Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

Announcement

- 3월 14일 (목) 휴강
- 보강 시간과 장소는 3월 19(화)에 공지

All about vectors!

1. vector란 무엇인가?
   상식적, 수학적, 물리적인 정의

2. vector의 표현방법
   기하학적 방법, 대수학적 방법, 단위벡터 이용하기

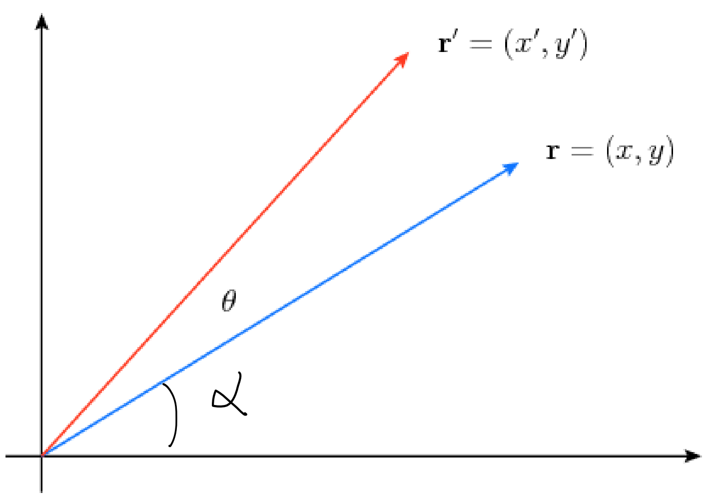
3. vector의 덧셈

4. vector의 곱셈
   - vector와 scalar의 곱셈
   - vector와 vector의 곱셈: scalar product,
     vector product

vector란 무엇인가?

1. 크기와 방향을 갖는 양.   N.b. scalar
2. vector 공간 안의 한 점과 일대일 대응을 시킬 수 있다.
3. 엄밀한 정의

   Euclid 공간에서 좌표를 회전시켰을 때 좌표성분이 변환하는 것처럼 변하는 숫자들의 집합.



$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

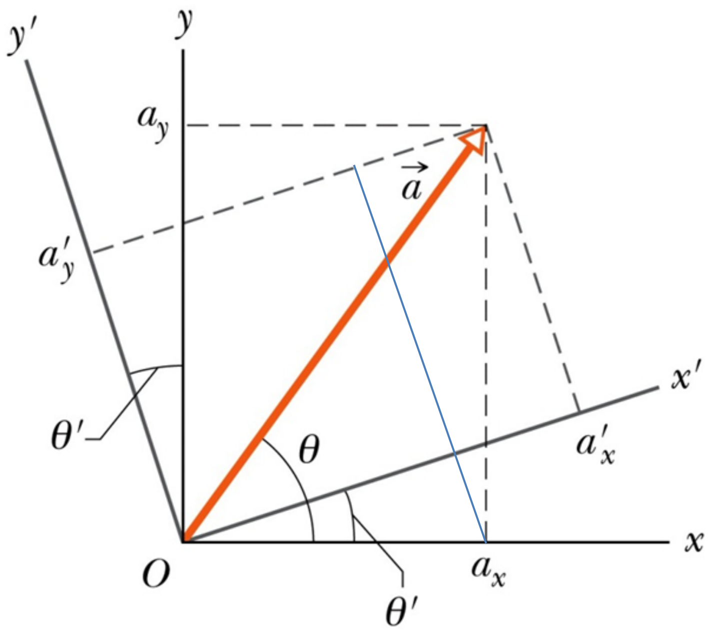
$x = r\cos\alpha,\ y = r\sin\alpha$

$x' = r\cos(\theta+\alpha)$

$= r\cos\alpha\cos\theta - r\sin\alpha\sin\theta$

$= x\cos\theta - y\sin\theta$

Coordinate rotation

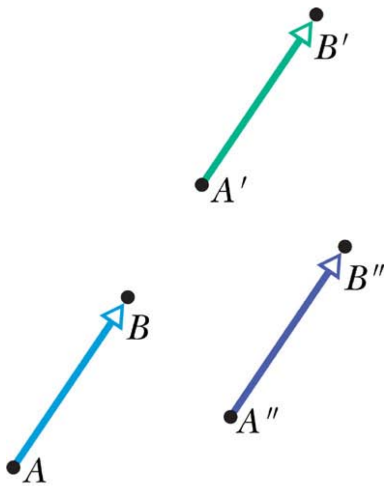


$$a'_x = a_x \cos\theta' + a_y \sin\theta'$$

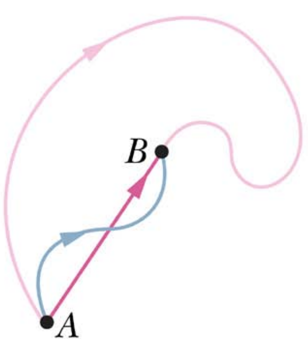
$$a'_y = -a_x \sin\theta' + a_y \cos\theta'$$

Vector의 표현방법

(1) Geometric method

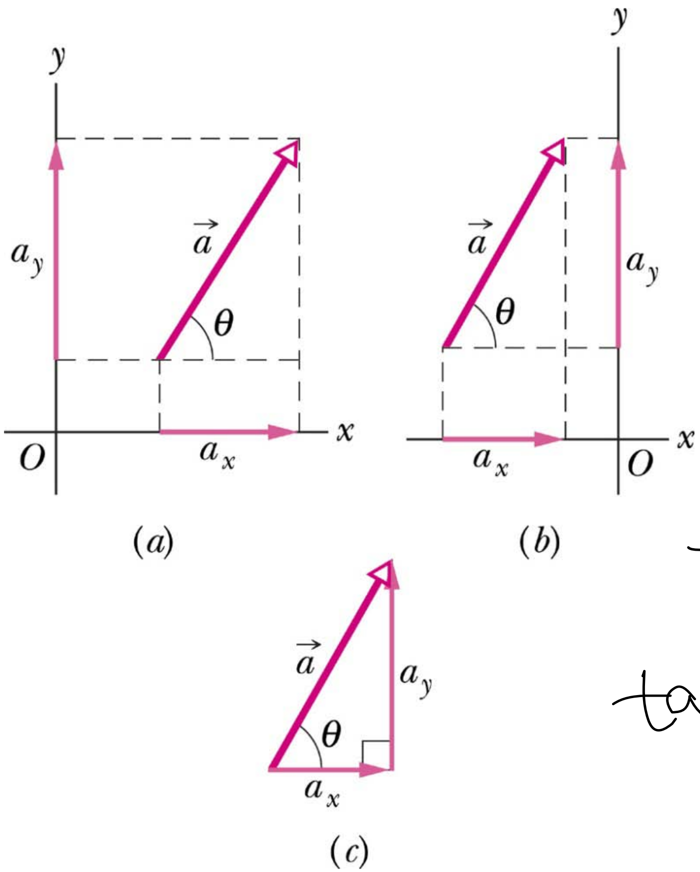


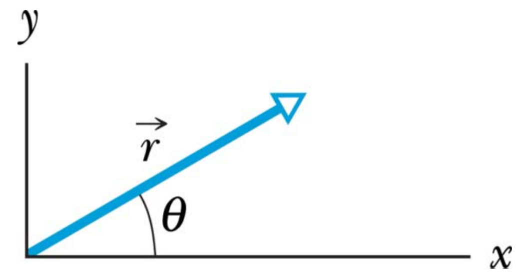
(a)



(b)

(2) Algebraic method





$$\theta = \tan^{-1}\frac{a_y}{a_x}$$

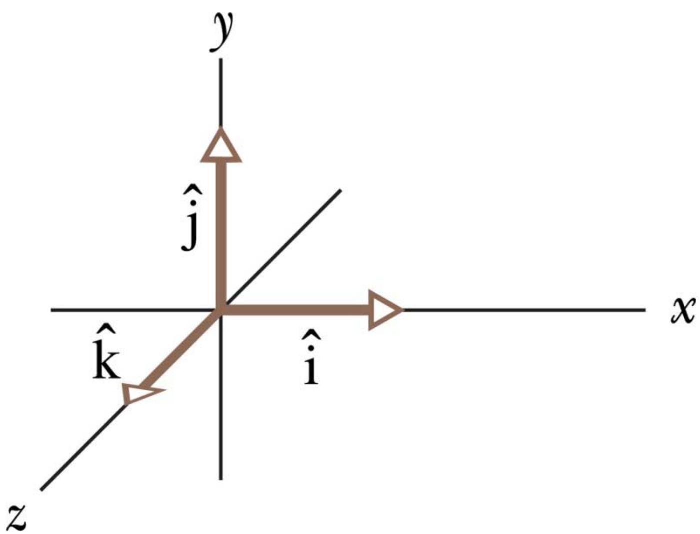
$$f^{-1}f(\theta) = \theta$$

$$\tan^{-1} \quad \tan\theta = \tan^{-1}\frac{a_y}{a_x}$$

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta,$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}$$

(3) Unit vectors

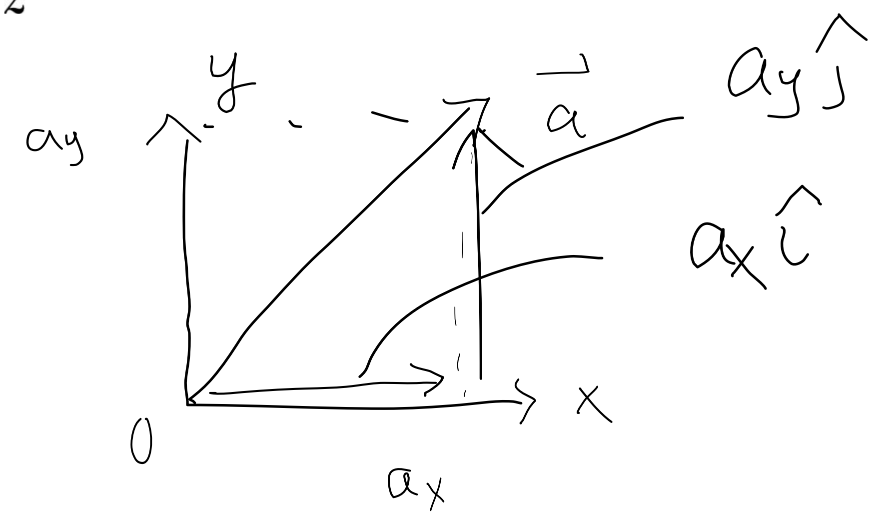


2D

$$\vec{a} = (a_x, a_y) = a_x\hat{i} + a_y\hat{j}$$

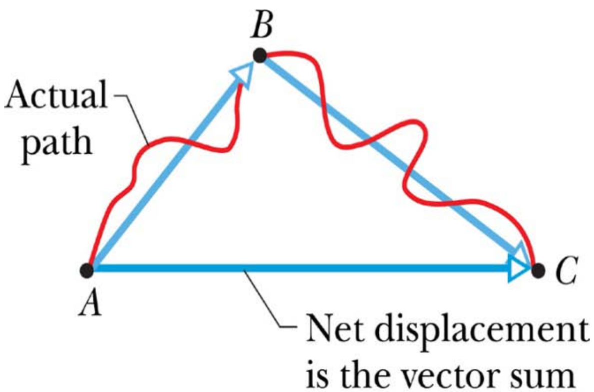
3D

$$\vec{a} = (a_x, a_y, a_z) = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

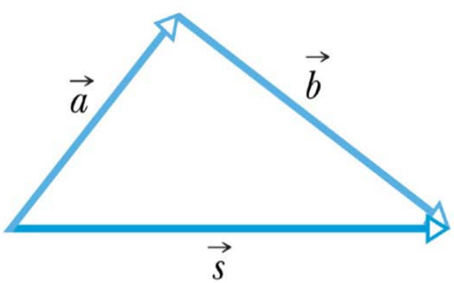


벡터의 덧셈

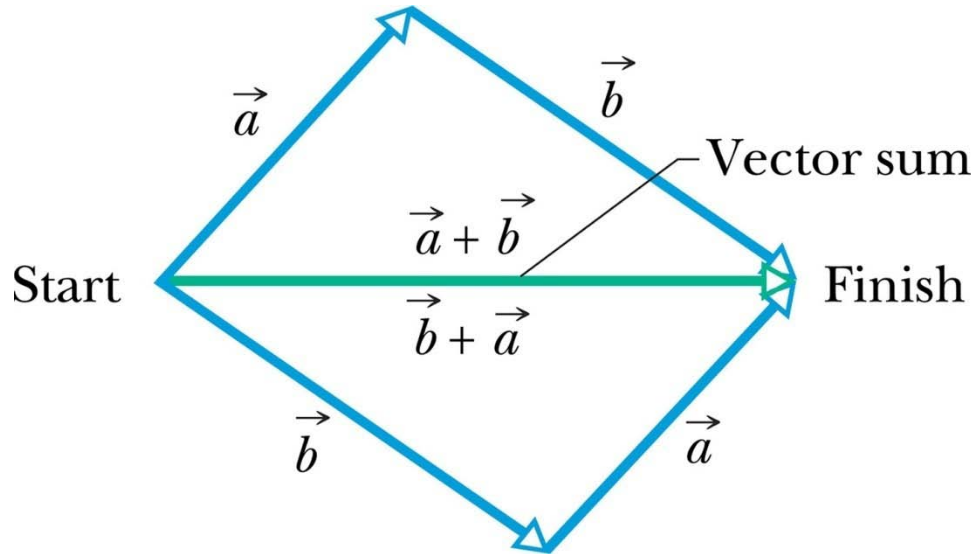
1. Geometric method, parallelogram method



(a)

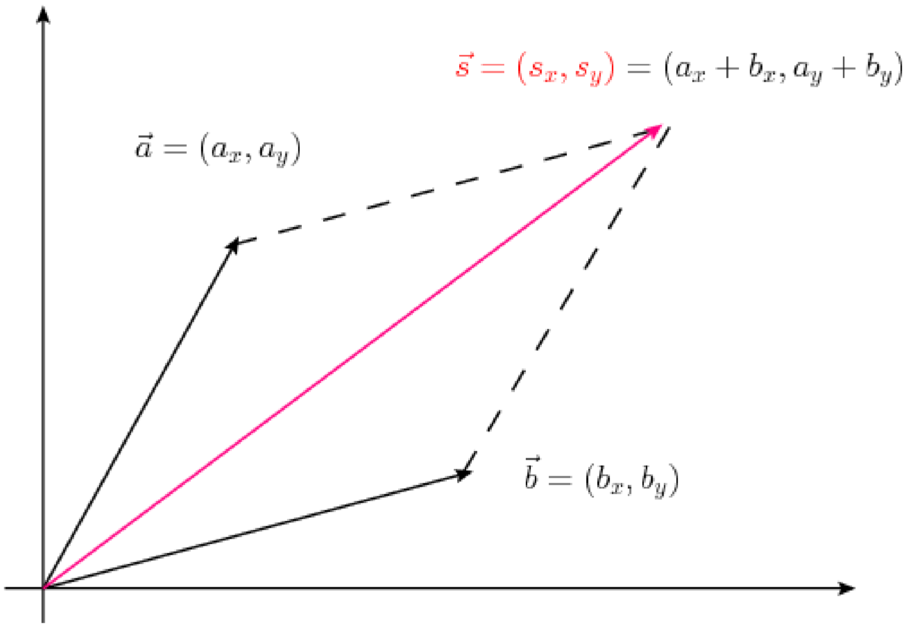


(b)



$$\vec{s} = \vec{a} + \vec{b}$$

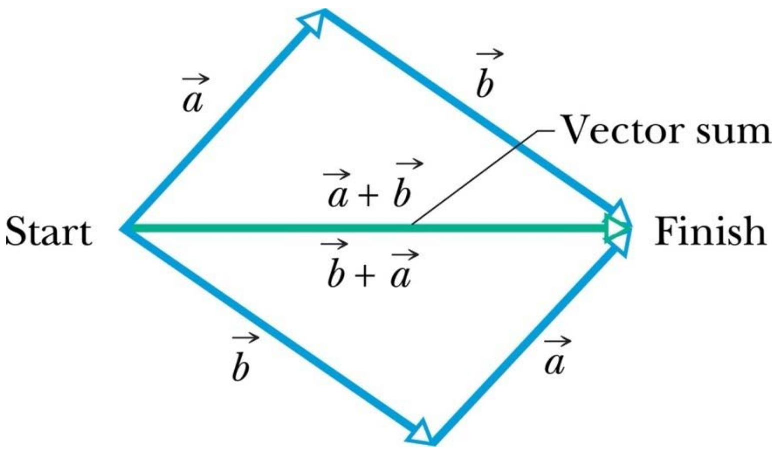
2. Algebraic method, adding components



$$\vec{a} = (a_x, a_y),\ \vec{b} = (b_x, b_y)$$

$$\vec{s} = (s_x, s_y) = (a_x + b_x, a_y + b_y)$$

vector 덧셈의 성질

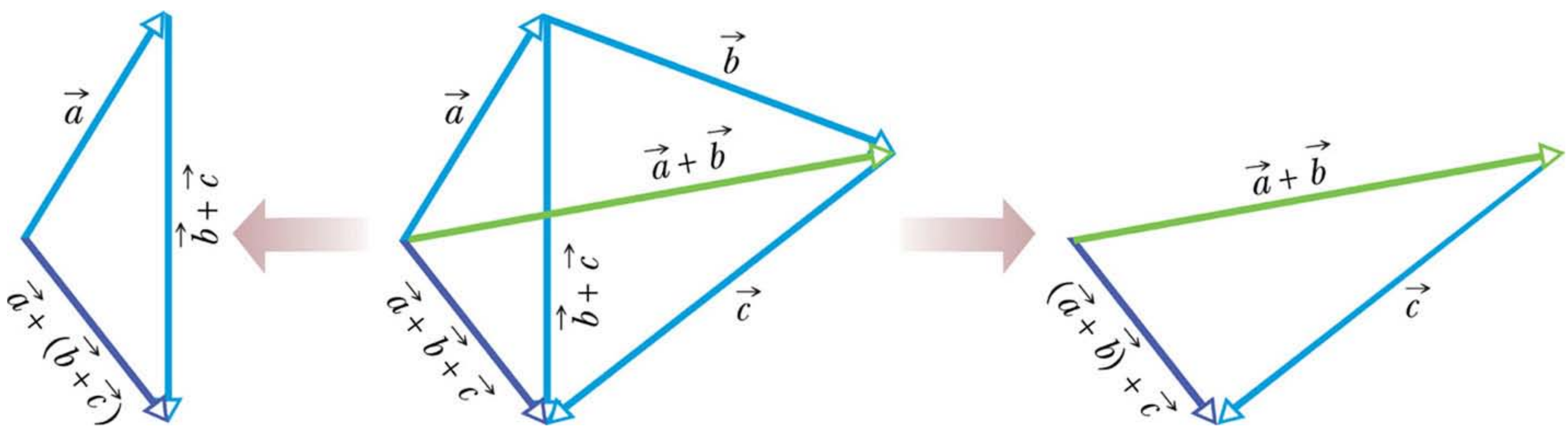


교환법칙

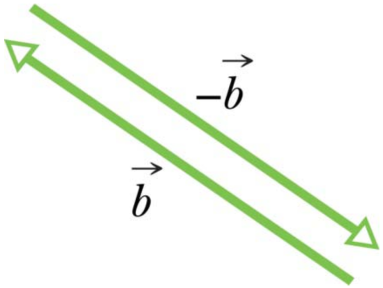
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

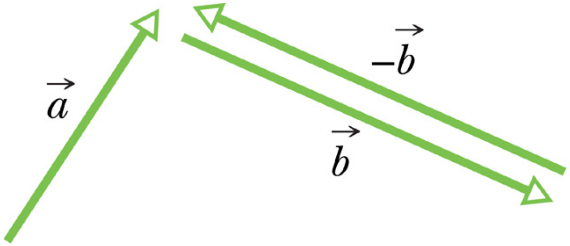
결합법칙

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$



vector의 뺄셈



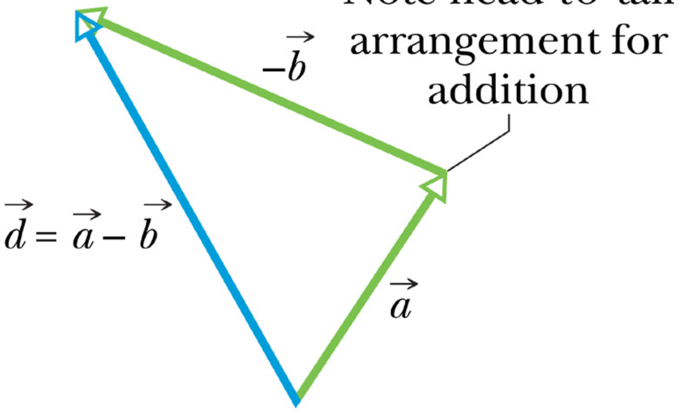


(a)

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

$$\vec{a} + (-\vec{a}) = \vec{0}$$

$$\vec{a} + \vec{0} = \vec{a}$$

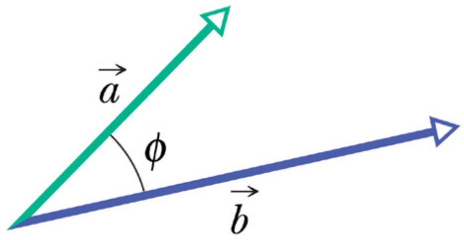


(b)

vector에 scalar 곱하기

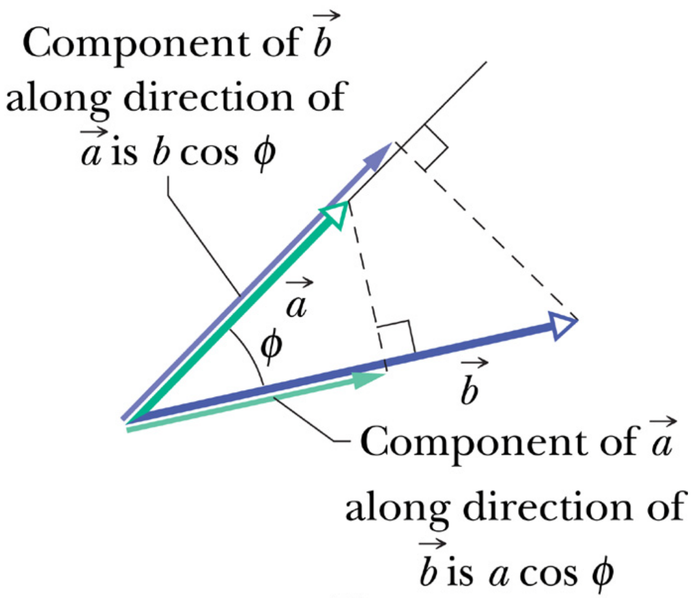
$$a \times \vec{b} = a\vec{b}$$

vector 곱하기: scalar product



(a)

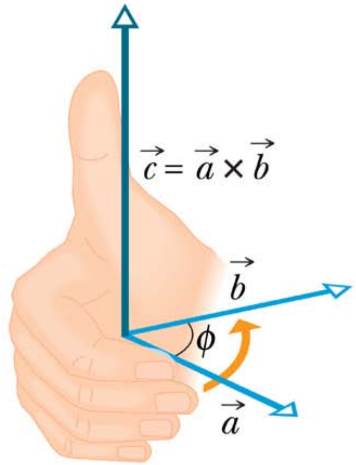
$$\vec{a} \cdot \vec{b} \equiv ab \cos \phi$$



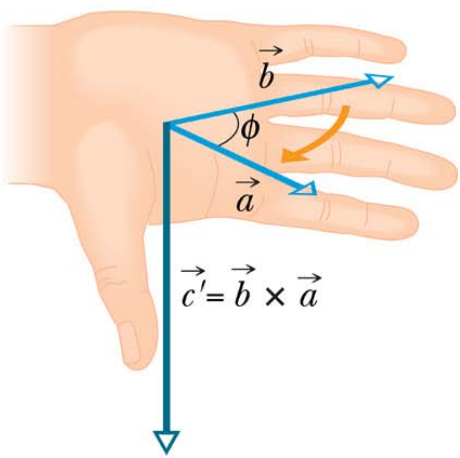
(b)

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \vec{b} \cdot \vec{a}, \\ \vec{a} \cdot (\vec{b} + \vec{c}) &= \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}, \\ \vec{a} \cdot (k\vec{b}) &= k(\vec{a} \cdot \vec{b}), \\ \vec{a} \cdot \vec{a} &= a^2. \end{aligned}$$

Vector 곱하기: vector product

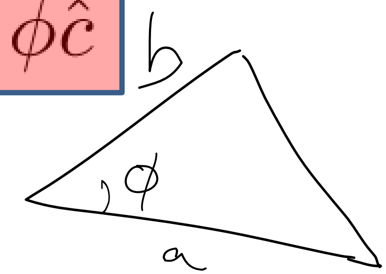


(*a*)



(*b*)

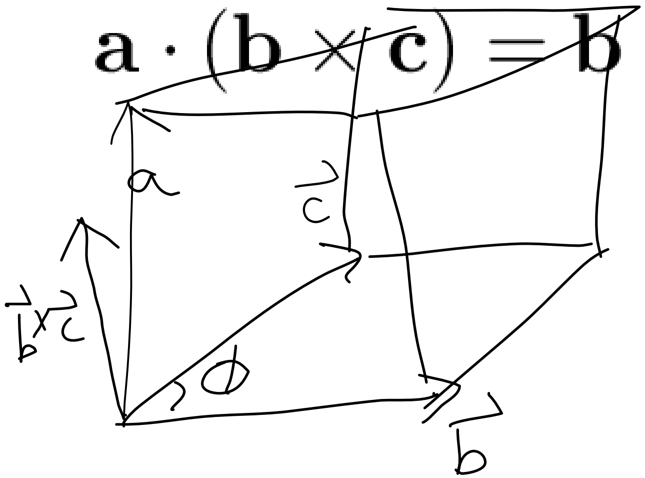
$$\vec{a} \times \vec{b} \equiv ab \sin\phi \hat{c}$$



$$
\begin{aligned}
\vec{a} \times \vec{b} &= -\vec{b} \times \vec{a}, \\
\vec{a} \times (\vec{b} + \vec{c}) &= \vec{a} \times \vec{b} + \vec{a} \times \vec{c}, \\
\vec{a} \times (k\vec{b}) &= k(\vec{a} \times \vec{b}), \\
\vec{a} \times \vec{a} &= 0.
\end{aligned}
$$

Vector product의 예

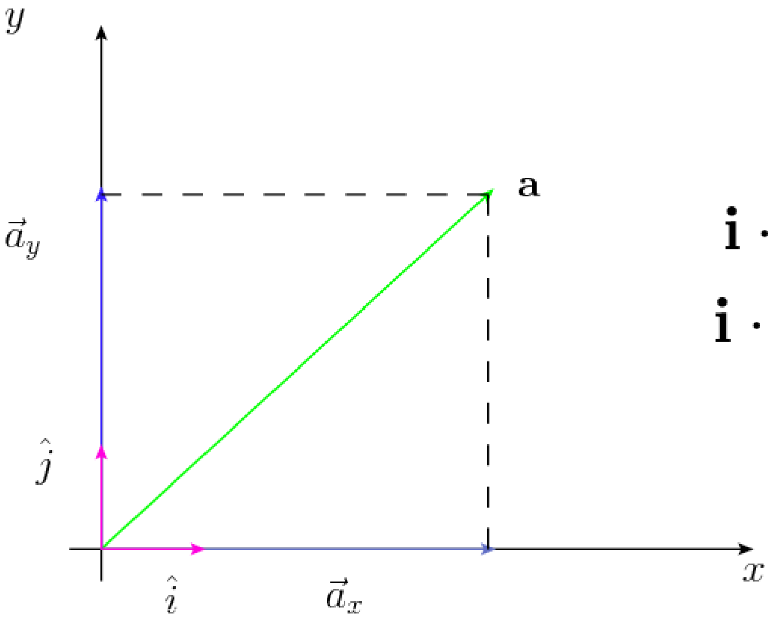
$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$$



Cyclic permutation

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \mathbf{b}(\mathbf{a}\cdot\mathbf{c}) - \mathbf{c}(\mathbf{a}\cdot\mathbf{b})$$

vector의 성분



$$\begin{aligned} \mathbf{i} &= \hat{\mathbf{x}} = (1,0,0) \\ \mathbf{j} &= \hat{\mathbf{y}} = (0,1,0) \\ \mathbf{k} &= \hat{\mathbf{z}} = (0,0,1) \end{aligned}$$

$$\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = 1\times 1\cos 90^\circ = 0$$

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1\times 1\cos 0^\circ = 1$$

$$\mathbf{i}\times\mathbf{i} = \mathbf{j}\times\mathbf{j} = \mathbf{k}\times\mathbf{k} = 0$$

$$\mathbf{i}\times\mathbf{j} = \mathbf{k},\ \mathbf{j}\times\mathbf{k} = \mathbf{i},\ \mathbf{k}\times\mathbf{i} = \mathbf{j}$$

$$\mathbf{a} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$$

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} = (a_x, a_y, a_z)$$

vector의 성분계산

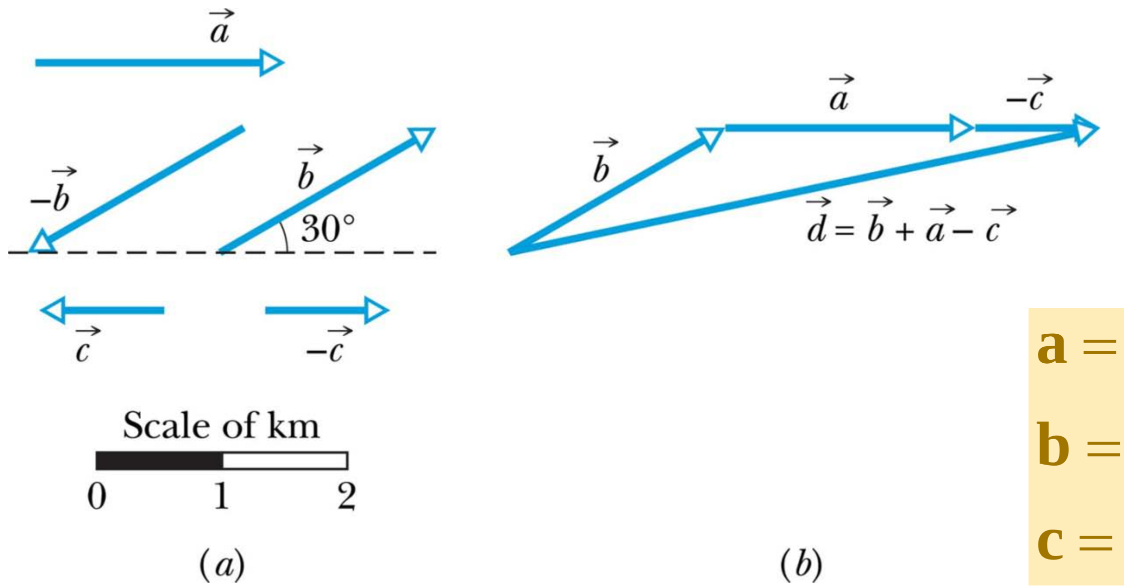
$$\mathbf{a}+\mathbf{b}=(a_x\mathbf{i}+a_y\mathbf{j})+(b_x\mathbf{i}+b_y\mathbf{j})=(a_x+b_x)\mathbf{i}+(a_y+b_y)\mathbf{j}$$

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= (a_x\mathbf{i}+a_y\mathbf{j})\cdot(b_x\mathbf{i}+b_y\mathbf{j}) \\ &= (a_xb_x)\mathbf{i}\cdot\mathbf{i}+(a_xb_y)\mathbf{i}\cdot\mathbf{j}+(a_yb_x)\mathbf{j}\cdot\mathbf{i}+(a_yb_y)\mathbf{j}\cdot\mathbf{j} \\ &= a_xb_x+a_yb_y\end{aligned}$$

$$\begin{aligned}\mathbf{a}\times\mathbf{b} &= (a_x\mathbf{i}+a_y\mathbf{j})\times(b_x\mathbf{i}+b_y\mathbf{j}) \\ &= (a_xb_x)\mathbf{i}\times\mathbf{i}+(a_xb_y)\mathbf{i}\times\mathbf{j}+(a_yb_x)\mathbf{j}\times\mathbf{i}+(a_yb_y)\mathbf{j}\times\mathbf{j} \\ &= a_xb_y\mathbf{k}+a_yb_x(-\mathbf{k}) \\ &= (a_xb_y-a_yb_x)\mathbf{k}\end{aligned}$$

$$
\begin{aligned}
\mathbf{a} \times \mathbf{b} &= (a_y b_z - a_z b_y)\mathbf{i} + (a_z b_x - a_x b_z)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k} \\
&= \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{pmatrix}
\end{aligned}
$$

Sample problem



$\mathbf{a} = 2.0\text{km, East}$

$\mathbf{b} = 2.0\text{km, } 30^\circ \text{ North of East}$

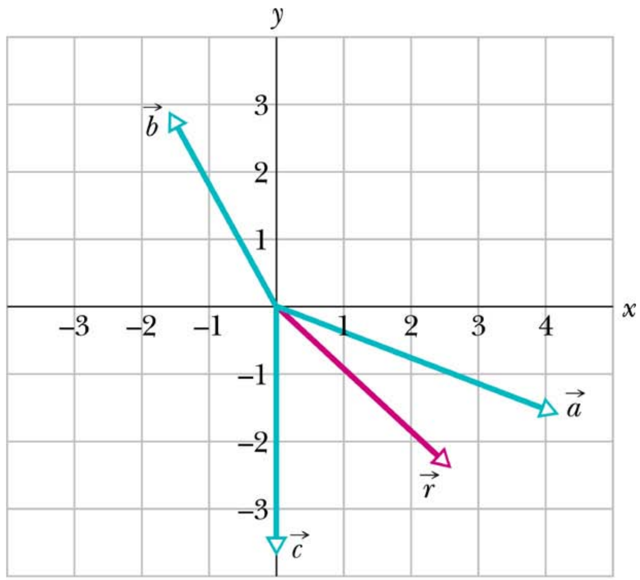
$\mathbf{c} = 1.0\text{km, West}$

$$d_x = a + b\cos 30^\circ - c = 4.732 \approx 4.7 \text{ (km)}$$

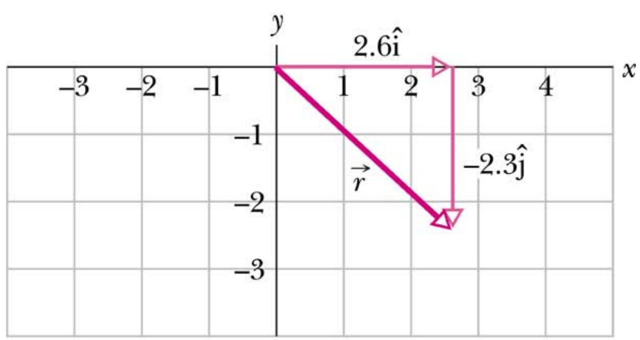
$$d_y = b\sin 30^\circ = 1.0 \text{ km}$$

$$d = \sqrt{d_x^2 + d_y^2} = \sqrt{4.7^2 + 1^2} = 4.8 \text{ (km)}$$

Sample problem



(a)



(b)

$$\begin{aligned}\mathbf{a} &= (4.2\text{ m})\mathbf{i} - (1.5\text{ m})\mathbf{j},\\ \mathbf{b} &= (-1.6\text{ m})\mathbf{i} + (2.9\text{ m})\mathbf{j},\\ \mathbf{c} &= (-3.7\text{ m})\mathbf{j},\\ \mathbf{r} &= \mathbf{a} + \mathbf{b} + \mathbf{c} = ?\end{aligned}$$

$$r_x = a_x + b_x + c_x = 2.6\text{m}$$

$$r_y = a_y + b_y + c_y = -2.3\text{m}$$

$$\mathbf{r} = (2.6\text{m})\mathbf{i} - (2.3\text{m})\mathbf{j}$$

$$r = \sqrt{(2.6)^2 + (-2.3)^2} = 3.5\text{m}$$

$$\theta = \tan^{-1}\frac{-2.3}{2.6} = -41^\circ$$

Example

$$\mathbf{a} = 3.0\mathbf{i} - 4.0\mathbf{j},\ \mathbf{b} = -2.0\mathbf{i} + 3.0\mathbf{k} : \phi = ?$$

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= ab\cos\phi \\ &= \sqrt{3.0^2 + 4.0^2}\cdot\sqrt{2.0^2 + 3.0^2}\cos\phi \\ &= 18\cos\phi\end{aligned}$$

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= (3.0\mathbf{i} - 4.0\mathbf{j})\cdot(-2.0\mathbf{i} + 3.0\mathbf{k}) \\ &= -6.0\mathbf{i}\cdot\mathbf{i} + 9.0\mathbf{i}\cdot\mathbf{k} + 8.0\mathbf{j}\cdot\mathbf{i} - 12.0\mathbf{j}\cdot\mathbf{k} \\ &= -6.0\end{aligned}$$

$$\phi = \cos^{-1}\frac{-6.0}{18.0} = 109^\circ$$

$$\mathbf{a} = 3.0\mathbf{i} - 4.0\mathbf{j},\ \mathbf{b} = -2.0\mathbf{i} + 3.0\mathbf{k} : \mathbf{c} = \mathbf{a} \times \mathbf{b} = ?$$

$$\begin{aligned}\mathbf{a}\times\mathbf{b} &= (3\mathbf{i}-4\mathbf{j})\times(-2\mathbf{i}+3\mathbf{k}) \\ &= -6\mathbf{i}\times\mathbf{i}+9\mathbf{i}\times\mathbf{k}+8\mathbf{j}\times\mathbf{i}-12\mathbf{j}\times\mathbf{k} \\ &= 9(-\mathbf{j})+8(-\mathbf{k})-12\mathbf{i} \\ &= -12\mathbf{i}-9\mathbf{j}-8\mathbf{k}\end{aligned}$$

$$\mathbf{c}\bullet\mathbf{a} = (-12\mathbf{i}-9\mathbf{j}-8\mathbf{k})\bullet(3\mathbf{i}-4\mathbf{j}) = -36\mathbf{i}\bullet\mathbf{i}+36\mathbf{j}\bullet\mathbf{j} = 0$$

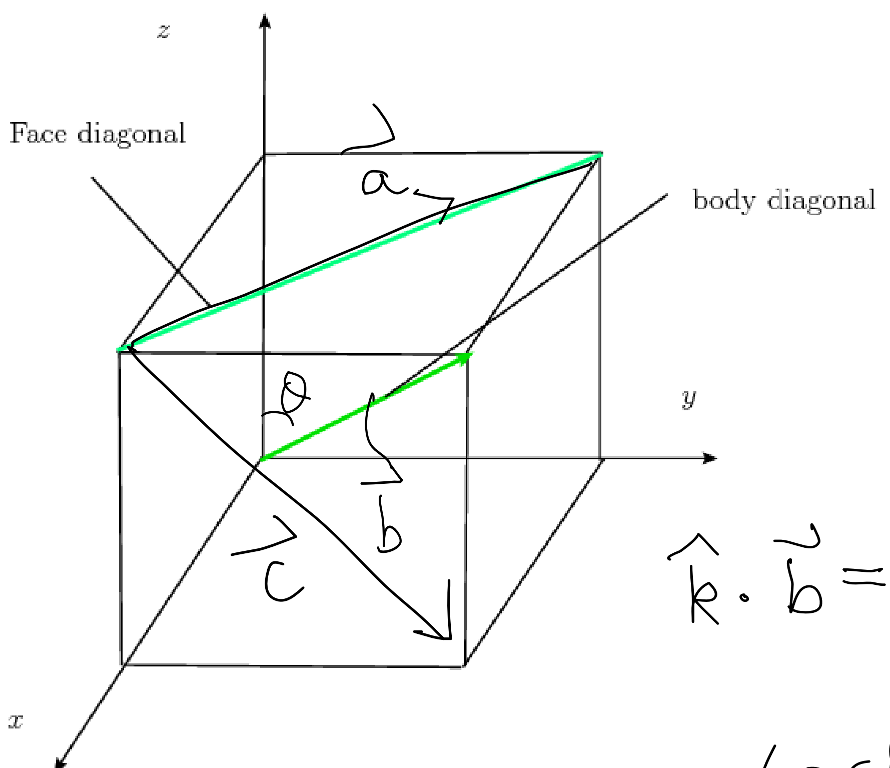
$$\mathbf{c}\bullet\mathbf{b} = (-12\mathbf{i}-9\mathbf{j}-8\mathbf{k})\bullet(-2\mathbf{i}+3\mathbf{k}) = 24\mathbf{i}\bullet\mathbf{i}-24\mathbf{k}\bullet\mathbf{k} = 0$$

$\vec{a} = -\hat{i} + \hat{j}$

$\vec{b} = \hat{j} - \hat{k}$

Problem

$\vec{a} \cdot \vec{b} = 2\cos\alpha = 1 \qquad \cos\alpha = \frac{1}{2}$



(a) Body diagonal, z축 사이의 각도

$\alpha = 60^\circ$

$\vec{b} = \hat{i} + \hat{j} + \hat{k}$

$\theta = \cos^{-1}\frac{1}{\sqrt{3}}$

(b) 이웃한 두 면의 face diagonal 사이의 각도

$\hat{k} \cdot \vec{b} = b\cos\theta = \sqrt{3}\cos\theta = 1$

$\cos\theta = \frac{1}{\sqrt{3}}$