

Quantum Mechanics 1

Assignment 3

Due: April 9 (Tuesday), 2013

1. A particle in a box with sides at $x = \pm a/2$ is described by the wave function at time $t = 0$

$$\psi(x) = \begin{cases} 0, & -\frac{a}{2} < x < 0 \\ \sqrt{\frac{2}{a}}, & 0 < x < \frac{a}{2}. \end{cases} \quad (1)$$

- (a) Find the eigenstates of the ground state and the first excited state for this system.
 - (b) What is the probability of observing the energy of the ground state, and the energy of the first excited state?
 - (c) It evolves in time according to the Schrödinger equation. Can it return to the original state at later times?
2. A particle is confined in a box with sides at $x = 0$ and $x = a$. Its initial wave function is given by

$$\psi(x) = A(\psi_1(x) + \sqrt{2}\psi_2(x)), \quad (2)$$

where $\psi_1(x)$ is the eigenfunction of the ground state, and $\psi_2(x)$ is the first excited state.

- (a) Compute A .
- (b) What are the probabilities of observing E_1 , and E_2 ? (E_1 and E_2 are the energies of the ground state and the first excited state.)
- (c) Compute $\langle x \rangle$ and $\langle p \rangle$.
- (d) What is the wave function at time t ?
- (e) What is $\langle x \rangle$ at time t ?

3. Consider the ground state of a particle in the box described in Problem 2. The right-hand side of the box is expanded abruptly to $x = b$. ($b > a$)
 - (a) What is the probability of this particle to be in the new ground state?
 - (b) What is the probability of being in the first excited state?
4. Consider a particle confined in a box with sides $x = 0$ and $x = a$.
 - (a) As a simple review, compute again the eigenfunctions and the corresponding energy eigenvalues.

Consider another particle confined in a box with sides $x = \pm a$.
 - (b) Out of the eigenfunctions of this system, which eigenfunctions satisfy the same boundary conditions as those in part (a)?
 - (c) By considering the relation in (b), and using a symmetry argument, can you relate the eigenfunctions in part (a) with part of the eigenfunctions in part (b)?