Mobile Communications (KECE425)

Lecture Note 17

5-7-2014

Prof. Young-Chai Ko

Summary

- Performance over Fading Channels
  - SER for PAM, PSK, and QAM over Rayleigh fading channels
- Diversity systems

Review

- Alternative representation of Q-function:

$$Q\left(x\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\theta}}\, d\theta$$

- Alternative representation of Q^2-function:

$$Q^2(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{4}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right)\, d\theta$$

- Symbol error rate of digitally modulated signals

1) M-PAM

$$P_s(e) = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6}{M^2-1}\gamma_s}\right) = 2\left(1-\frac{1}{M}\right)\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left[-\frac{g_{pam}\gamma_s}{\sin^2\theta}\right] d\theta$$

where $g_{pam} = \frac{3}{M^2-1}$.

2) M-PSK

$$P_s(e) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \exp\left[-\frac{\gamma_s \sin^2\frac{\pi}{M}}{\sin^2\phi}\right] d\phi$$

3) M-QAM

$$\begin{aligned} P_s(e) &= 4\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\gamma_s}\right) - 4\left(1-\frac{1}{\sqrt{M}}\right)^2 Q^2\left(\sqrt{\frac{3}{M-1}\gamma_s}\right) \\ &= \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}} \exp\left[-\frac{g_{qam}\gamma_s}{\sin^2\phi}\right] d\phi - \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2 \int_0^{\frac{\pi}{4}} \exp\left[-\frac{g_{qam}\gamma_s}{\sin^2\phi}\right] d\phi \end{aligned}$$

where $g_{qam} = \frac{3}{2(M-1)}$.

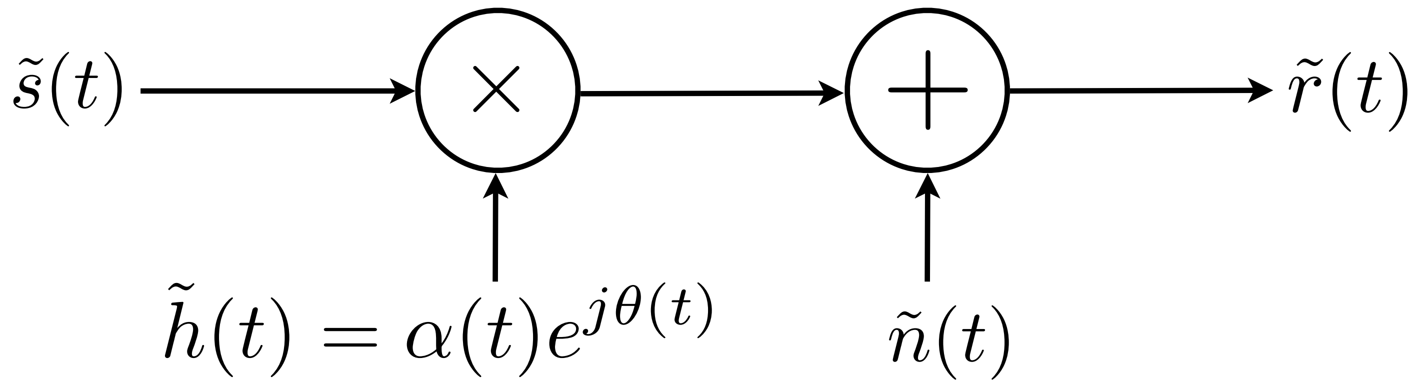
4) Binary Differential PSK (BDPSK)

$$P_b(e) = \frac{1}{2} e^{-E_b/N_0}$$

Performance over Fading Channels

Flat Fading Channel Model

- Flat fading channel model



- Received signal:

$$\tilde{r}(t) = \tilde{s}(t)\tilde{h}(t) + \tilde{n}(t)$$

- SNR of received signal, γ:

$$\gamma = \frac{|\tilde{h}(t)|^2 E_s}{N_0} = \frac{\alpha^2(t) E_s}{N_0}$$

Statistics of Received SNR over Fading Channels

- Rayleigh channels
  - PDF of γ:

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$$

  - CDF of γ:

$$P_\gamma(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}}}$$

  - Moment generating function (MGF) of γ:

$$\mathcal{M}_\gamma(s) = \int_0^\infty e^{s\gamma} p_\gamma(\gamma)\, d\gamma = (1 - s\bar{\gamma})^{-1}$$

- Ricean channels

  - PDF of γ:

$$p_\gamma(\gamma) = \frac{K+1}{\bar{\gamma}} \exp\left[-K - \frac{(K+1)\gamma}{\bar{\gamma}}\right] I_0\left(2\sqrt{\frac{K(K+1)}{\bar{\gamma}}\gamma}\right)$$

  - CDF of γ:

$$P_\gamma(x) = \Pr[\gamma \le x] = \int_0^x p_\gamma(\gamma)\, d\gamma$$

$$= \int_0^x \frac{K+1}{\bar{\gamma}} \exp\left[-K - \frac{(K+1)\gamma}{\bar{\gamma}}\right] I_0\left(2\sqrt{\frac{K(K+1)}{\bar{\gamma}}\gamma}\right) d\gamma$$

$$= 1 - Q_1\left(\sqrt{2K}, \sqrt{\frac{2(1+K)}{\bar{\gamma}}x}\right)$$

where $Q_1(a,b)$ is called Marcum Q function.

- Generalized Marcum Q-function $Q_m(a,b)$, is defined as

$$\begin{aligned} Q_m(a,b) &= \int_b^\infty x \left(\frac{x}{a}\right)^{m-1} e^{-(x^2+a^2)/2} I_{m-1}(ax)\,dx \\ &= Q_1(a,b) + e^{-(a^2+b^2)/2} \sum_{k=1}^{m-1} \left(\frac{b}{a}\right)^k I_k(ab) \end{aligned}$$

- Marcum Q function, $Q_1(a,b)$, is defined as

$$\begin{aligned} Q_1(a,b) &= \int_b^\infty x e^{-\frac{x^2+a^2}{2}} I_0(ax)\,dx \\ &= e^{-\frac{a^2+b^2}{2}} \sum_{k=0}^{\infty} \left(\frac{a}{b}\right)^k I_k(ab), \quad b \geq a \geq 0 \end{aligned}$$

- MGF of γ:

$$\begin{aligned}\mathcal{M}_\gamma(s) &= \int_0^\infty p_\gamma(\gamma) e^{s\gamma}\, d\gamma \\ &= \int_0^\infty \frac{K+1}{\bar{\gamma}} \exp\left[-K - \frac{(K+1)\gamma}{\bar{\gamma}}\right] I_0\left(2\sqrt{\frac{K(K+1)}{\bar{\gamma}}\gamma}\right) e^{s\gamma}\, d\gamma \\ &= \frac{(K+1)}{\bar{\gamma}} e^{-K} \int_0^\infty \exp\left[-\left(\frac{K+1}{\bar{\gamma}} - s\right)\gamma\right] I_0\left(2\sqrt{\frac{K(K+1)}{\bar{\gamma}}\gamma}\right) d\gamma\end{aligned}$$

It is known from the integration table that

$$\int_0^\infty x e^{-\alpha x^2} I_0(\beta x)\, dx = \frac{1}{2\alpha} \exp\left[-\frac{\beta^2}{4\alpha}\right]$$

Then we can show that

$$\mathcal{M}_\gamma(s) = \frac{1+K}{1+K-s\bar{\gamma}} \exp\left(\frac{s\bar{\gamma}K}{1+K-s\bar{\gamma}}\right)$$

- Nakagami-m channels
  - PDF of γ:

$$p_\gamma(\gamma) = \frac{\left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1}}{\Gamma(m)} e^{-\frac{m\gamma}{\bar{\gamma}}}$$

  - CDF of γ:

$$\begin{aligned} P_\gamma(x) &= \Pr[\gamma \leq x] = \int_0^x p_\gamma(\gamma)\, d\gamma = \int_0^x \frac{\left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1}}{\Gamma(m)} e^{-\frac{m\gamma}{\bar{\gamma}}}\, d\gamma \\ &= 1 - \frac{\Gamma\left(m, \frac{m}{\bar{\gamma}}\gamma\right)}{\Gamma(m)} \end{aligned}$$

where $\Gamma(\alpha, x)$ is called incomplete Gamma function defined as

$$\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1}\, dt$$

- MGF of γ:

$$\mathcal{M}_\gamma(s) = \int_0^\infty p_\gamma(\gamma) e^{s\gamma}\, d\gamma = \int_0^\infty \frac{\left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1}}{\Gamma(m)} e^{-\frac{m\gamma}{\bar{\gamma}}} e^{s\gamma}\, d\gamma$$

$$= \frac{\left(\frac{m}{\bar{\gamma}}\right)^m}{\Gamma(m)} \int_0^\infty \gamma^{m-1} e^{-\left(\frac{m}{\bar{\gamma}}-s\right)\gamma}\, d\gamma$$

Using the following identity:

$$\int_0^\infty x^{\nu-1} e^{-\mu x}\, dx = \mu^{-\nu}\Gamma(\nu)$$

we obtain the MGF of γ as

$$\mathcal{M}_\gamma(s) = \left(1 - \frac{s\bar{\gamma}}{m}\right)^{-m}$$

Symbol Error Rate over Fading Channels

- SER over fading channels:

$$P_s(e) = \int_0^\infty P_s(e|\gamma)\, p_\gamma(\gamma) d\gamma$$

where $P_s(e|\gamma)$ is the SER given γ (or SER over AWGN channel) and $p_\gamma(\gamma)$ is the PDF of the SNR, γ.

BER of BPSK over Fading Channels

- For example for BPSK, $P_s(e|\gamma) = Q\left(\sqrt{2\gamma}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{\gamma}{\sin^2\phi}}\, d\phi$.

$$
\begin{aligned}
P_s(e) &= \int_0^{\infty} Q(\sqrt{2\gamma})p_\gamma(\gamma)\, d\gamma \;=\; \int_0^{\infty}\left[\frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{\gamma}{\sin^2\phi}}\, d\phi\right] p_\gamma(\gamma)\, d\gamma \\
&= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left[\int_0^{\infty} e^{-\frac{\gamma}{\sin^2\phi}} p_\gamma(\gamma)\, d\gamma\right] d\phi \\
&= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{M}_\gamma\left(-\frac{1}{\sin^2\phi}\right) d\phi
\end{aligned}
$$

- SER of BPSK

$$P_s(e) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{M}_\gamma\left(-\frac{1}{\sin^2\phi}\right) d\phi$$

* For Rayleigh fading, MGF of γ is $\mathcal{M}_\gamma(s) = (1 - s\bar{\gamma})^{-1}$

  Then we have

$$P_s(e) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}}{\sin^2\phi}\right)^{-1} d\phi = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{\bar{\gamma} + \sin^2\phi}\right) d\phi$$

* For Nakagami-m fading, MGF of γ is $\mathcal{M}_\gamma(s) = \left(1 - \frac{s\bar{\gamma}}{m}\right)^{-m}$

  Then we have

$$P_s(e) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + \frac{\bar{\gamma}}{m\sin^2\phi}\right)^{-m} d\phi = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{m\sin^2\phi}{\bar{\gamma} + m\sin^2\phi}\right)^{m} d\phi$$

Some Useful Integrations

- Define $I_n(c)$ as

$$I_n(c) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{\sin^2\phi + c}\right)^n \, d\phi$$

  - It has been shown that

$$I_n(c) = \frac{1}{2} - \left[\frac{1}{2} - A(c)\right]\sum_{i=0}^{n-1}\left(\begin{array}{c} 2i \\ i \end{array}\right)[A(c)]^i\,[1 - A(c)]^i$$

where

$$A(c) = \frac{1}{2}\left[1 - \sqrt{\frac{c}{1+c}}\right]$$

  - For $n = 1$, $I_1(c)$ is

$$I_1(c) = \frac{1}{2}\left[1 - \sqrt{\frac{c}{1+c}}\right]$$

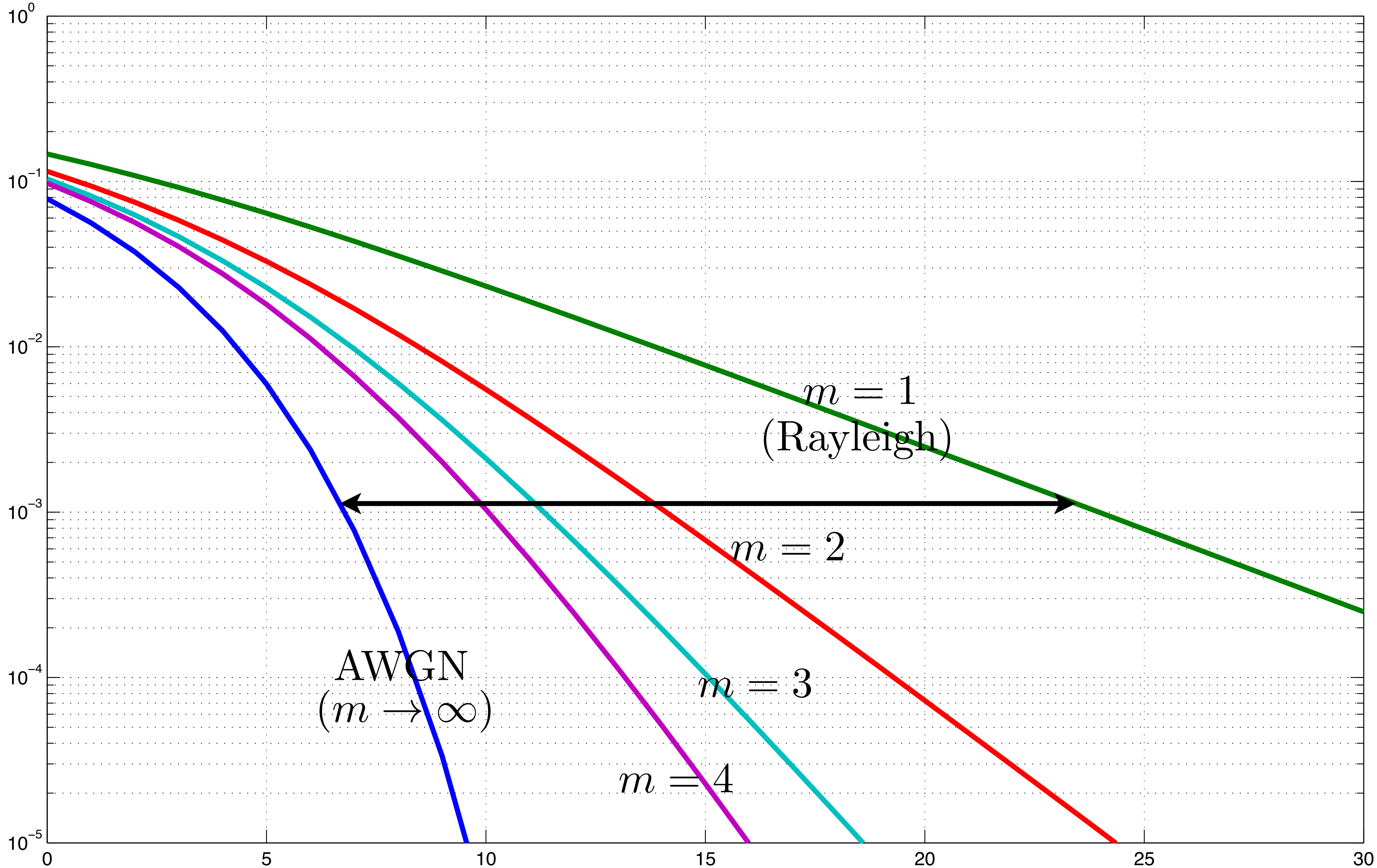
- SER over Rayleigh fading channel

$$P_s(e) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{\bar{\gamma}+\sin^2\phi}\right) d\phi = I_1(\bar{\gamma}) = \frac{1}{2}\left[1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right]$$

- SER over Nakagami-m fading channel

$$\begin{aligned} P_s(e) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{m\sin^2\phi}{\bar{\gamma}+m\sin^2\phi}\right)^m d\phi = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\phi}{\frac{\bar{\gamma}}{m}+\sin^2\phi}\right)^m d\phi \\ &= I_m(\bar{\gamma}/m) \end{aligned}$$

- BER of BPSK over Nakagami-m fading channels



Average SER:Performance of *M*-PSK

- Conditional SER of M-PSK given γ

$$P_s(e|\gamma) = \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} \exp\left(-\frac{g_{psk}\gamma}{\sin^2\phi}\right)\, d\phi, \text{ where } g_{psk} = \sin^2\left(\frac{\pi}{M}\right).$$

- Average SER:

$$\begin{aligned} P_s(e) &= \int_0^{\infty} P_s(e|\gamma)p_\gamma(\gamma)\, d\gamma \\ &= \frac{1}{\pi}\int_0^{\frac{M-1}{M}\pi} M_\gamma\left(-\frac{g_{psk}}{\sin^2\phi}\right)\, d\phi \end{aligned}$$

Average SER:Performance of *M*-QAM

- Average SER given γ:

$$
\begin{aligned}
P_s(e) &= 4\left(1-\frac{1}{\sqrt{M}}\right)\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{g_{qam}\gamma}{\sin^2\phi}\right)\,d\phi \\
&\quad -4\left(1-\frac{1}{\sqrt{M}}\right)^2\frac{1}{\pi}\int_0^{\frac{\pi}{4}}\exp\left(-\frac{g_{qam}\gamma}{\sin^2\phi}\right)\,d\phi
\end{aligned}
$$

where $g_{qam}=\dfrac{3}{2(M-1)}$

- Average SER:

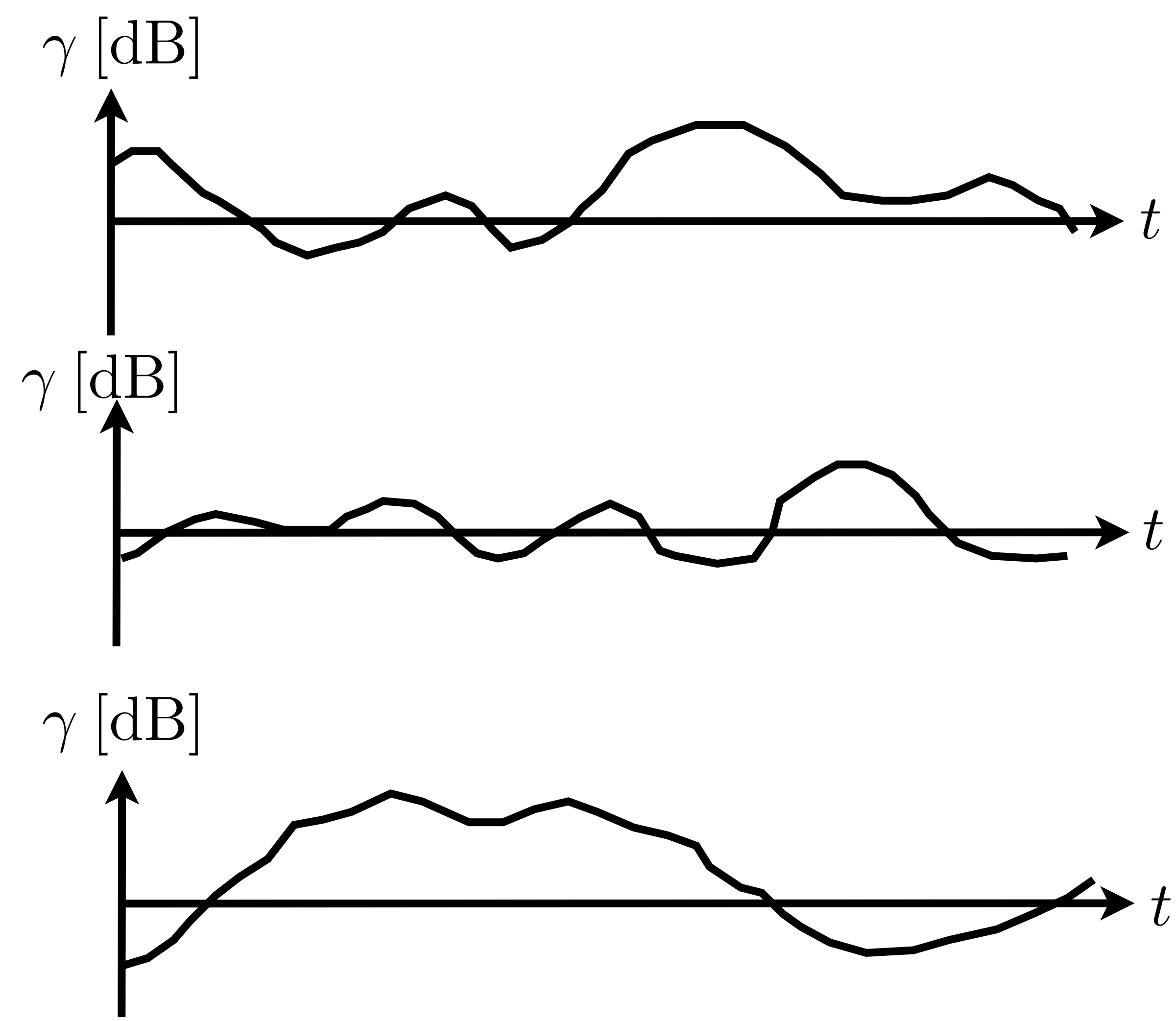
$$
\begin{aligned}
P_s(e) &= \frac{\pi}{4}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\frac{\pi}{2}} M_\gamma\left(-\frac{g_{qam}}{\sin^2\phi}\right)\,d\phi \\
&- \frac{\pi}{4}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\frac{\pi}{4}} M_\gamma\left(-\frac{g_{qam}}{\sin^2\phi}\right)\,d\phi
\end{aligned}
$$

Part III. Diversity Techniques

- Receive diversity
- Transmit diversity
- Transmit-Receive diversity

Concept and Intuition of Diversity Systems

- Concept
  - Receiving redundantly the same information bearing signals over two or more fading channels
- Intuition
  - Take advantage of low probability of occurrence of deep fades in all diversity branches

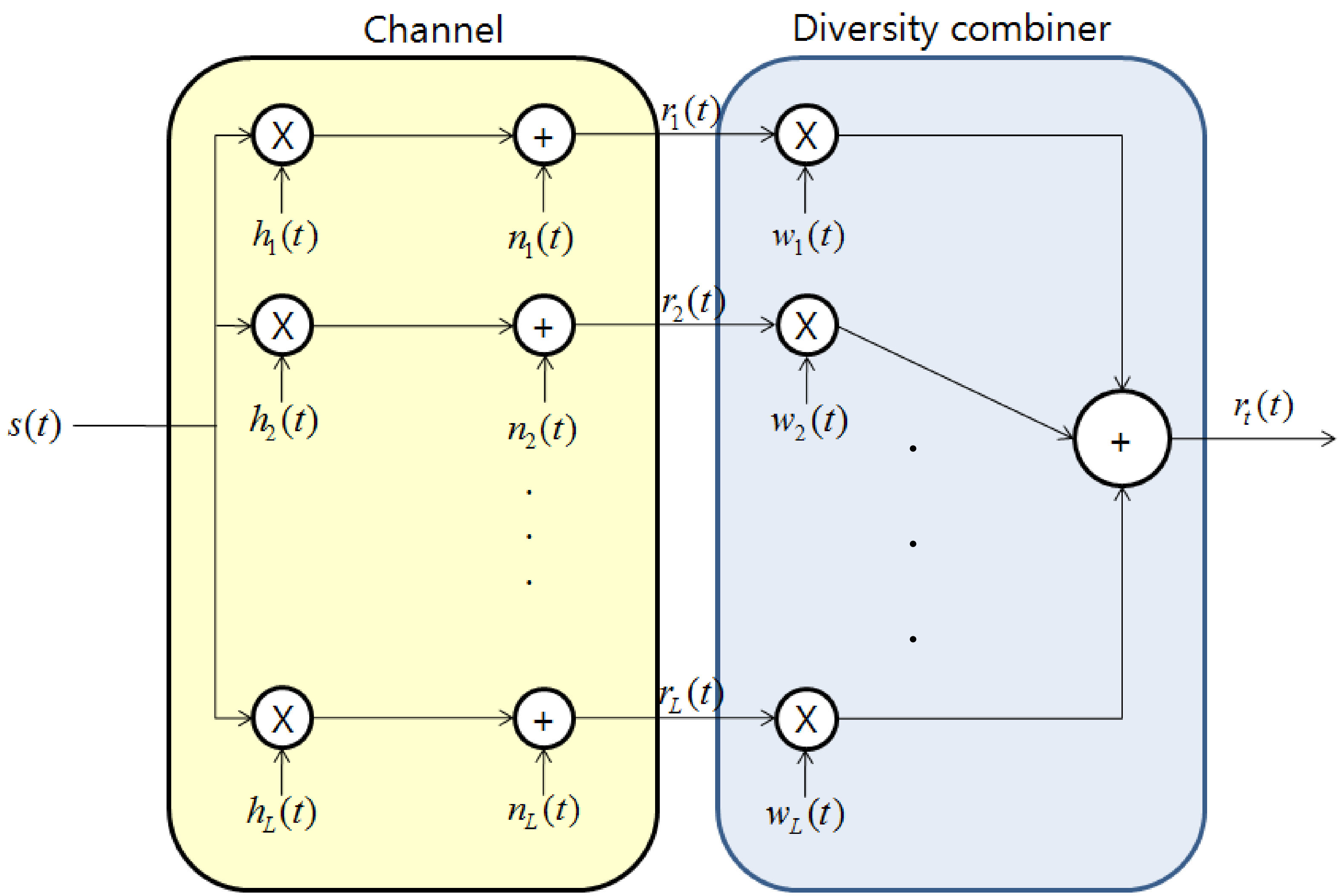


Receiver Diversity

- Maximal ratio combining
- Equal gain combining
- Selection combining
- Switched combining

Linear Receiver Antenna Diversity Combining

- Block diagram



Combined received signal

$$
\begin{aligned}
r_t(t) &= \sum_{l=1}^{L} w_l(t) r_l(t) \\
&= \sum_{l=1}^{L} w_l(t)(h_l(t)s(t) + n_l(t))
\end{aligned}
$$

Maximal Ratio Combining

- MRC maximizes the received signal-to-noise ratio.
  - What is the optimum weight vector to maximize the SNR of the combined signal $r_t(t)$?
- Optimum weight vector

$$r_t(t) = \sum_{l=1}^{L} w_l r_l(t) = \sum_{l=1}^{L} w_l h_l(t) s(t) + \sum_{l=1}^{L} w_l n_l((t)$$

  - Combined output SNR

$$\gamma_t = \frac{|\sum_{l=1}^{L} w_l h_l|^2 E_s}{\sum_{l=1}^{L} |w_l|^2 N_0} = \frac{E_s}{N_0} \frac{|\sum_{l=1}^{L} w_l h_l|^2}{\sum_{l=1}^{L} |w_l|^2}$$

  - Cauchy-Schwartz inequality

$$\left|\sum_{l=1}^{L} w_l h_l\right|^2 \leq \left|\sum_{l=1}^{L} w_l\right|^2 \left|\sum_{l=1}^{L} h_l\right|^2$$

Equality hold iff $w_l = c h_l^*(t)$ with an arbitrary constant value of c.

- Using the optimal weight vector, we have

$$\gamma_t \leq \frac{E_s}{N_0}\sum_{l=1}^{L}|h_l|^2 = \frac{E_s}{N_0}\sum_{l=1}^{L}\alpha_l^2 = \sum_{l=1}^{L}\gamma_l$$

where $\gamma_l = \frac{\Omega_l E_s}{N_0}$, that is, SNR at each branch.

- Maximal ratio combining

$$\begin{aligned}\gamma_t &= \sum_{l=1}^{L}\gamma_l \\ w_l &= c h_l^*(t) \text{ for an arbitrary constant value of } c\end{aligned}$$

Received Output SNR of MRC

- Received output SNR of MRC

$$\gamma_t = \sum_{l=1}^{L} \gamma_l$$

  - Average received output SNR

$$\bar{\gamma}_t = \sum_{l=1}^{L} \bar{\gamma}_l$$

    - If $\Omega_l = \Omega$ for $l = 1, \ldots, L$ (identical channels) and hence , $\bar{\gamma}_l = \bar{\gamma}$

$$\bar{\gamma}_t = L\bar{\gamma}$$