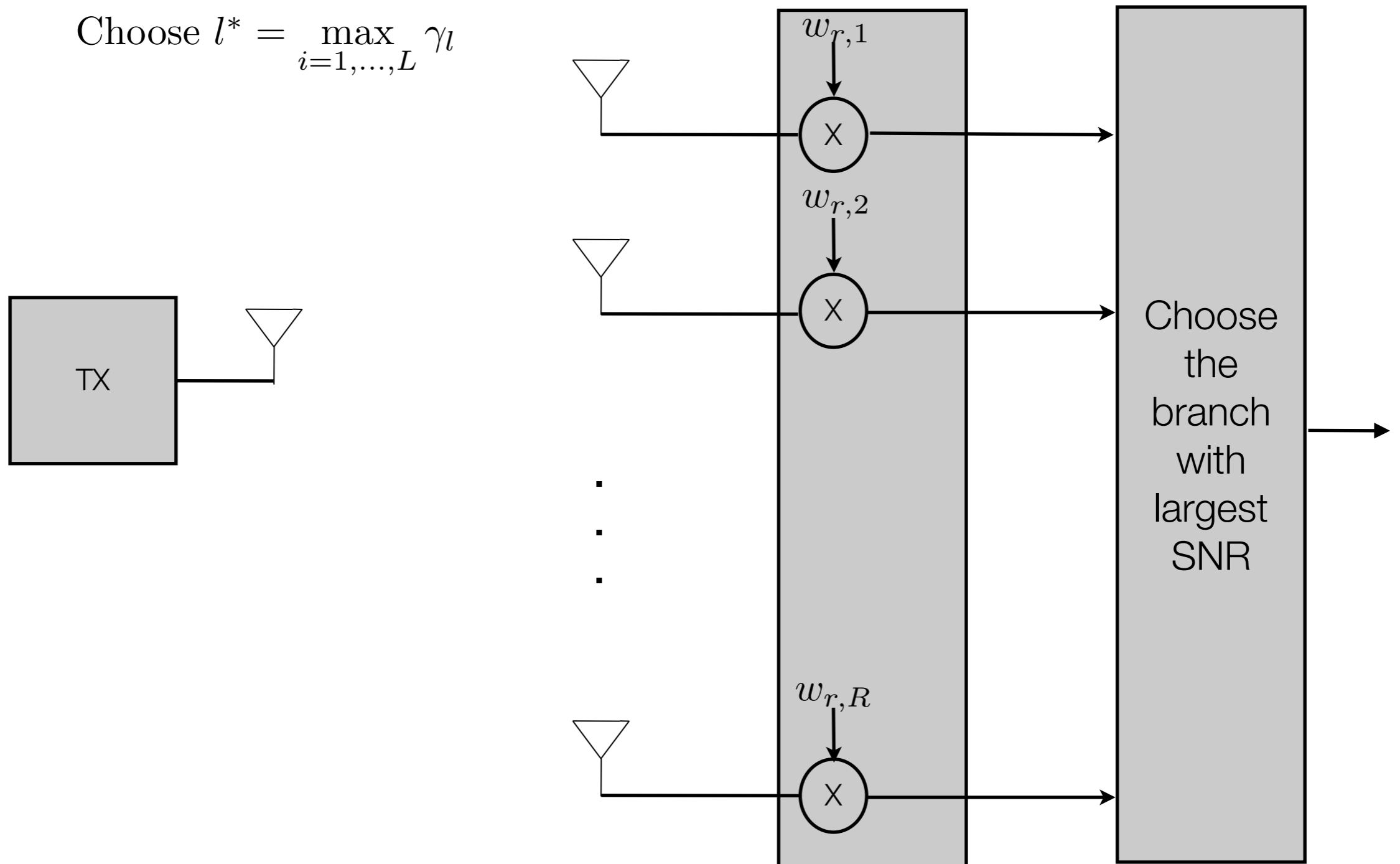
For i.i.d. Rayleigh case, the average output SNR can be shown as

$$\bar{\gamma}_{\text{EGC}} = \bar{\gamma} \left[1 + (L - 1) \frac{\pi}{4} \right]$$

Selection Combining

- Selection combining chooses the branch with the largest SNR

$$\text{Choose } l^* = \max_{i=1, \dots, L} \gamma_l$$



Combined output SNR

$$\gamma_{\text{sc}} = \max\{\gamma_1, \gamma_2, \dots, \gamma_L\}$$

CDF

$$P_{\gamma_{\text{sc}}}(\gamma) = \Pr[\gamma_1 \leq \gamma, \gamma_2 \leq \gamma, \dots, \gamma_L \leq \gamma]$$

$$= \prod_{l=1}^L P_{\gamma_l}(\gamma) \quad (\text{independent case})$$

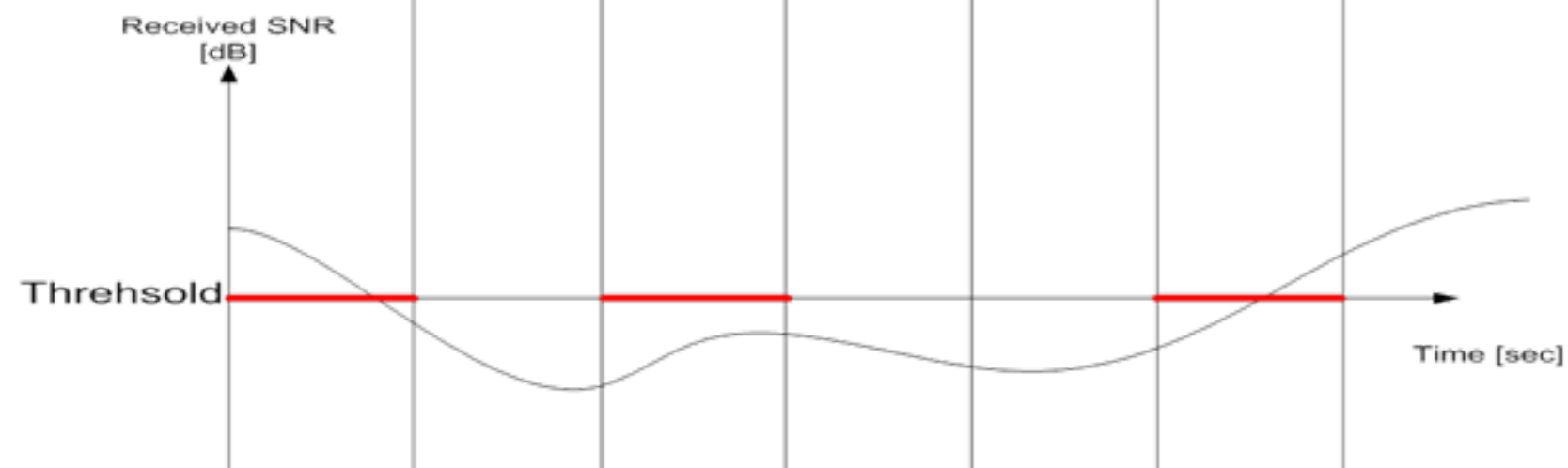
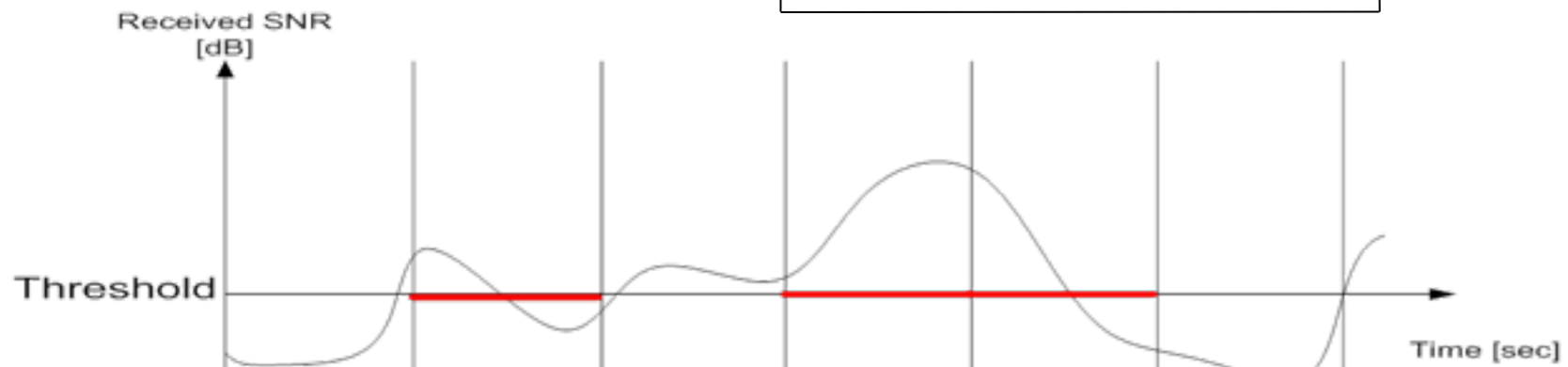
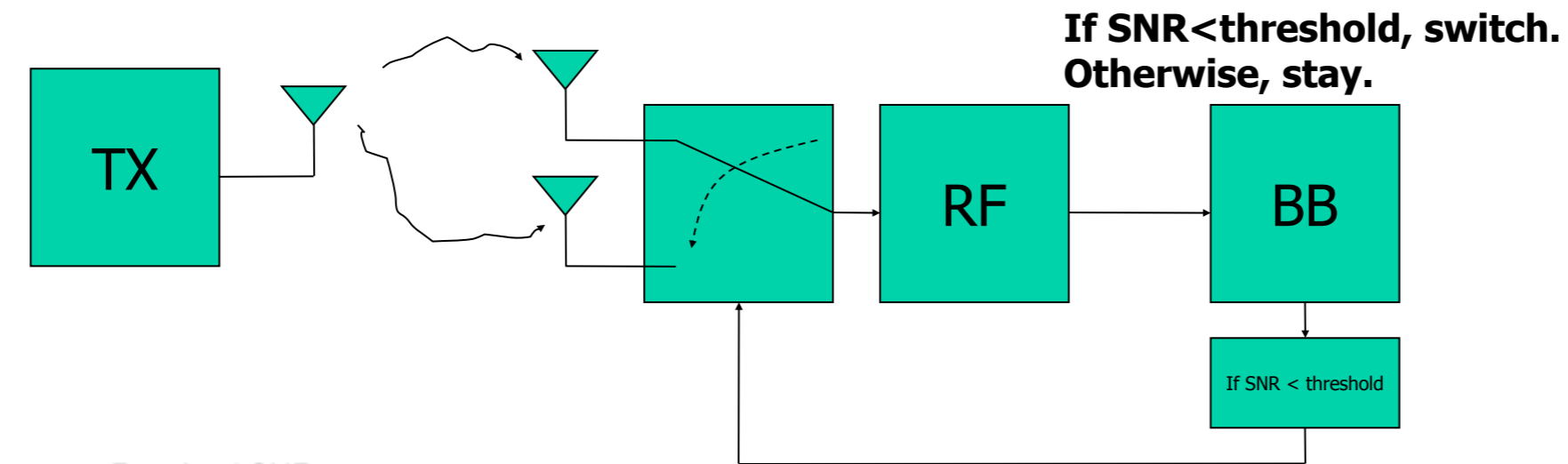
$$= \prod_{l=1}^L \left[1 - e^{-\gamma/\bar{\gamma}_l} \right] \quad (\text{independent Rayleigh case})$$

$$= \left[1 - e^{-\gamma/\bar{\gamma}} \right]^L \quad (\text{i.i.d. Rayleigh case})$$

PDF

$$p_{\gamma_{sc}}(\gamma) = \frac{L}{\bar{\gamma}} \left[1 - e^{-\gamma/\bar{\gamma}} \right]^{L-1} e^{-\gamma/\bar{\gamma}}$$

Switched Combining



CDF of γ_{SSC}

$$P_{\gamma_{SSC}}(\gamma) = \begin{cases} \Pr[(\gamma_1 \leq \gamma_T) \text{ and } (\gamma_2 \leq \gamma)] & \gamma < \gamma_T \\ \Pr[(\gamma_T \leq \gamma_1 \leq \gamma) \text{ or } (\gamma_1 \leq \gamma_T \text{ and } \gamma_2 \leq \gamma)], & \gamma \geq \gamma_T \end{cases}$$

which can be expressed in terms of the CDF of the individual branches $P_\gamma(\gamma)$

$$P_{\gamma_{SSC}}(\gamma) = \begin{cases} P_\gamma(\gamma_T)P_\gamma(\gamma), & \gamma < \gamma_T \\ P_\gamma(\gamma) - P_\gamma(\gamma_T) + P_\gamma(\gamma)P_\gamma(\gamma_T), & \gamma \geq \gamma_T \end{cases}$$

PDF of γ_{SSC}

$$p_{\gamma_{SSC}}(\gamma) = \frac{dP_{\gamma_{SSC}}(\gamma)}{d\gamma} = \begin{cases} P_\gamma(\gamma_T)p_\gamma(\gamma), & \gamma < \gamma_T \\ (1 + P_\gamma(\gamma_T))p_\gamma(\gamma), & \gamma \geq \gamma_T \end{cases}$$

MGF of SSC

$$\mathcal{M}_{\gamma_{ssc}}(s) = P_{\gamma}(\gamma_T)\mathcal{M}_{\gamma}(s) + \int_{\gamma_T}^{\infty} p_{\gamma}(\gamma)e^{s\gamma} d\gamma$$

For Rayleigh case, we can show the MGF as

$$\mathcal{M}_{\gamma_{ssc}}(s) = (1 - s\bar{\gamma})^{-1} (1 - e^{-\gamma_T/\bar{\gamma}} + e^{-(1-s\bar{\gamma})\gamma_T/\bar{\gamma}})$$

Average output SNR of SSC

$$\begin{aligned}\bar{\gamma}_{\text{SSC}} &= P_{\gamma}(\gamma_T) \int_0^{\infty} \gamma p_{\gamma}(\gamma) d\gamma + \int_{\gamma_T}^{\infty} \gamma p_{\gamma}(\gamma) d\gamma \\ &= P_{\gamma}(\gamma_T) \bar{\gamma} + \int_{\gamma_T}^{\infty} \gamma p_{\gamma}(\gamma) d\gamma\end{aligned}$$

Rayleigh case

$$\bar{\gamma}_{\text{SSC}} = \bar{\gamma} \left(1 + \frac{\gamma_T}{\bar{\gamma}} e^{-\gamma_T/\bar{\gamma}} \right)$$

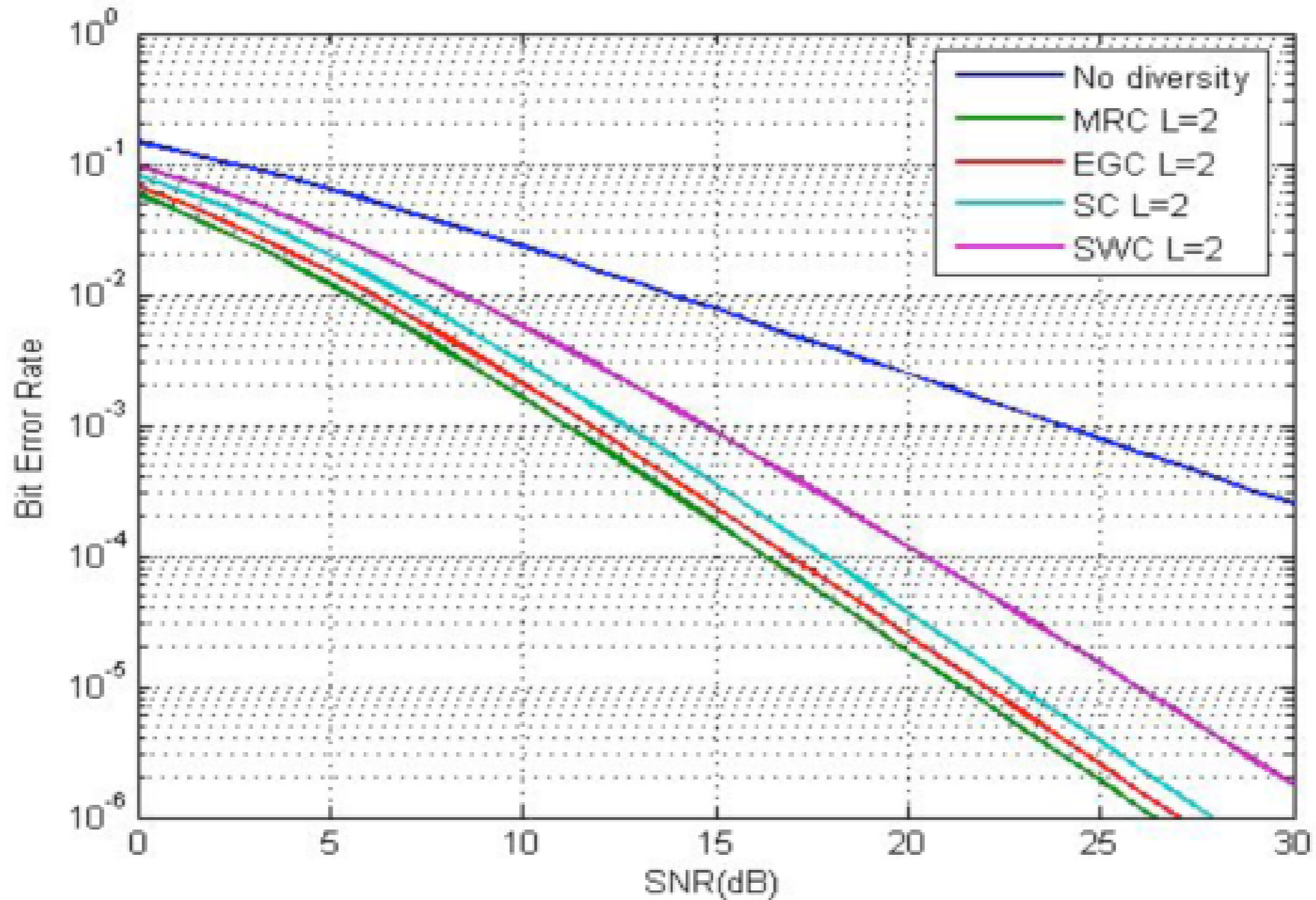
Optimal threshold

$$\max_{\gamma_T} \bar{\gamma}_{\text{SSC}}$$

$$\frac{d\bar{\gamma}_{\text{SSC}}}{d\gamma_T} = e^{-\gamma_T/\bar{\gamma}} - \frac{\gamma_T}{\bar{\gamma}} e^{-\gamma_T/\bar{\gamma}} = 0$$

$$\gamma_T^* = \bar{\gamma}$$

Comparison of BER Performance of Dual Diversity

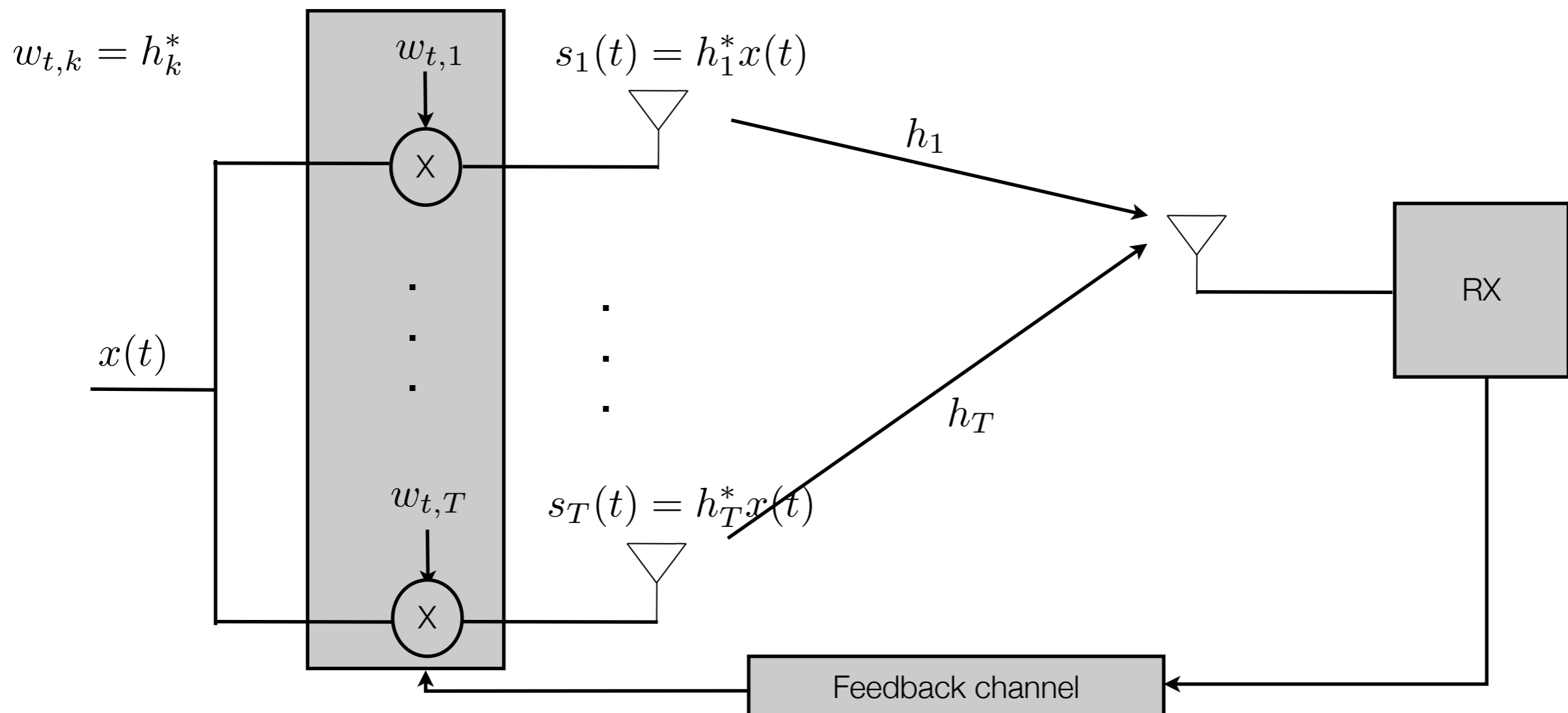


Transmit Diversity

- Two different transmit diversity schemes
 - Channel known at the transmitter
 - Channel unknown at the transmitter

Channel Known at the Transmitter

- Channel state information (CSI) at the transmitter
 - Transmitter knows the CSI, that is, $h_l(t)$, so we have $s_l(t) = h_l^*(t)x(t)$.



- The received signal at the receiver is

$$r(t) = \sum_{l=1}^L h_l(t)s_l(t) + n(t) = \sum_{l=1}^L |h_l|^2 x(t)$$

which gives the same diversity gain as the receiver MRC.

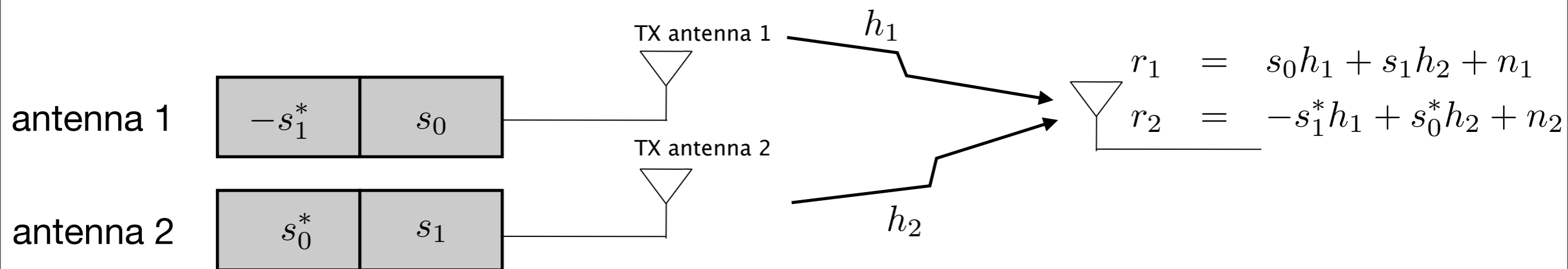
- Remark

- ~ There is a transmit power constraint such that

$$\sum_{l=1}^L E[|s_l(t)|^2] \leq P_t$$

Channel Unknown at the Transmitter: Alamouti Scheme

- Alamouti scheme (2x1 case)



[Space-time block code (STBC)]

	antenna1	antenna2
time t	s_0	s_1
time t+T	$-s_1^*$	s_0^*

■ QPSK example

$$\begin{aligned}
 00 &\rightarrow s^1 = 1 + j \\
 01 &\rightarrow s^2 = 1 - j \\
 11 &\rightarrow s^3 = -1 + j \\
 10 &\rightarrow s^4 = -1 - j
 \end{aligned}$$

[Space-time block code (STBC)]

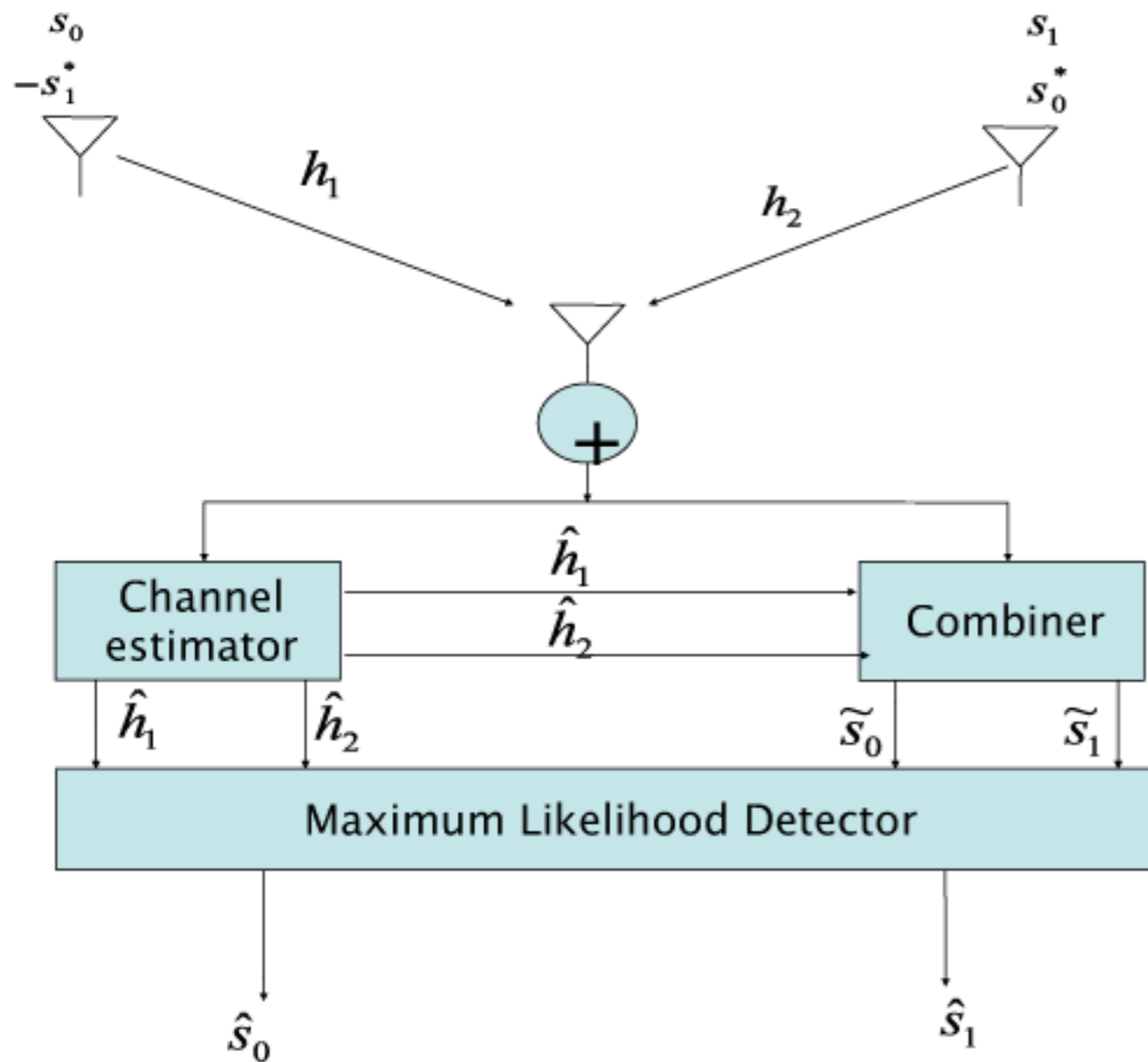
	antenna1	antenna2
time t	s_0	s_1
time t+T	$-s_1^*$	s_0^*

data: 01001011... $\implies s_0 s_1 s_2 s_4 \dots = s^2 s^1 s^4 s^3 \dots$

time	ant1	ant2
T	$1 - j$	$1 + j$
$2T$	$-1 + j$	$1 + j$
$3T$	$-1 - j$	$-1 + j$
$4T$	$1 + j$	$-1 + j$
	\vdots	

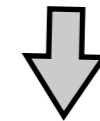
$$\begin{aligned}
 r_1 &= s_0 h_1 + s_1 h_2 + n_1 \\
 r_2 &= -s_1^* h_1 + s_0^* h_2 + n_2 \\
 r_3 &= s_2 h_1 + s_3 h_2 + n_1 \\
 r_4 &= -s_3^* h_1 + s_2^* h_2 + n_2
 \end{aligned}$$

■ Detection of space-time block coding signal



$$r_1 = s_0 h_1 + s_1 h_2 + n_1$$

$$r_2 = -s_1^* h_1 + s_0^* h_2 + n_2$$



$$v_1 = h_1^* r_1 + h_2 r_2^*$$

$$v_2 = h_2^* r_1 - h_1 r_2^*$$



$$v_1 = (|h_1|^2 + |h_2|^2) s_0 + h_1^* n_1 + h_2 n_2^*$$

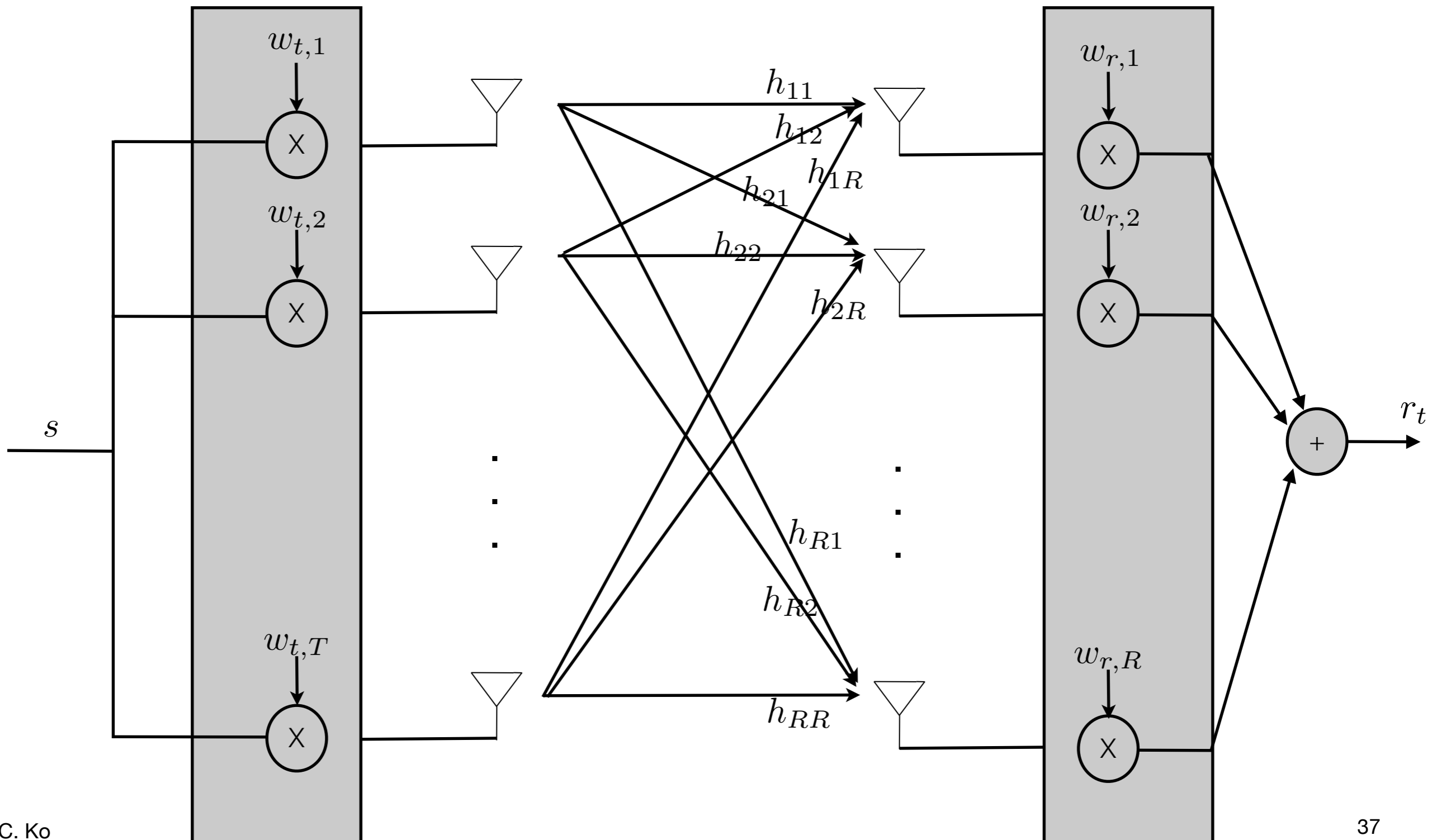
$$v_2 = (|h_1|^2 + |h_2|^2) s_1 + h_2^* n_1 - h_1 n_2^*$$

■ Received SNR

$$\gamma_t = \frac{(|h_1|^2 + |h_2|^2)E_s}{2N_0}$$

- ~ where the factor of 2 comes from the fact that s_i is transmitted using half the total symbol energy E_s .

Maximal Ratio Transmission (Transmit-Receive Diversity)



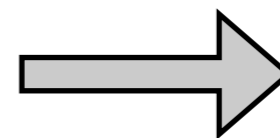
Channel Matrix of Multiple-Input Multiple Output Antenna (MIMO) Systems

- Channel matrix for $T \times R$ antenna systems

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1R} \\ h_{21} & h_{22} & \cdots & h_{2R} \\ \vdots & \vdots & \vdots & \vdots \\ h_{T1} & h_{T2} & \cdots & h_{TR} \end{bmatrix}$$

- Received signal

$$\begin{aligned} r_1 &= (w_{t,1}h_{11} + w_{t,2}h_{12} + \cdots + w_{t,T}h_{1T})s + n_1 \\ r_2 &= (w_{t,1}h_{21} + w_{t,2}h_{22} + \cdots + w_{t,T}h_{2T})s + n_2 \\ &\vdots \\ r_R &= (w_{t,1}h_{R1} + w_{t,2}h_{R2} + \cdots + w_{t,T}h_{RT})s + n_R \end{aligned}$$



$$\mathbf{r} = \mathbf{H}\mathbf{w}_t s + \mathbf{n}$$

Optimal Weight for MIMO-MRC

■ Combined signal

$$r_t = \mathbf{w}_R^H \mathbf{r}$$

~ Optimal receive weight vector can be easily shown to be given as

$$\mathbf{w}_R = \alpha \mathbf{H} \mathbf{w}_t$$

~ In this case, the received SNR can be written as

$$r_t = \alpha \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t s + \alpha \mathbf{w}_t^H \mathbf{H}^H \mathbf{n}$$

~ SNR of the combined signal r_t and the optimum weight

$$\mu = \frac{1}{\sigma_n^2} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t \quad \longrightarrow \quad \mathbf{w}_t^{\text{opt}} = \max_{\mathbf{w}_t} \mu$$

- Find the optimal weight vector \mathbf{w}_t to maximize the SNR μ .

~ Recall Rayleigh-Ritz theorem

$$\mathbf{x}^H \mathbf{A} \mathbf{x} \leq \|\mathbf{x}\| \lambda_{\max}$$

where \mathbf{A} is the hermitian matrix, \mathbf{x} is any non-zero complex vector and λ_{\max} is the largest eigenvalue of \mathbf{A} .

- Equality holds if and only if \mathbf{x} is the eigenvector corresponding to λ_{\max} .

~ Based on Rayleigh-Ritz theorem. we can find the optimal weight vector, we can find the optimal weight vector as

$$\mathbf{w}_t = \sqrt{\Omega} \mathbf{U}_{\max}$$

where \mathbf{U}_{\max} is the eigenvector corresponding to the largest eigenvalue of the quadratic form

$$F = \mathbf{H}^H \mathbf{H}$$

- Combined SNR using the optimal weight vector

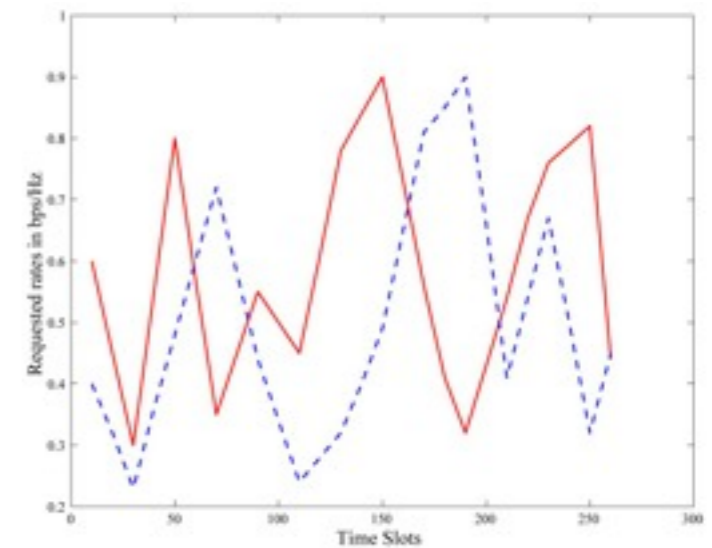
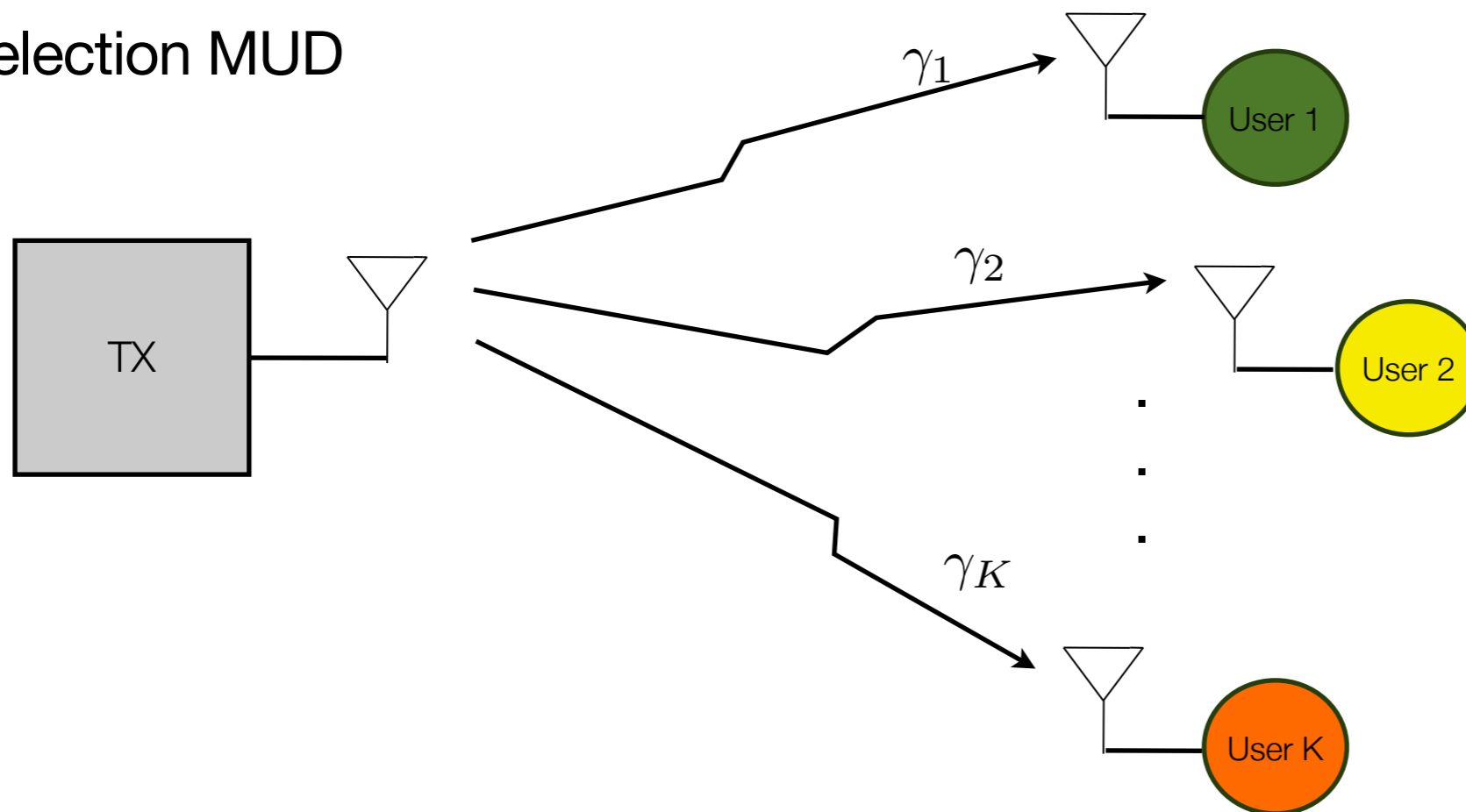
$$\mu = \frac{\Omega}{\sigma_n^2} \lambda_{\max}$$

Multi-User Opportunistic Diversity (MUD)

■ Motivation

- ~ In cellular system, a user can have one or two antennas. Hence, obtaining the diversity gain is limited.

■ Selection MUD



Choose the user which has the largest SNR among K users!!!

Selected user k

$$k^* = \max_k(\gamma_1, \gamma_2, \dots, \gamma_K)$$

Ergodic channel capacity

$$C = E[\log_2(1 + \gamma_k^*)]$$

$$= \int_0^{\infty} \log_2(1 + \gamma_k^*) p_{\gamma_k^*}(\gamma_k^*) d\gamma_k$$

