Mobile Communications (KECE425)

Lecture Note 4

03-12-2014

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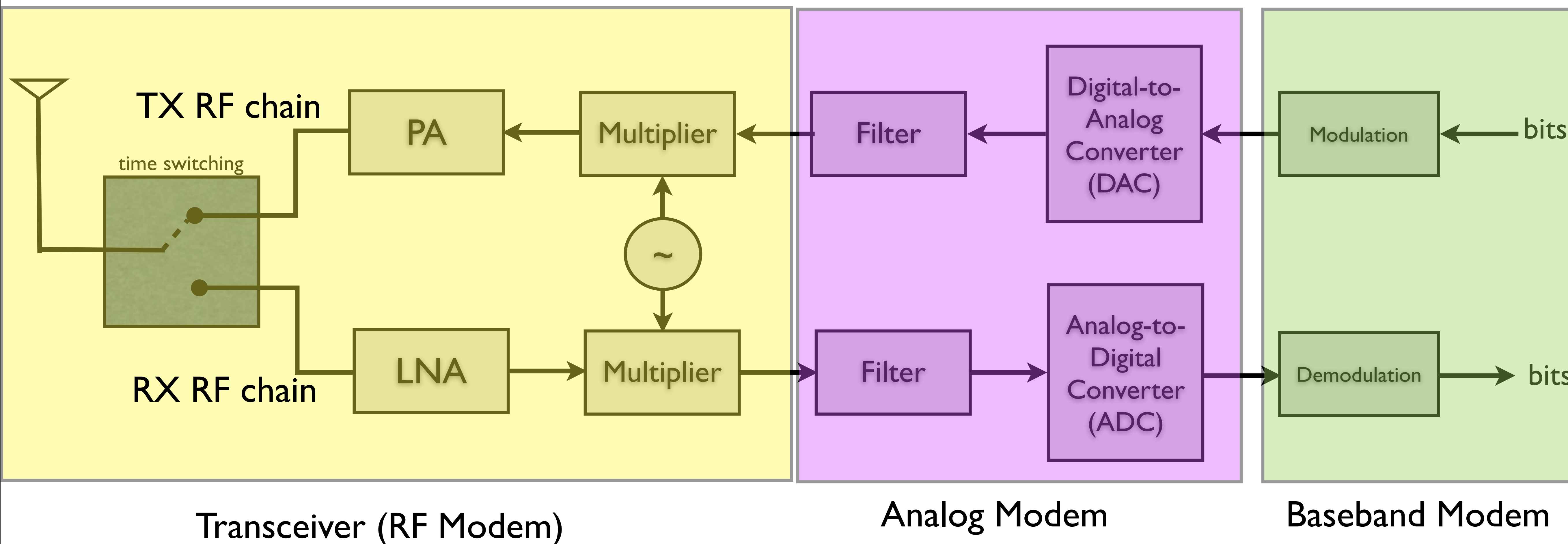
Summary

- Receiver Sensitivity
- Coverage (maximum allowable path loss)
- Outage probability

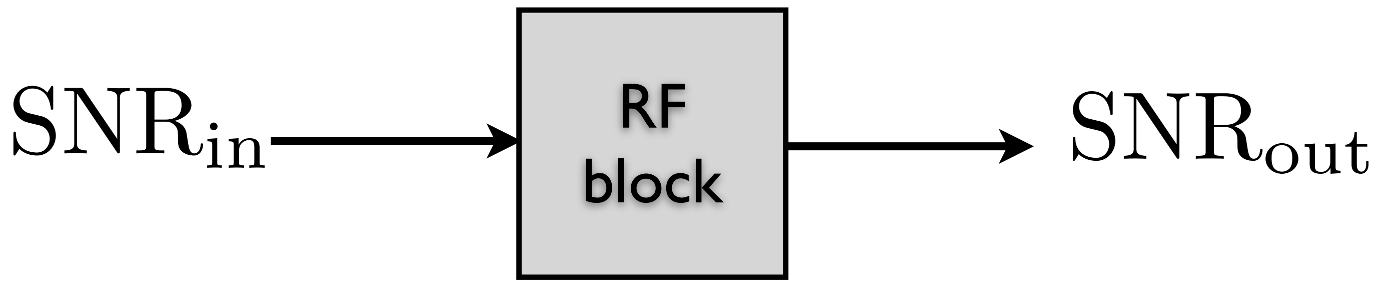
MODEM Architecture



- RF block is characterized by "noise figure" and "gain"

Noise Figure

- Noise factor F is defined as



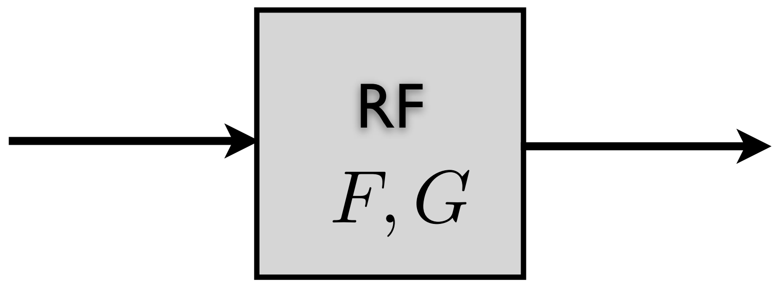
$$F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$$

- Noise figure NF is defined as

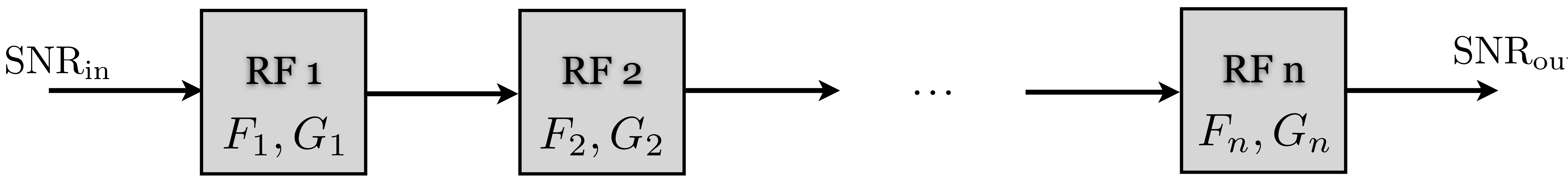
$$NF = 10\log_{10}(F) = 10\log_{10}\left(\frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}\right)$$

$$= \mathrm{SNR_{in,dB}} - \mathrm{SNR_{out,dB}}$$

- RF block is characterized by "noise figure" and "gain"



- Friis' formula



$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots + \frac{F_n - 1}{G_1 G_2 G_3 \cdots G_{n-1}}$$

$$F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$$

Noise

- Power spectral density of thermal noise is kT_0 and is at room temperature (17°C = 290° K)

$$kT_0 = -174\,\mathrm{dB/Hz}$$

  - where $k = 1.38 \times 10^{-23} Ws/K$ is the Boltzmann's constant

- Total input noise power to the receiver

$$N = kT_0 B_w F$$

  - where B_w is the receiver noise bandwidth and F is the noise figure

- Noise power spectral density at the receiver

$$N_0 = kT_0 F\,\mathrm{dB/Hz}$$

Received Carrier Power and CNR

- Effective received carrier power

$$\Omega_p = \frac{\Omega_t G_T G_R}{L_{R_X} L_P}$$

$$\text{where } L_p = \left(\frac{\lambda}{4\pi d}\right)^{-\beta}$$

- Received carrier-to-noise ratio

$$\Gamma = \frac{\Omega_p}{N} = \frac{\Omega_t G_T G_R}{k T_0 B_w F L_{R_X} L_p}$$

$$\begin{aligned}
\Omega_t &= \text{transmitted carrier power} \\
G_T &= \text{transmitter antenna gain} \\
L_p &= \text{path loss} \\
G_R &= \text{receiver antenna gain} \\
\Omega_p &= \text{received signal power} \\
T_0 &= \text{receiving system noise temperature in degrees Kelvin} \\
B_w &= \text{receiver noise bandwidth} \\
N_0 &= \text{white noise power spectral density} \\
R_c &= \text{modulated symbol rate} \\
k &= 1.38 \times 10^{-23} Ws/K \text{ Boltzmann's constant} \\
F &= \text{Noise figure, typically to 5 to 6dB} \\
L_{R_X} &= \text{receiver implementation loss}
\end{aligned}$$

Link Budget

- Modulated symbol energy-to-noise ratio

$$\frac{E_c}{N_0} = \Gamma \times \frac{B_w}{R_c}$$

- Modulated symbol energy-to-noise ratio

$$\frac{E_c}{N_0} = \frac{\Omega_t G_T G_R}{k T_0 R_c F L_{R_x} L_p}$$

  - or in decibel unit

$$\begin{aligned}(E_c/N_0)_{(\mathrm{dB})} &= \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} \\ &- kT_{0(\mathrm{dBm})/\mathrm{Hz}} - R_{c(\mathrm{dBHz})} - F_{(\mathrm{dB})} - L_{R_x(\mathrm{dB})} - L_{p(\mathrm{dB})}\end{aligned}$$

CNR and SNR

- Carrier-to-noise power ratio

$$\Gamma = \frac{\Omega_p}{N} = \frac{\Omega_p}{N_0 B_w} = \frac{E_c/T_c}{N_0 B_w} = \frac{E_c R_c}{N_0 B_w}$$

where T_c is the symbol rate, $T_c = 1/R_c$.

- Ratio of modulated symbol energy to noise power spectral density

$$\frac{E_c}{N_0} = \Gamma \frac{B_w}{R_c}$$

Receiver Sensitivity

- Definition of receiver sensitivity

$$S_{R_x} = L_{R_x} k T_0 F (E_c/N_0) R_c$$

  - or in decibel unit as

$$S_{R_x(\mathrm{dBm})} = L_{R_x(\mathrm{dB})} + kT_{0(\mathrm{dBm})/\mathrm{Hz}} + F_{(\mathrm{dB})} + (E_c/N_0)_{(\mathrm{dB})} + R_{c(\mathrm{dBHz})}$$

  - Previously we defined the link budget as

$$\begin{aligned}(E_c/N_0)_{(\mathrm{dB})} &= \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} \\ &- kT_{0(\mathrm{dBm})/\mathrm{Hz}} - R_{c(\mathrm{dBHz})} - F_{(\mathrm{dB})} - L_{R_x(\mathrm{dB})} - L_{p(\mathrm{dB})}\end{aligned}$$

  - Then we have

$$S_{R_x(\mathrm{dBm})} = \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} - L_{p(\mathrm{dB})}$$

- Path loss $L_{p(\mathrm{dB})}$:

$$L_{p(\mathrm{dB})} \quad = \quad \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} - S_{R_x(\mathrm{dBm})}$$

which is maximum allowable path loss to satisfy the receiver sensitivity.

  - Hence, we can say the maximum allowable path loss as

$$L_{p,\max(\mathrm{dB})} = \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} - S_{R_X(\mathrm{dBm})}.$$

- Example
  - Acceptable link quality (minimum required E_c/N_0) is given. (ex. 17 dB)
  - Substitute this value into the receiver sensitivity equation.
  - Solving for $L_{p(\mathrm{dB})}$ will give maximum allowable path loss.

Example

- Calculate the receiver sensitivity for the following case:
  - BPSK with required BER of 10^{-5}.
  - $R_c = 10$ Gbps
  - $L_{R_x} = 3$ dB
  - $F = 6$ dB

- Solution:
  - Required E_c/N_0:

$$P_b(e) = Q\left(\sqrt{2\frac{E_c}{N_0}}\right) = 10^{-5}$$

which gives $\frac{E_c}{N_0} = \frac{1}{2}\left(Q^{-1}(10^{-5})\right)^2 = 9.0991 \rightarrow\ 9.59\,\text{dB}$

- Noise spectral density of thermal noise:

$$kT_0 = -174\,\mathrm{dBm/Hz}$$

- Receiver sensitivity:

$$\begin{aligned} S_{R_x(\mathrm{dBm})} &= L_{R_x} + kT_{0(\mathrm{dBm})} + F_{(\mathrm{dB})} + R_{c(\mathrm{dB})} + \left(\frac{E_c}{N_0}\right)_{(\mathrm{dB})} \\ &= 3 - 174 + 6 + 100 + 9.59 = -55.41\,(\mathrm{dBm}) \end{aligned}$$

Maximum Allowable Path Loss

- Maximum allowable path loss

$$L_{p,\max\,(\mathrm{dB})} = \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} - S_{R_X\,(\mathrm{dBm})}.$$

- Example: Calculate the maximum allowable distance for the following case:
  - $\Omega_t = 10$ mW =10 dBm
  - $G_t = G_r = 3$ dB
- Solution
  - Maximum allowable path loss:

$$\begin{aligned} L_{\max(\mathrm{dB})} &= \Omega_{t(\mathrm{dBm})} + G_{t(\mathrm{dB})} + G_{t(\mathrm{dB})} - S_{Rx(\mathrm{dBm})} \\ &= 10 + 3 + 3 - (-55.41) = 71.4(\mathrm{dB}) \end{aligned}$$

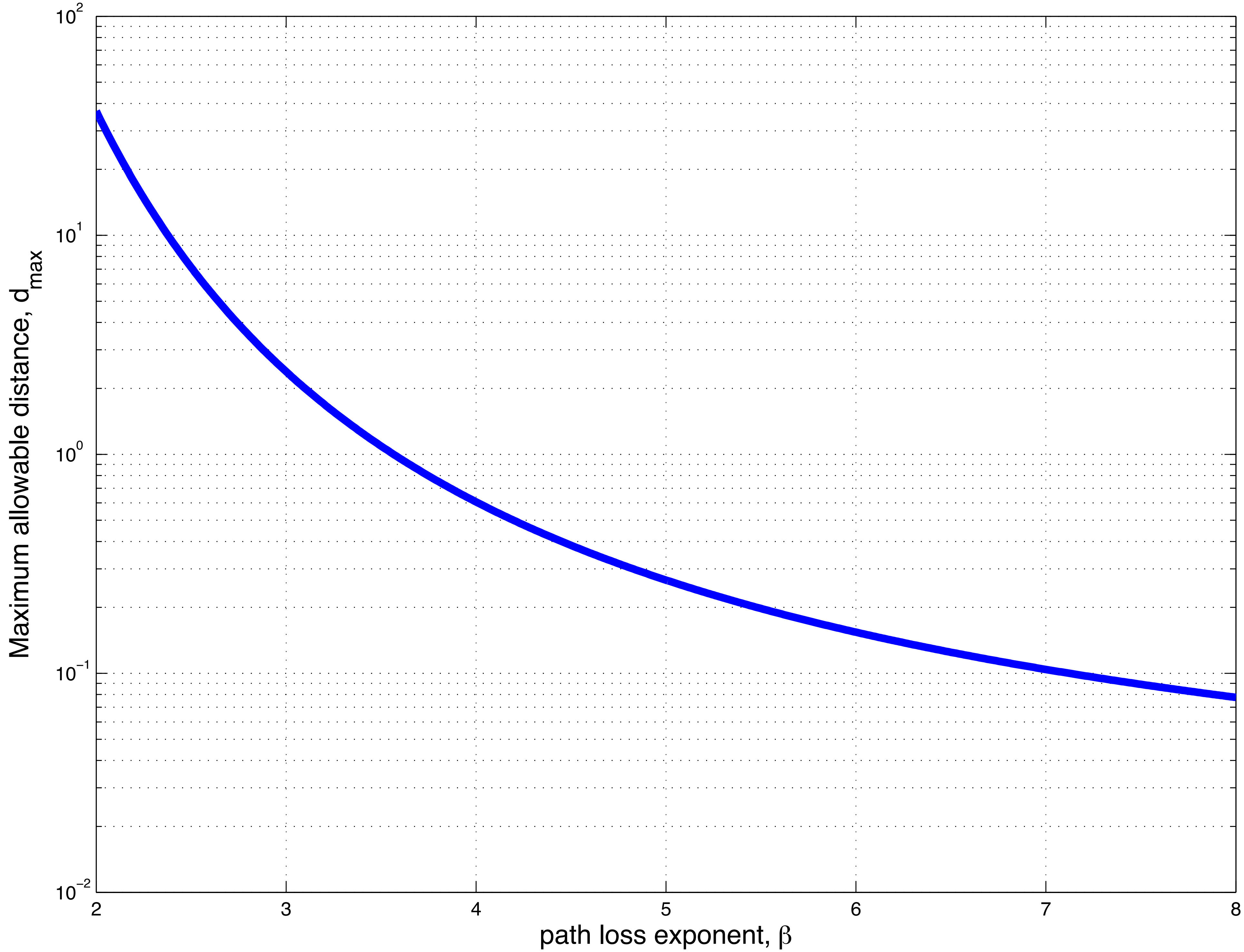
- Note that

$$L_{p,\max} = \left(\frac{\lambda_c}{4\pi d_{\max}} \right)^{-\beta}$$

$$\implies \quad d_{\max} = \frac{\lambda_c}{4\pi} (L_{p,\max})^{\frac{1}{\beta}}$$

- For $f_c = 2.4$ GHz, that is, $\lambda_c = c/f_c = \frac{3 \cdot 10^8}{2.4 \cdot 10^9} = 0.125\text{m}$, and $\beta = 2$, we have

$$d_{\max} = \sqrt{10^{71.4/10}} \cdot \frac{0.125}{4\pi} = 36.9573\text{m}$$



Maximum allowable distance, d_{max}
path loss exponent, β
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
2
3
4
5
6
7
8

Outage Probability

- Carrier-to-noise ratio

$$\Gamma = \frac{\text{Carrier power}}{\text{Noise power}}$$

  - Thermal noise outage probability

$$O_N = \Pr[\Gamma < \Gamma_{\text{th}}]$$

- Carrier-to-interference ratio

$$\Lambda = \frac{\text{Carrier power}}{\text{Interference power}}$$

  - Co-channel interference outage probability

$$O_I = \Pr[\Lambda < \Lambda_{\text{th}}]$$

- Overall outage due to both thermal noise and co-channel interference

$$O = \Pr[\Gamma < \Gamma_{\text{th}} \text{ or } \Lambda < \Lambda_{\text{th}}]$$

- Edge noise outage probability

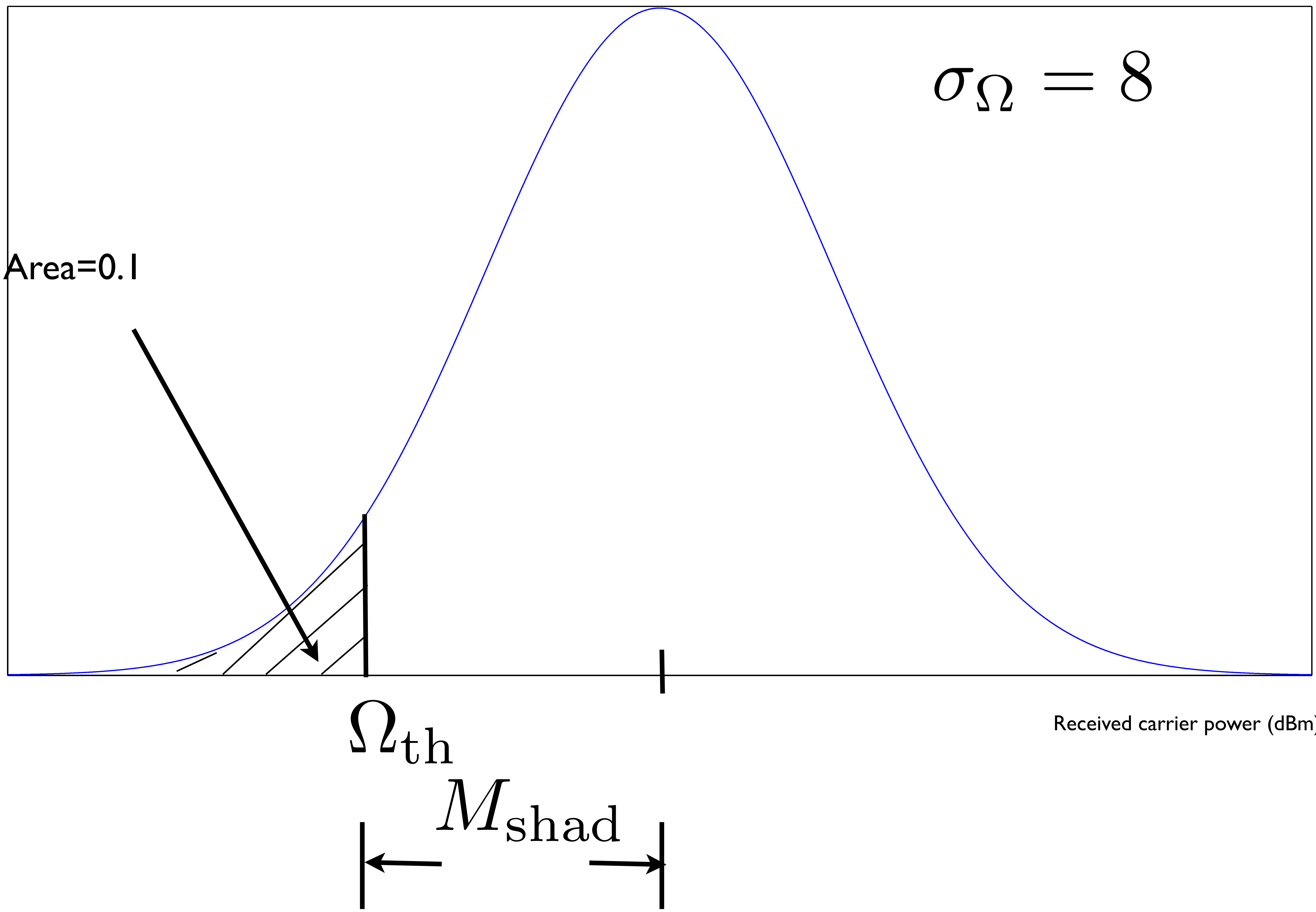
$$\begin{aligned} O_N(R) &= P(\Omega_{p(\text{dBm})(R)} < \Omega_{\text{th(dBm)}}) \\ &= \int_{-\infty}^{\Omega_{\text{th(dBm)}}} \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ -\frac{(x - \mu_{\Omega_{p(\text{dBm})}}(R)^2}{2\sigma_\Omega^2} \right\} dx \end{aligned}$$

$$= Q\left(\frac{M_{\text{shad}}}{\sigma_\Omega}\right)$$

where $M_{\text{shad}} = \mu_{\Omega_{p(\text{dBm})}} - \Omega_{\text{th(dBm)}}$ is called *Shadow margin*.

- Example
  - Suppose that we wish to have $O_N(R) = 0.1$. Determine the Shadow margin M_{shad}.

- Solution
  - We solve
    $$0.1 = Q\left(\frac{M_{\text{shad}}}{\sigma_\Omega}\right),$$
    which gives
    $$\frac{M_{\text{shad}}}{\sigma_\Omega} = Q^{-1}(0.1) = 1.28$$
  - For $\sigma_\Omega = 8$ dB, the required shadow margin is
    $$M_{\text{shad}} = 1.28 \times 8 = 10.24\,\text{dB}$$



$\sigma_\Omega = 8$
Area=0.1
Ω_{th}
Received carrier power (dBm)
M_{shad}

Area Outage Probability

- Area outage probability averaged over area of a cell:

$$O_N = \frac{1}{\pi R^2}\int_0^R O(r) 2\pi r \, dr = Q(X) - \exp\{XY + Y^2/2\} Q(X+Y)$$

where

$$X = \frac{M_{\text{shad}}}{\sigma_\Omega}$$

$$Y = \frac{2\sigma_\Omega}{\beta\zeta}$$

$$\zeta = \frac{10}{\ln 10}$$