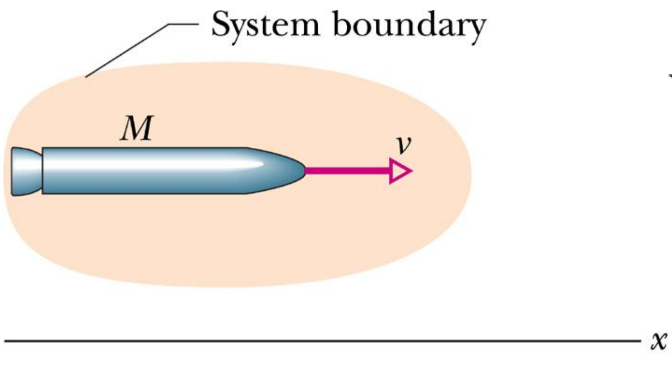
Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8^{th} and 9^{th} Ed.
- The rest is made by me.

질량이 변하는 계: 로켓



(a)

외부의 힘이 없으므로 운동량이 보존된다. $\mathbf{P}_i = \mathbf{P}_f$

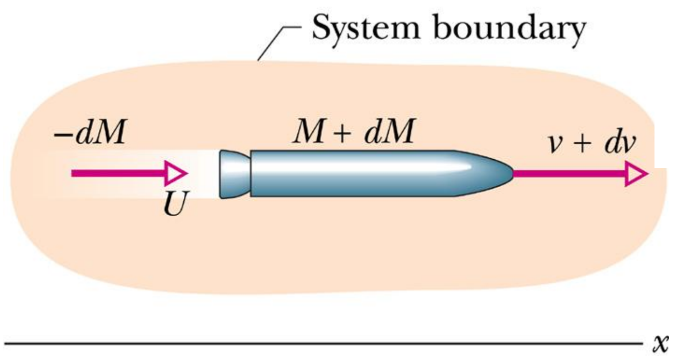
$$\boxed{Mv = (-dMU) + (M + dM)(v + dv)}$$

dt 후의 운동량

로켓과 분출된 연료의 상대속도 v_{rel}을 도입하면

로켓의 속도 = 로켓의 연료에 대한 상대속도
+ 연료의 속도

$$(v + dv) = v_{\mathrm{rel}} + U \rightarrow U = v + dv - v_{\mathrm{rel}}$$



(b)

$$Mv = -dM(v + dv - v_{rel}) + (M + dM)(v + dv)$$

$$0 = \cancel{-dMv} + v_{rel}\,dM + M\,dv + \cancel{v\,dM}$$

따라서

$$-dM\,v_{\text{rel}} = M\,dv$$

가 되므로

$$-\frac{dM}{dt}v_{\text{rel}} = M\frac{dv}{dt}$$

이다. $dM/dt = -R$로 일정하다면

thrust

$$Rv_{\text{rel}} = M\frac{dv}{dt}$$

를 얻는다. 따라서

$$dv = -v_{\text{rel}}\frac{dM}{M}$$

이므로

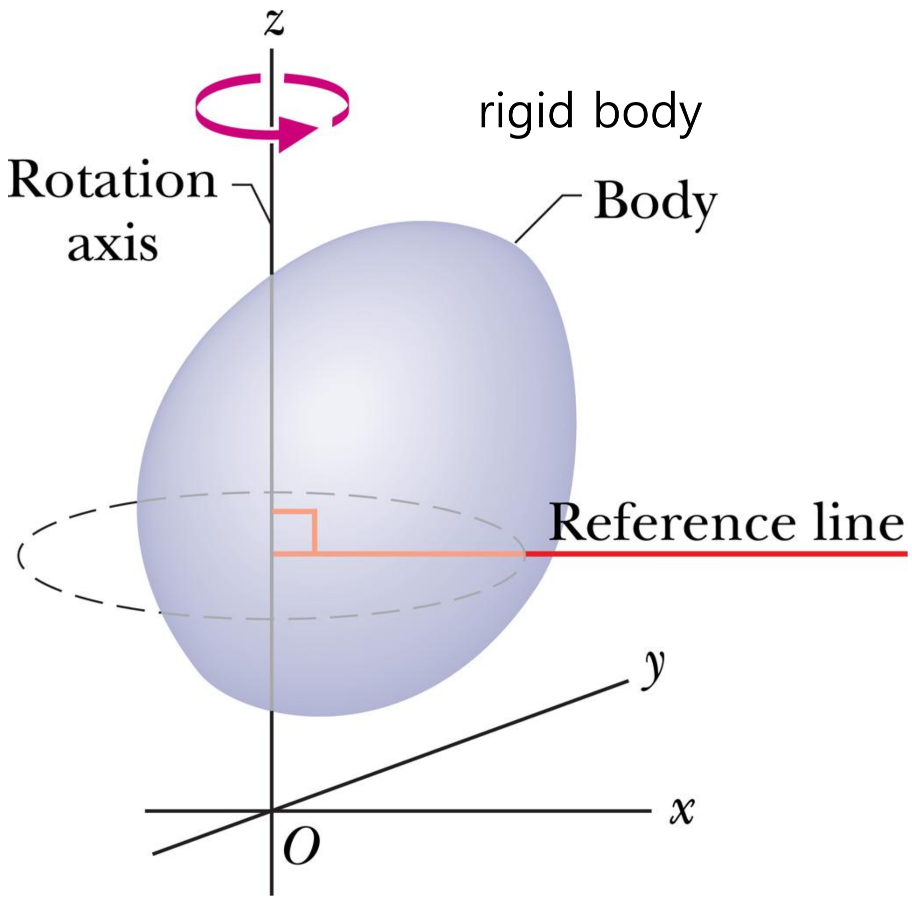
$$v_f - v_i = \int_{v_i}^{v_f} dv = -v_{\text{rel}}\int_{M_i}^{M_f}\frac{dM}{M} = v_{\text{rel}}\ln\frac{M_i}{M_f}$$

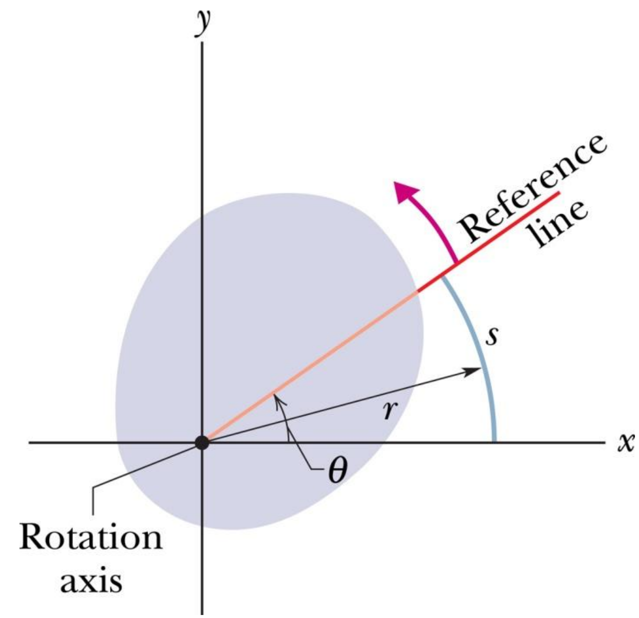
를 얻는다.

Chapter 9 Circular Motion

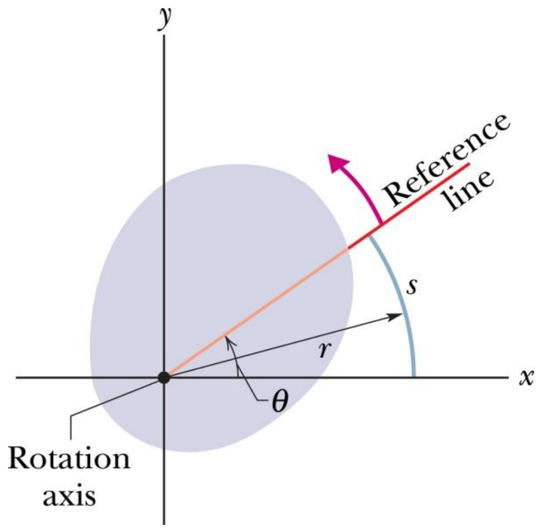
Rotational variables

rigid body





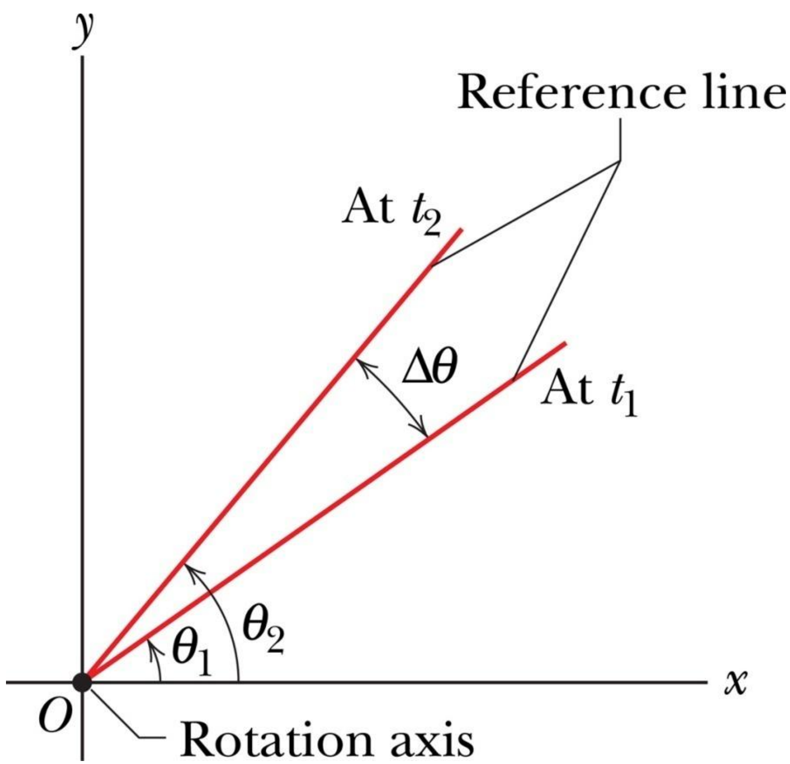
translation		rotation	
displacement	$\mathbf{r}$	angular D	θ
velocity	$\mathbf{v}$	angular V	ω
Accel.	$\mathbf{a}$	Angular A.	α



Angular position $\theta = \dfrac{s}{r}$ (in radian)

$$1 \text{ rev} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

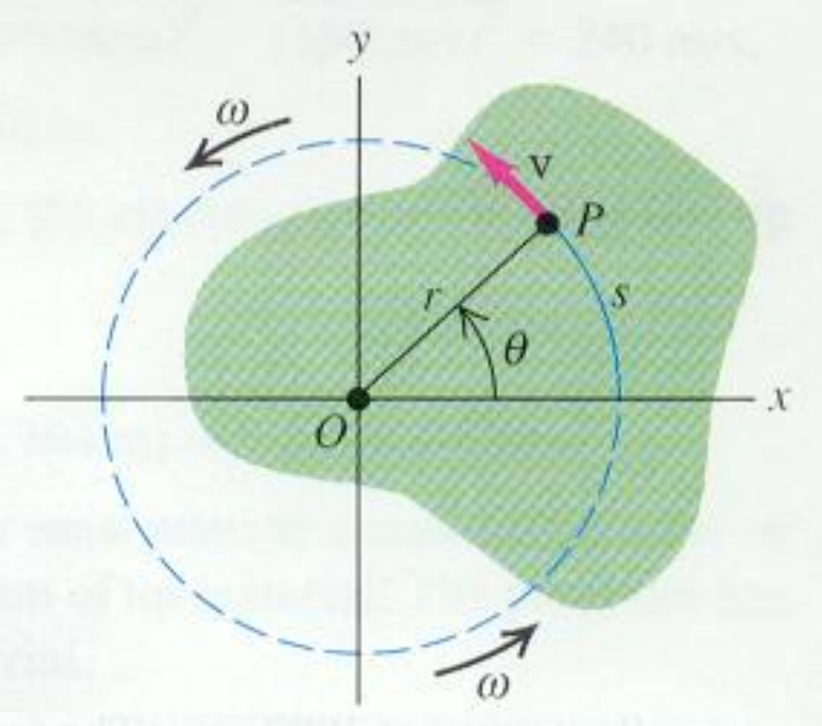
$$1 \text{ rad} = 57.3^\circ$$

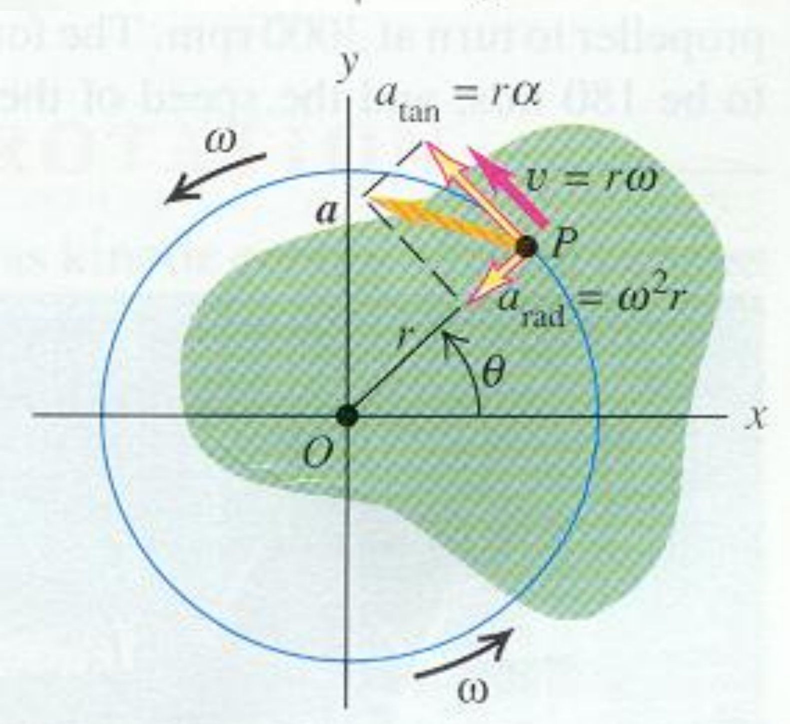


Angular displacement: $\Delta\theta = \theta_2 - \theta_1$

Angular velocity: $\omega = \lim_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

Angular acceleration: $\alpha = \lim_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t} = \dfrac{d\omega}{dt}$





$$s = r\theta$$

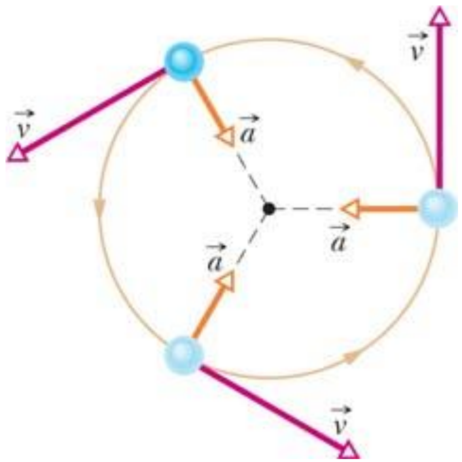
$$v = \frac{ds}{dt} = \frac{rd\theta}{dt} = r\omega$$

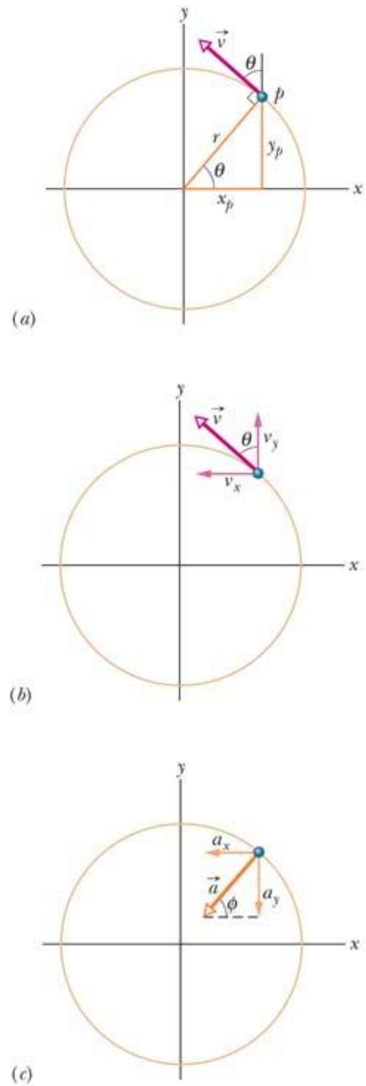
$$a_t = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r\alpha$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$\boxed{\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r}$$

Uniform circular motion





$$
\begin{aligned}
\vec{v} &= v_x\mathbf{i} + v_y\mathbf{j} = -v\sin\theta\mathbf{i} + v\cos\theta\mathbf{j} \\
&= \left(-\frac{vy_p}{r}\right)\mathbf{i} + \left(\frac{vx_p}{r}\right)\mathbf{j}
\end{aligned}
$$

$$
\begin{aligned}
\vec{a} &= \frac{d\vec{v}}{dt} \\
&= \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\mathbf{i} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\mathbf{j} \\
&= -\left(\frac{v^2}{r}\cos\theta\right)\mathbf{i} + \left(-\frac{v^2}{r}\sin\theta\right)\mathbf{j}
\end{aligned}
$$

Centripetal acc.

$$a = \frac{v^2}{r}$$

period

$$T = \frac{2\pi r}{v}$$

$$\boxed{a = \frac{v^2}{r} \qquad \tan\phi = \tan\theta}$$

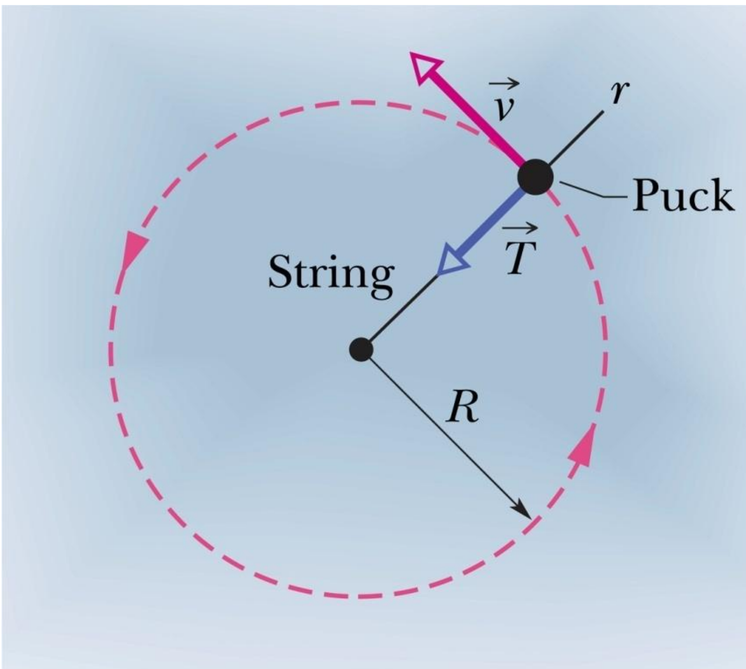
각운동에서의 변수들을 선운동에서의 변수들이 만족하는 방정식과 매우 비슷한 방정식을 만족한다.

$$x \to \theta, \quad v \to \omega, \quad a \to \alpha$$

표 1: 선운동과 회전운동에서 변수들 사이의 관계

선운동 방정식	빠진 변수	빠진 변수	각운동 방정식
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_o t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Uniform circular motion



$$v = R\omega \rightarrow a = R\omega^2$$

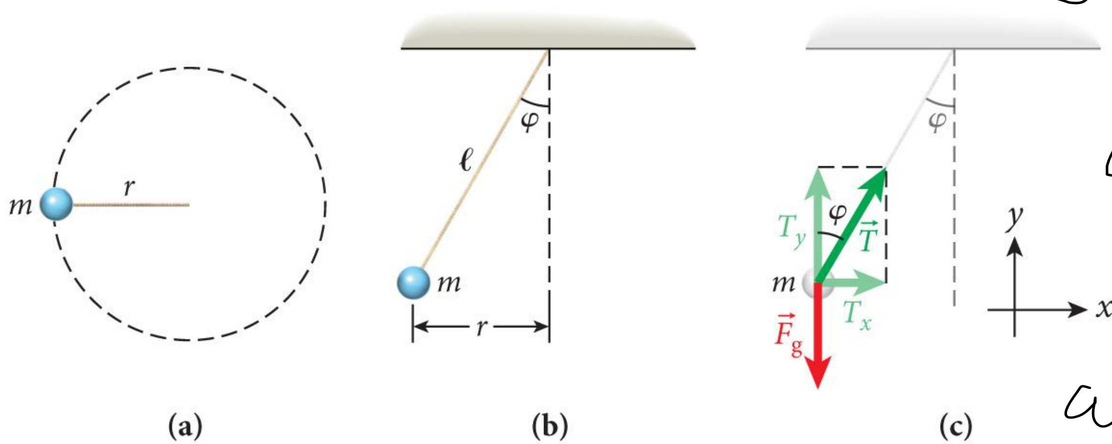
Centripetal acceleration $$a = \frac{v^2}{R}$$

Centripetal force $$F = m\frac{v^2}{R}$$

Source:

tension, gravity, friction

Conical pendulum



(a) (b) (c)

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

$$\omega T = 2\pi$$

$$T\cos\phi = mg,$$
$$T\sin\phi = mr\omega^2.$$

$$r = l\sin\phi \to T = ml\omega^2$$

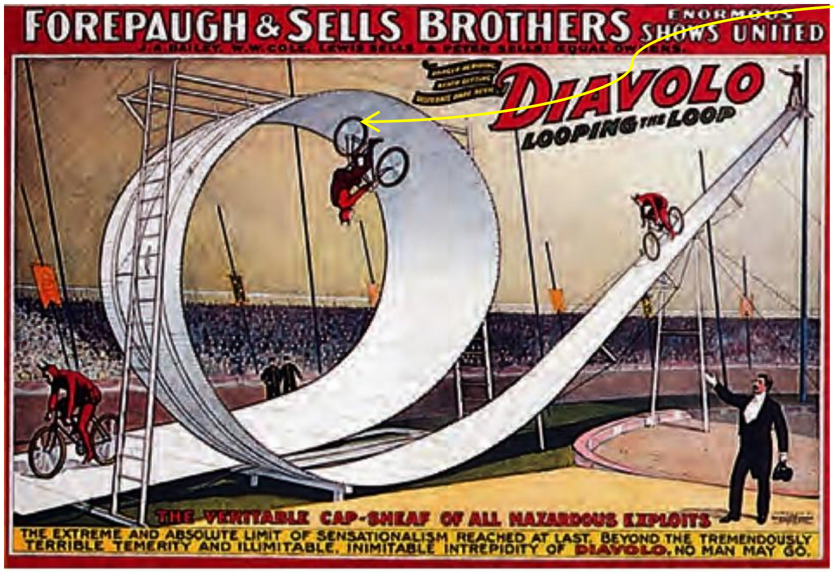
$$ml\omega^2\cos\phi = mg$$

$$\omega^2 = \frac{g}{l\cos\phi} \to \omega = \sqrt{\frac{g}{l\cos\phi}}$$

$$T = \frac{2\pi r}{v_{\text{''} r\omega}} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\phi}{g}}$$

Sample problem

$v_{\cancel{max}}^{min} = ?$



(a)

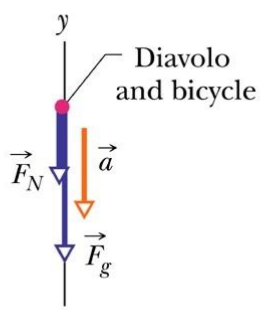
$$F_N + mg = \frac{mv^2}{R}$$

$$F_N \geq 0$$

$$mg$$

$$g \geq \frac{v^2}{R}$$

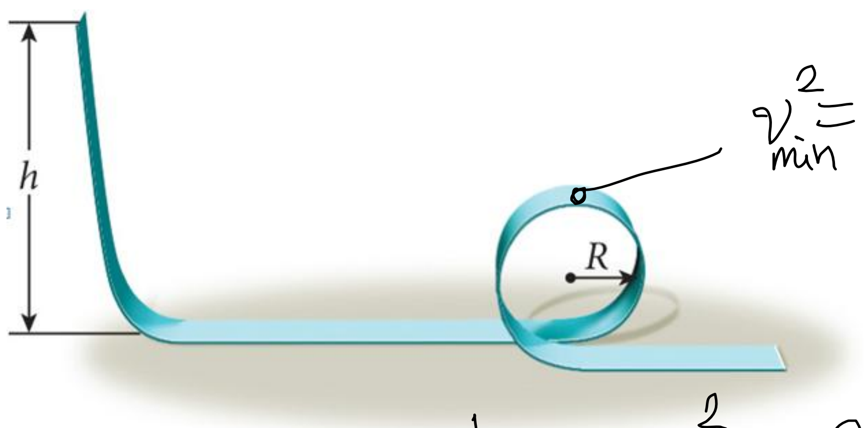
$$v_{min} = \sqrt{Rg}$$



(b)

Prob. 9.21

$h_{\min} = ?$



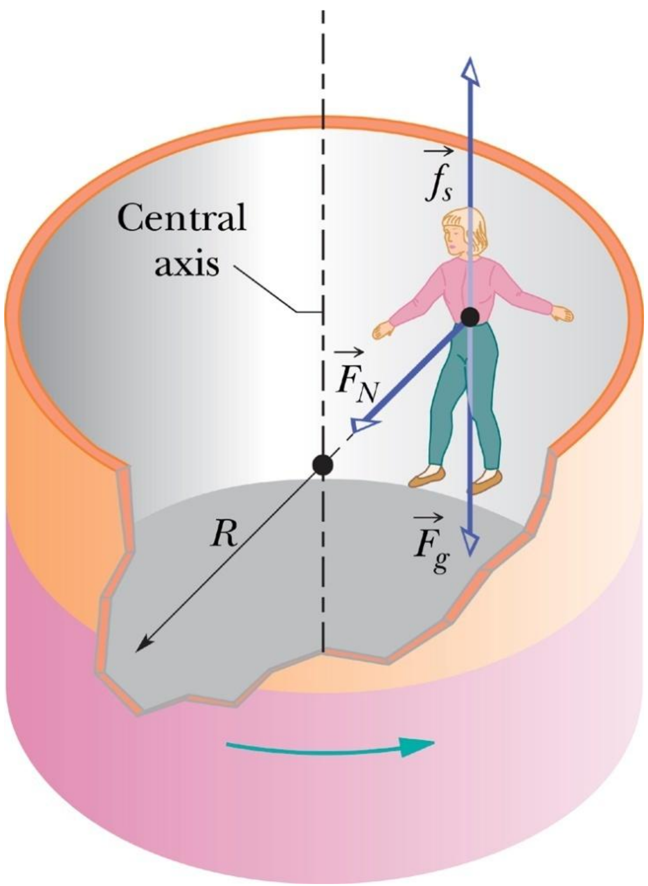
$v_{min}^2 = Rg$

$$\cancel{m}gh_{min} = \frac{1}{2}\cancel{m}v_{min}^2 + 2\cancel{m}gR$$

$$= \frac{5}{2}Rg$$

$$h_{min} = \frac{5}{2}R$$

Sample problem



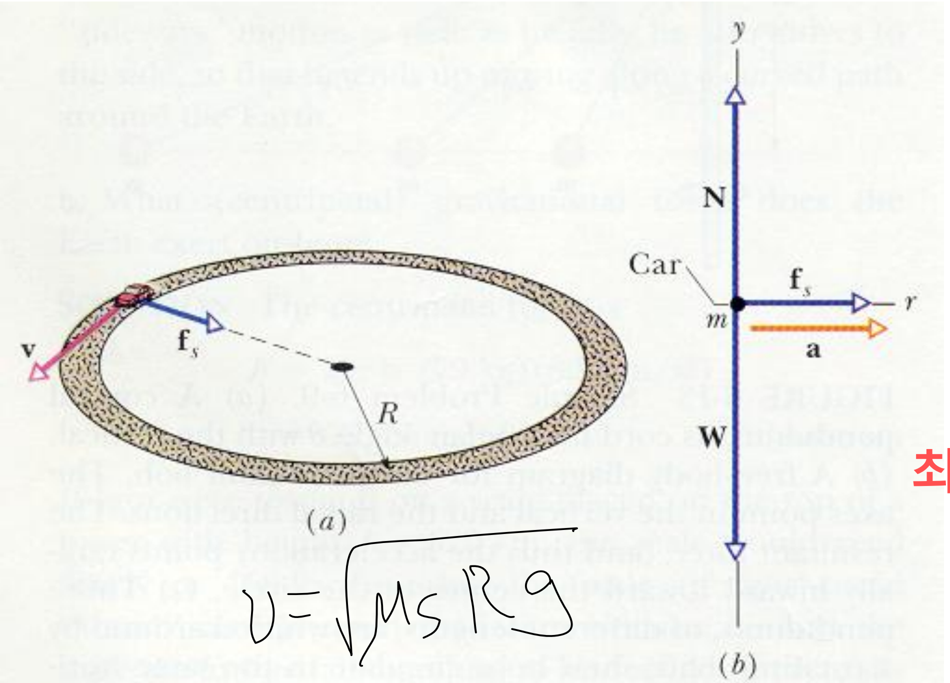
$$F_N = m\frac{v^2}{R}$$

$$f_s = \mu_s F_N = mg$$

$$\mu_s \frac{v^2}{R} = g$$

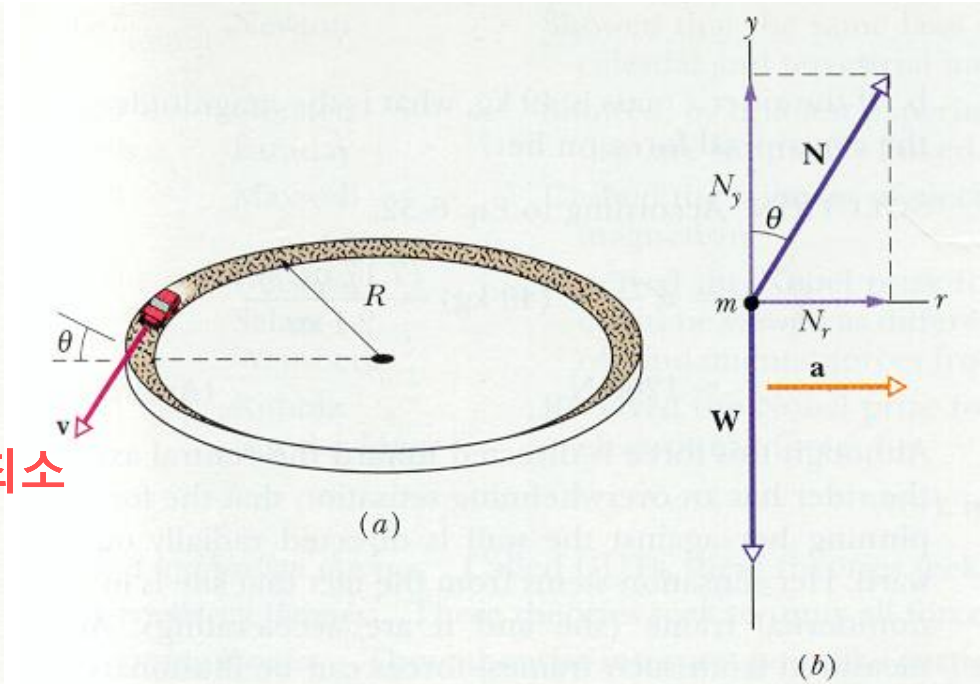
$$v = \sqrt{\frac{Rg}{\mu_s}}$$

Sample problem



$v = \sqrt{\mu_s R g}$

최소



$$y:\ N - W(mg) = 0$$

$$r:\ \frac{mv^2}{R} - f_s = 0$$

$$\boxed{\mu_s = \frac{mv^2}{mgR} = 0.21}$$ ➔최소

$N\cos\theta = mg$

$$y:\ N\cos\theta = mg$$

$N\sin\theta = \frac{mv^2}{R}$

$v = \sqrt{Rg\tan\theta}$

$$r:\ N\sin\theta = \frac{mv^2}{R}$$

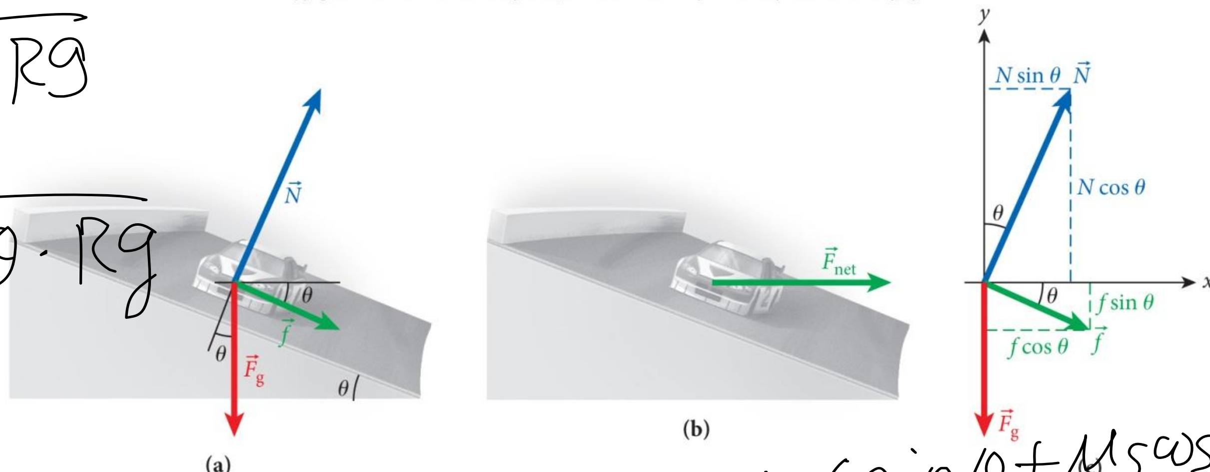
$\tan\theta = \frac{v^2}{Rg}$

$$\boxed{\tan\theta = \frac{v^2}{gR} \Rightarrow \theta = 12^\circ}$$

SP 9.2 NASCAR racing

$$v = \sqrt{\mu_s R g}$$

$$v = \sqrt{\tan\theta \cdot R g}$$



$$N\cos\theta = f\sin\theta + mg$$

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

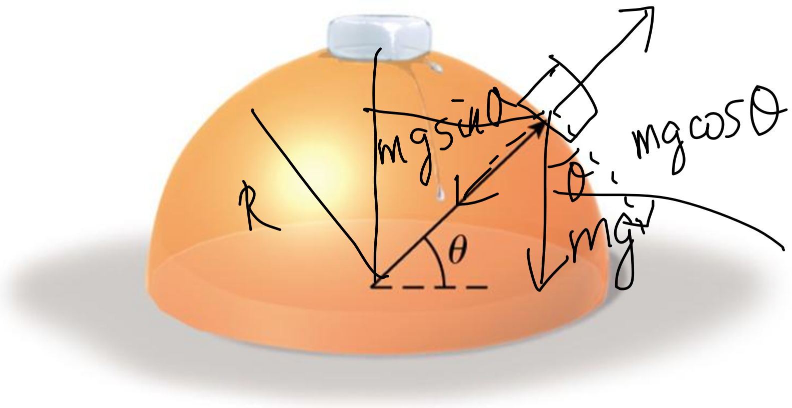
$$f = \mu_s N$$

$$N(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{R}$$

$$N(\cos\theta - \mu_s\sin\theta) = mg$$

$$v = \sqrt{Rg\,\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}$$

Prob. 9.24



$$\cancel{m}g\sin\theta = \frac{\cancel{m}v^2}{R}$$

$$= 2g(1-\sin\theta)$$

$$h = R\sin\theta$$

$$= \frac{2}{3}R$$

$$\sin\theta = 2 - 2\sin\theta$$

$$mgR = \frac{1}{2}mv^2 + mgR\sin\theta$$

$$3\sin\theta = 2$$

$$\cancel{m}v^2 = 2gR(1-\sin\theta)$$

$$\sin\theta = \frac{2}{3}$$