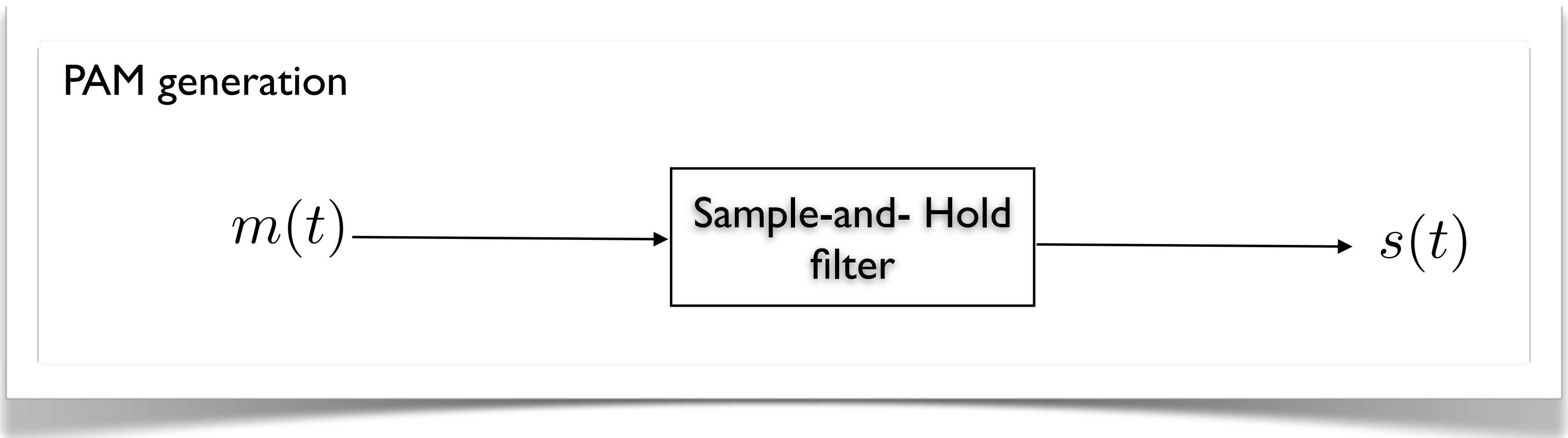
KECE321 Communication Systems I

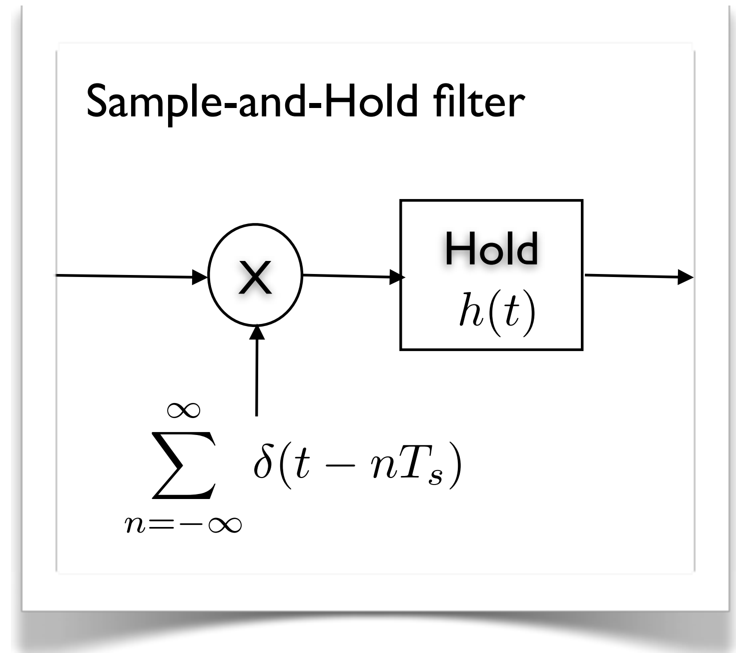
(Haykin Sec. 5.1 - Sec. 5.2)

Lecture #20, May 21, 2012

Prof. Young-Chai Ko

Pulse Amplitude Modulation





$$h(t) = \mathrm{rect}\left(\frac{t-\frac{T}{2}}{T}\right) = \begin{cases} 1, & 0 < t < T \\ \frac{1}{2}, & t = 0,\ \ t = T, \\ 0, & \text{otherwise} \end{cases}$$

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t-nT_s)$$

$$s(t) = m_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_s)$$

Signal representation of PAM signal in time-domain

$$\begin{aligned} m_\delta(t) * h(t) &= \int_{-\infty}^{\infty} m_\delta(t) h(t-\tau)\, d\tau \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s)\delta(\tau - nT_s) h(t-\tau)\, d\tau \\ &= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s) h(t-\tau)\, d\tau \\ &= \sum_{n=-\infty}^{\infty} m(nT_s) h(t - nT_s) \end{aligned}$$

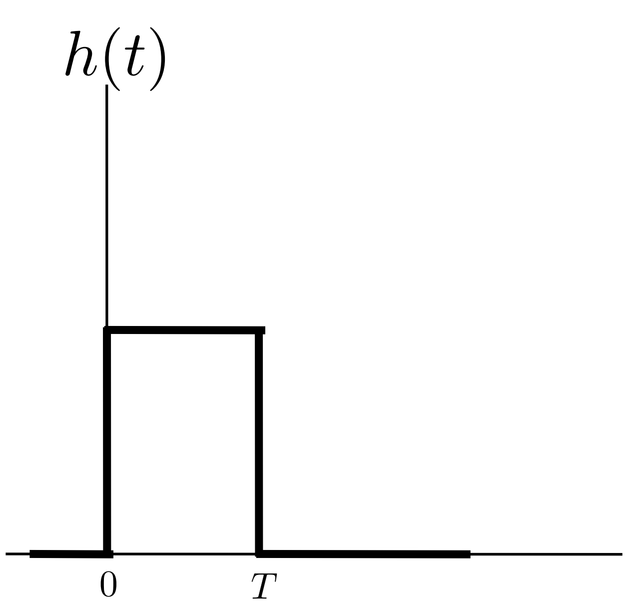
Signal representation of PAM signal in frequency-domain

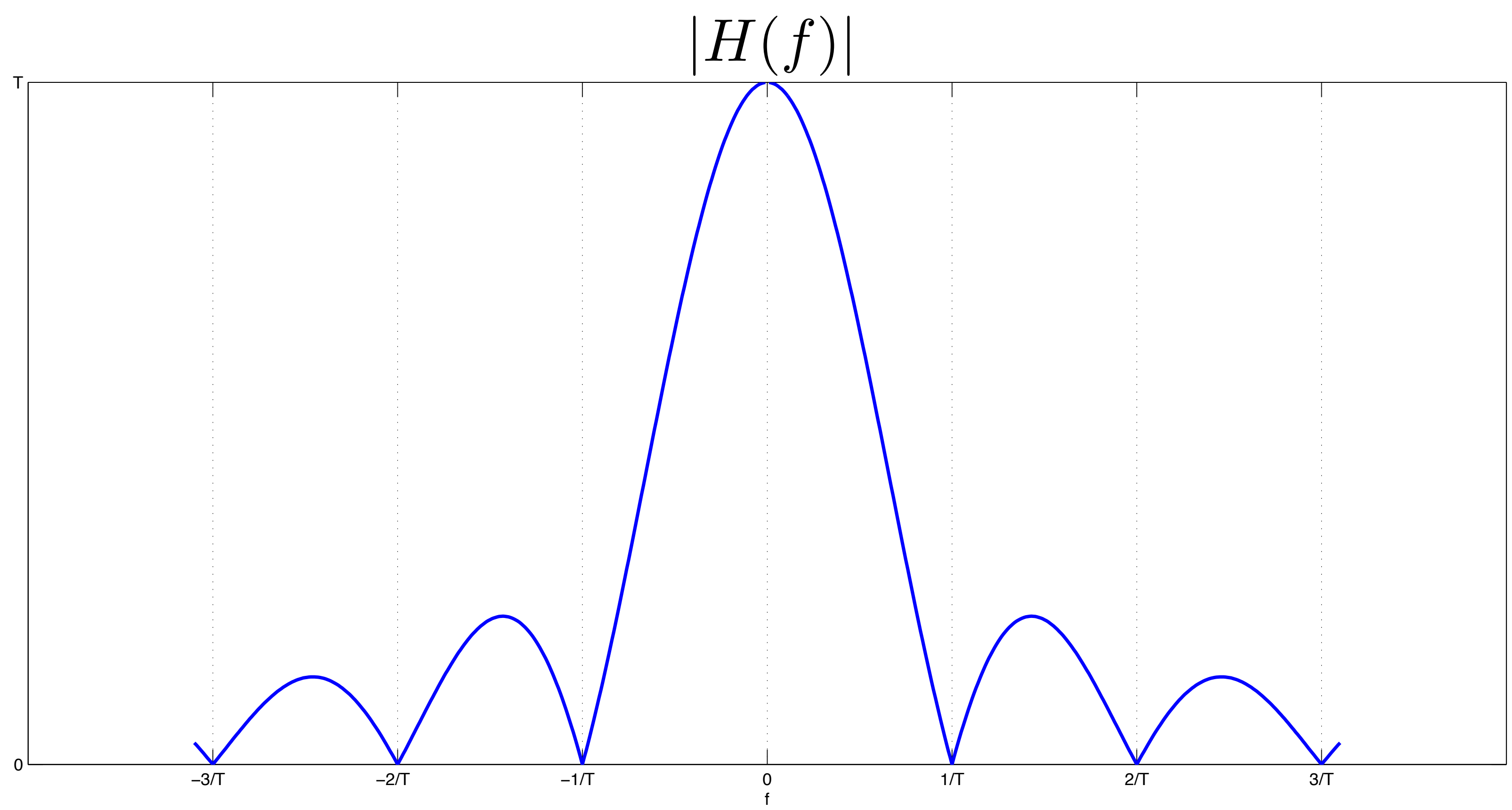
$$\begin{aligned} S(f) &= M_\delta(f) H(f) \\ &= \left(f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \right) H(f) \\ &= f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) H(f) \end{aligned}$$

where $H(f) = T\text{sinc}(fT)\exp(-j\pi f T)$

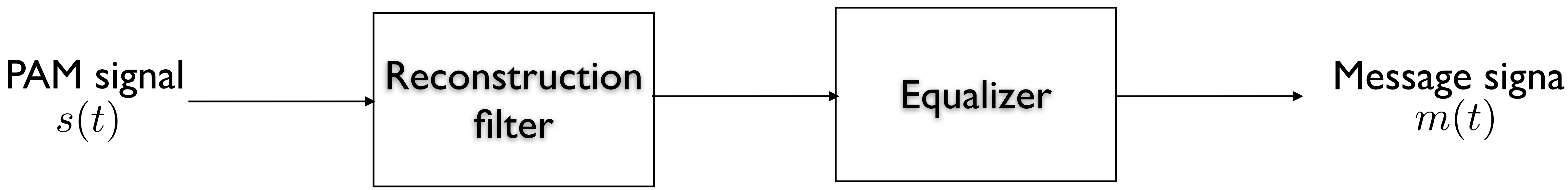
Aperture effect

- Employing the rectangular pulse results in amplitude distortion as well as a delay of $T/2$.
  - This is referred to as the *aperture effect*.





Equalization to correct the amplitude distortion



- Frequency response of ideal equalizer

$$\frac{1}{|H(f)|} = \frac{1}{T\text{sinc}(fT)} = \frac{\pi f}{\sin(\pi f T)}$$

Pulse-Position Modulation

- PDM (Pulse-duration modulation)
  - Pulse-width or pulse-length modulation
  - The samples of the message signal are used to vary the duration of individual pulses.
  - PDM is wasteful of power
- PPM (pulse-position modulation)
  - The position of a pulse relative to its unmodulated time of occurrence is varied in accordance with the message signal

$$s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s))$$

$$g(t) = 0, \quad |t| > (T_s/2) - k_p|m(t)|_{\max}, \qquad \Rightarrow \quad k_p|m(t)|_{\max} < T_2/2$$

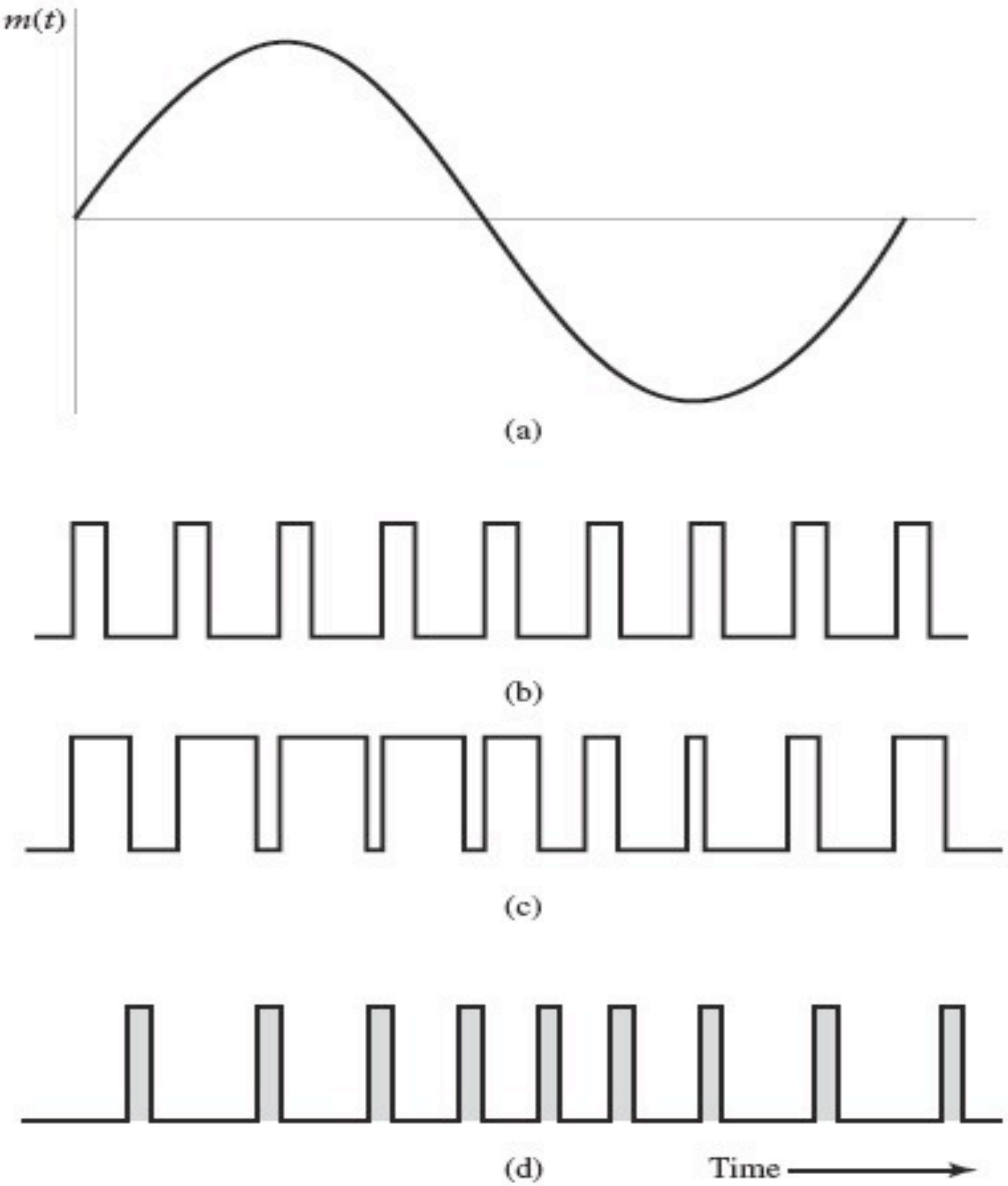


FIGURE 5.8 Illustration of two different forms of pulse-time modulation for the case of a sinusoidal modulating wave. (*a*) Modulating wave. (*b*) Pulse carrier. (*c*) PDM wave. (*d*) PPM wave.

[Ref: Haykin & Moher, Textbook]

Completing the Transition from Analog to Digital

- The advantages offered by digital pulse modulation
  - Performance
    - *Digital pulse modulation permits the use of regenerative repeaters, when placed along the transmission path at short enough distances, can practically eliminate the degrading effects of channel noise and signal distortion.*
  - Ruggedness
    - *A digital communication system can be designed to withstand the effects of channel noise and signal distortion*
  - Reliability
    - *Can be made highly reliable by exploiting powerful error-control coding techniques.*
  - Security
    - *Can be made highly secure by exploiting powerful encryption algorithms*
  - Efficiency
    - *Inherently more efficient than analog communication system in the tradeoff between transmission bandwidth and signal-to-noise ratio*
  - System integration
    - *To integrate digitized analog signals with digital computer data*

Quantization Process

- Amplitude quantization
  - The process of transforming the sample amplitude $m(nT_s)$ of a baseband signal $m(t)$ at time $t = nT_s$ into a discrete amplitude $\nu(nT_s)$ taken from a finite set of possible levels.

$$I_k : \{m_k < m \le m_{k+1}\}, \qquad k = 1, 2, \ldots, L$$

  - Representation level (or Reconstruction level)
    - The amplitudes $\nu_k,\ k = 1, 2, 3, \ldots, L$
  - Quantum (or step-size)
    - The spacing between two adjacent representation levels

$$v = g(m)$$

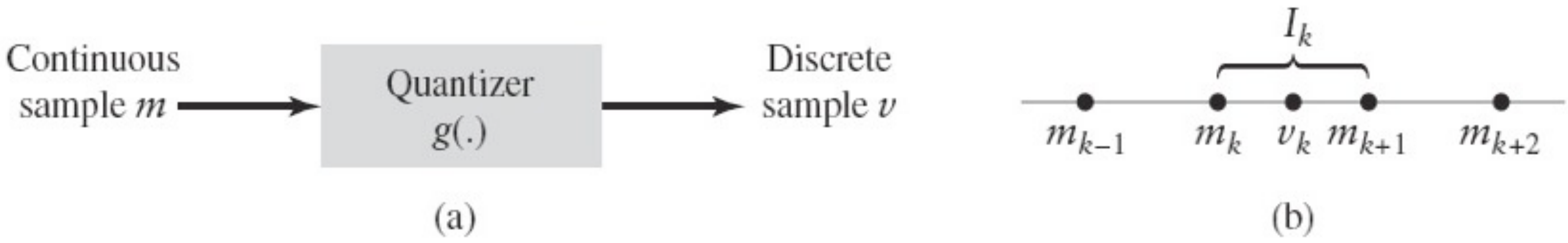


FIGURE 5.9 Description of a memoryless quantizer.

[Ref: Haykin & Moher, Textbook]

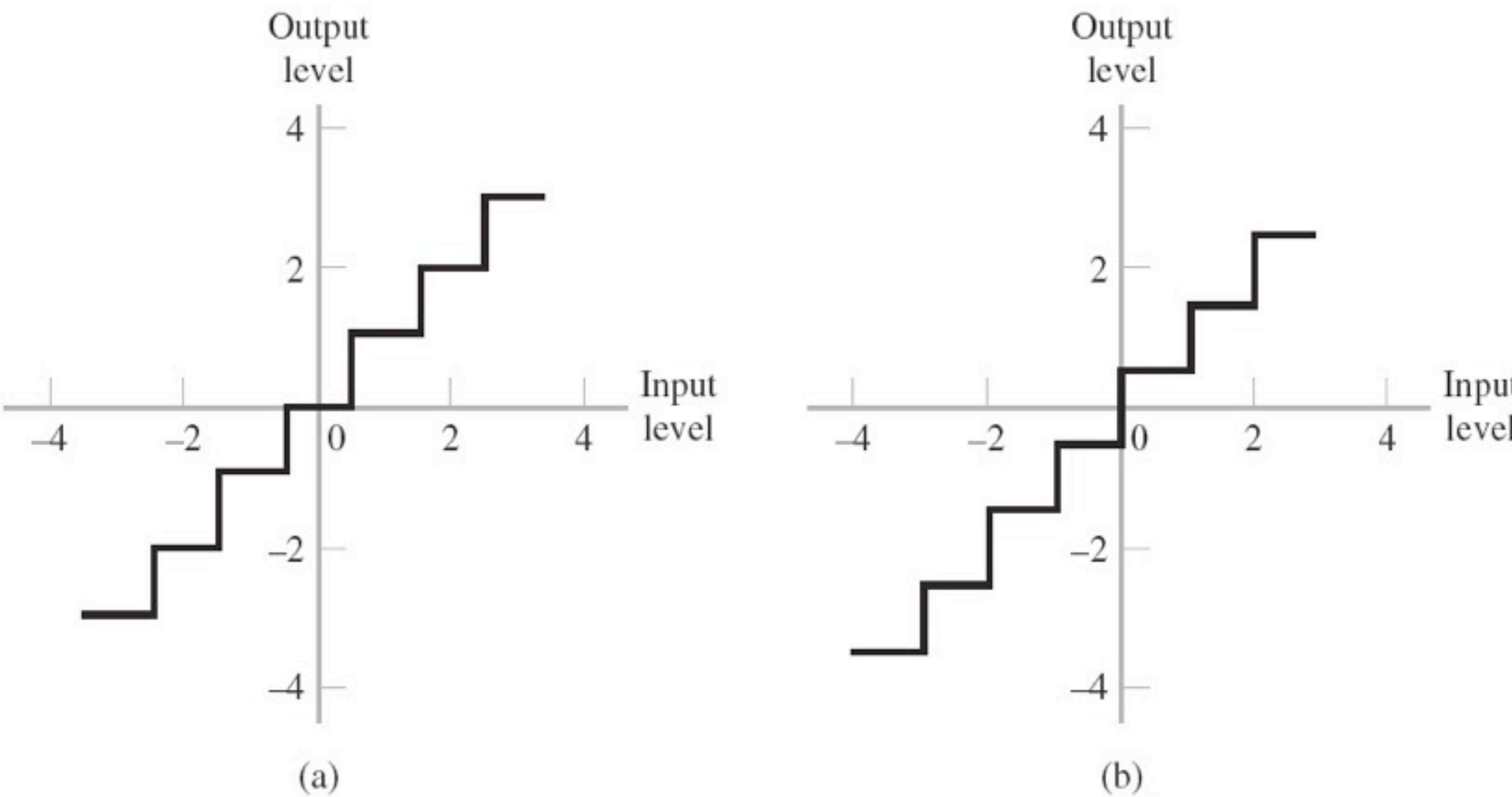


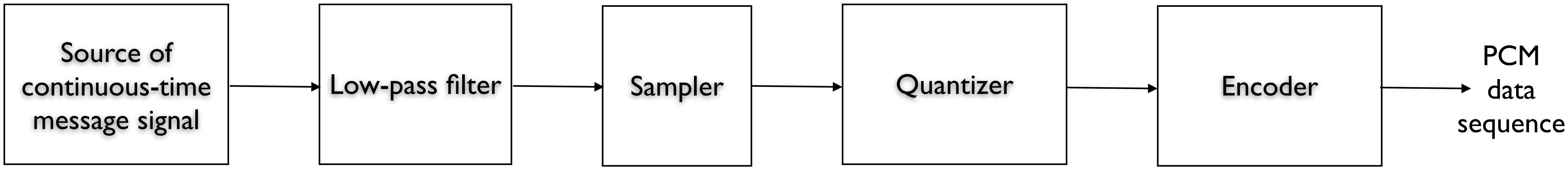
FIGURE 5.10 Two types of quantization: (*a*) midtread and (*b*) midrise.

[Ref: Haykin & Moher, Textbook]

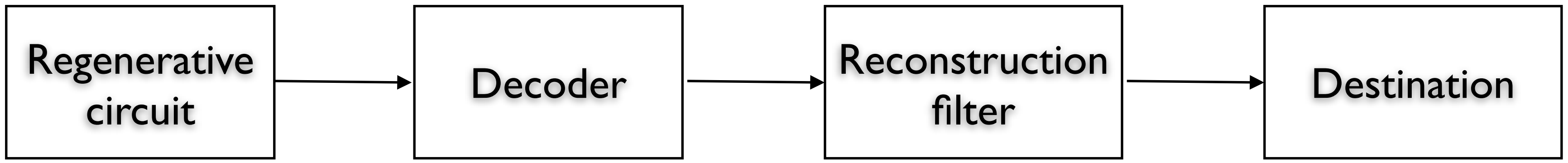
Pulse-Code Modulation (PCM)

- PCM is the most basic form of digital pulse modulation.
- In PCM, a message signal is represented by a sequence of coded pulses, which is accomplished by representing the signal in discrete form in both time and amplitude.
- The basic operations performed in the transmitter of a PCM system
  - Sampling
  - Quantization
  - Encoding

- Block diagram of the transmitter

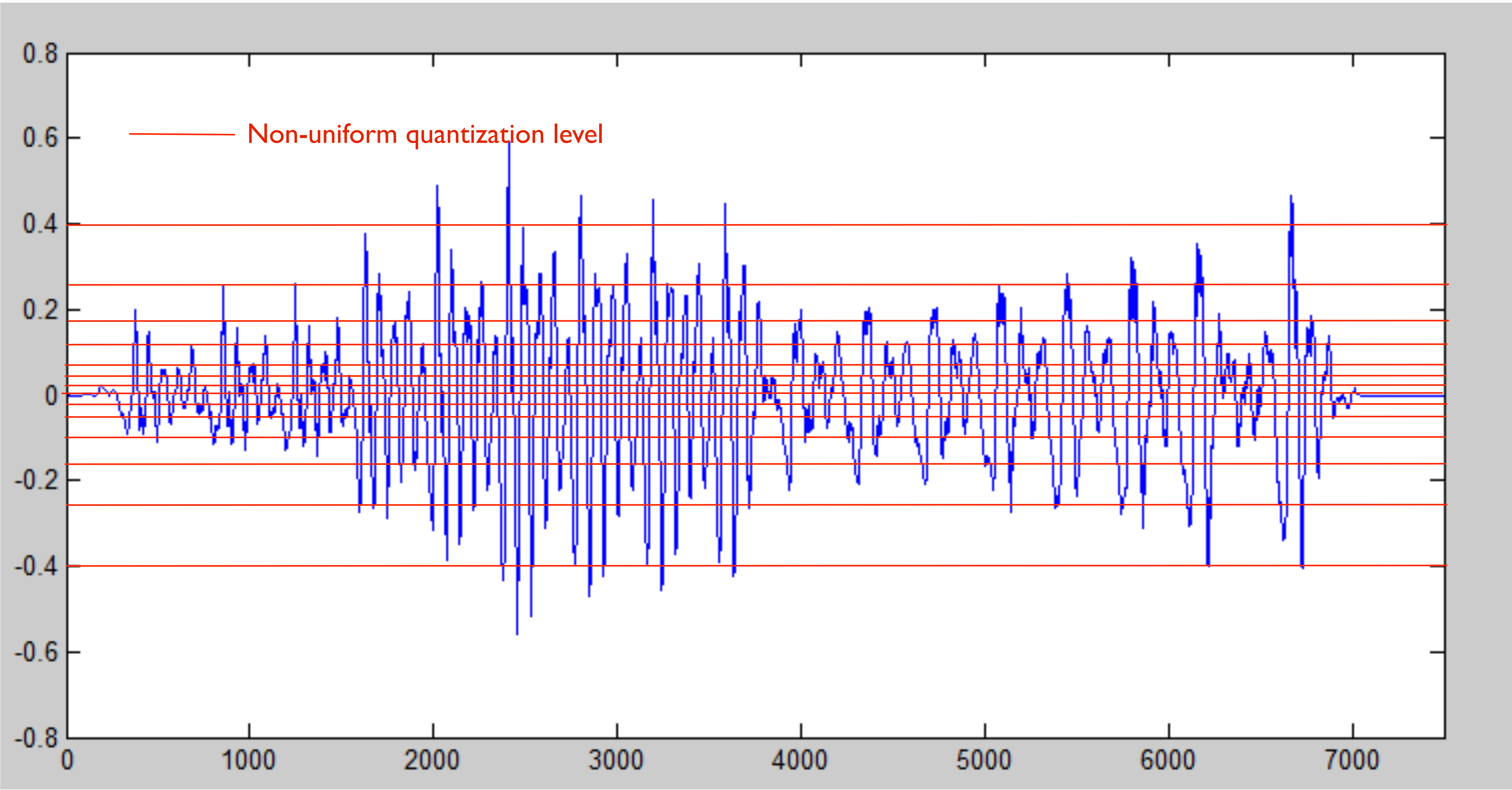


- Block diagram of the receiver



- Operations in the transmitter
  - Sampling
    - The incoming message signal is sampled with a train of rectangular pulses.
    - The reduction of the continuously varying message signal to a limited number of discrete values per second
  - Nonuniform quantization
    - Different step size per each level is used.
    - Frequent level of the signal has small step size while infrequent level has relatively large step size.

Typical voice signal



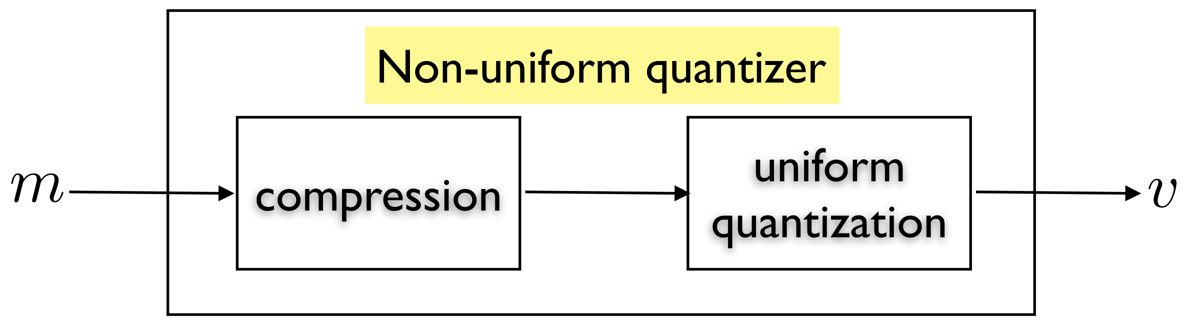
Nonuniform quantizer

- The use of a nonuniform quantizer is equivalent to passing the message signal through a compressor and then applying the compressed signal to a uniform quantizer.

- Compression law

  - μ - *law*

$$|v| = \frac{\ln(1+\mu|m|)}{\ln(1+\mu)}$$

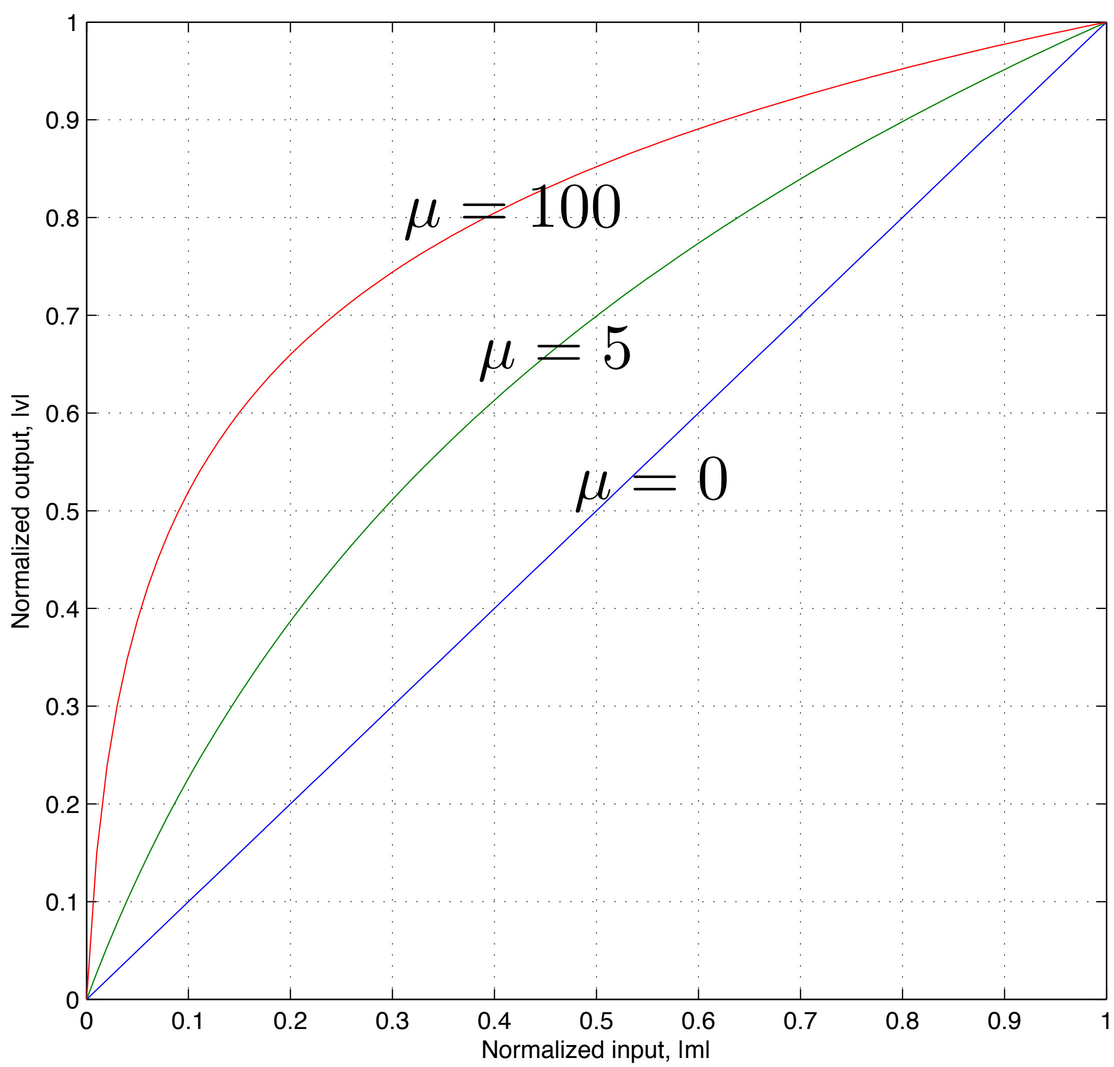


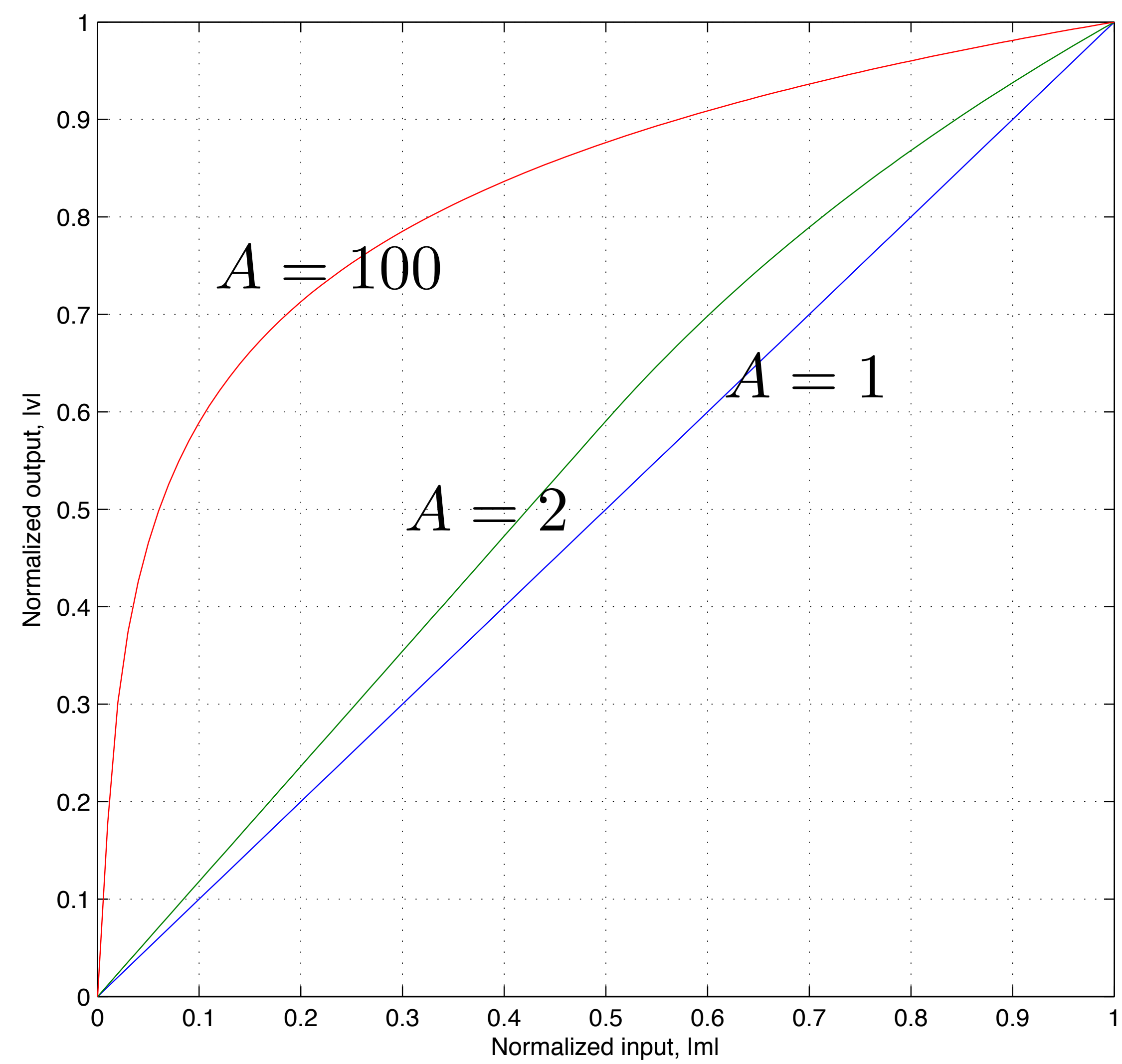
  - A - *law*

$$|v| = \begin{cases} \frac{A|m|}{1+\ln A}, & 0 \le |m| \le \frac{1}{A} \\ \frac{1+\ln(A|m|)}{1+\ln A}, & \frac{1}{A} \le |m| \le 1 \end{cases}$$

Two different compressions

- In practical system, $\mu = 255$ and $A = 100$ or near those values are often used.





Step size of two different compressors

$$\mu - law: \quad \frac{d|m|}{d|v|} = \frac{\ln(1+\mu)}{\mu}(1+\mu|m|) \quad , \quad A - law: \quad \frac{d|m|}{d|v|} = \begin{cases} \frac{1+\ln A}{A}, & 0 \le |m| \le \frac{1}{A} \\ (1+\ln A)|m|, & \frac{1}{A} \le |m| \le 1 \end{cases}$$