KECE321 Communication Systems I

(Haykin Sec. 3.1-Sec. 3.6)

Lecture #10, April 9, 2012

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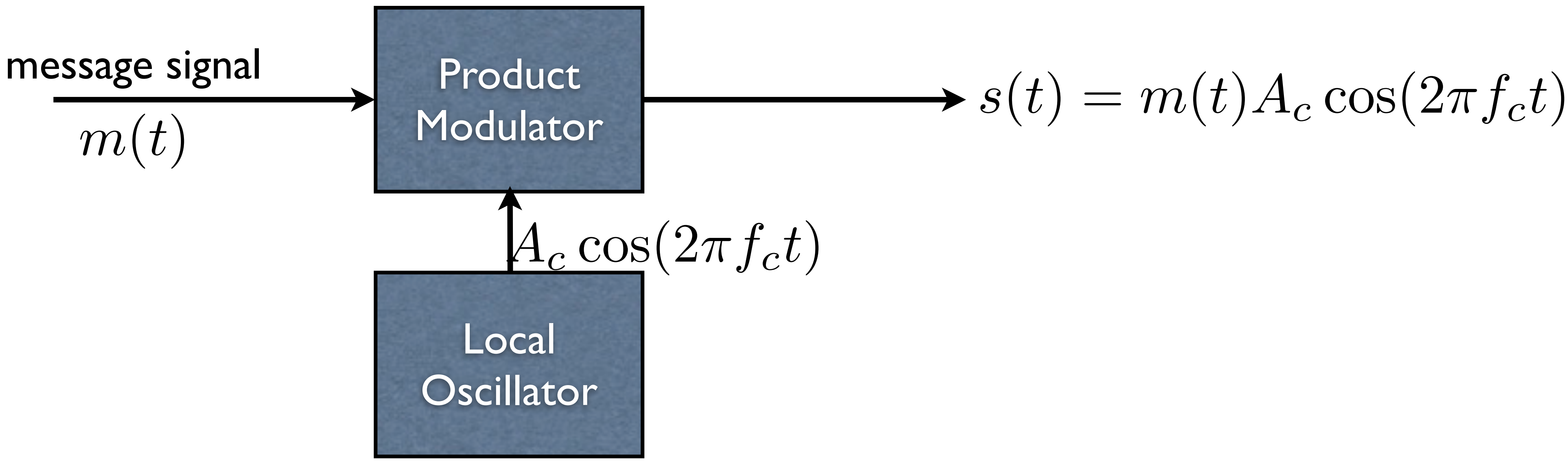
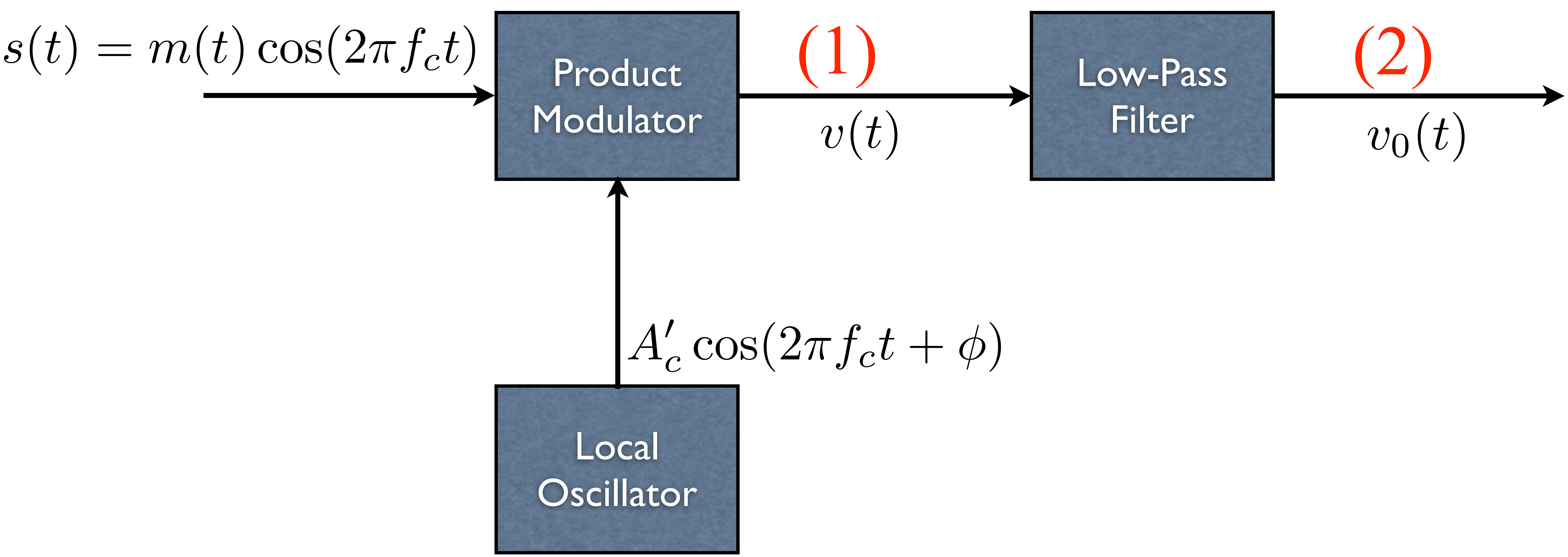
Summary

Summary

- Amplitude modulation
  - Modification of AM
    - Double Sideband-Suppressed Carrier Modulation
      - Costas Receiver
    - Single-Sideband Modulation
    - Vestigial Sideband Modulation

DSB-SC Modulator and Coherent Detector





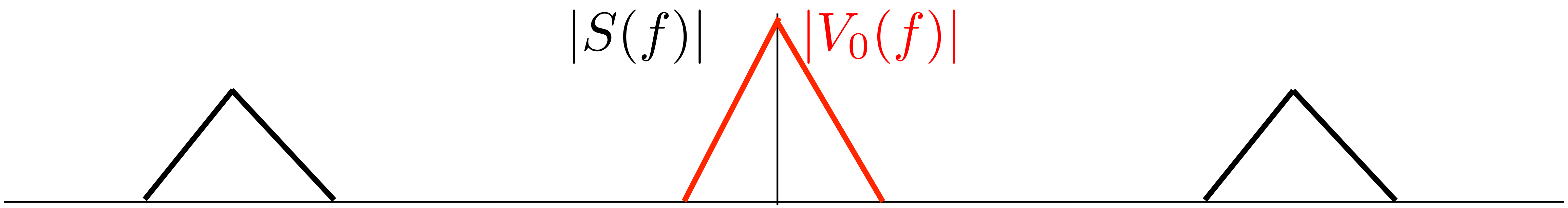
- Signal at the output of the product modulator in the coherent detector

$$\begin{aligned} v(t) &= s(t)A_c' \cos(2\pi f_c t + \phi) \\ &= A_c A_c' \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) \\ &= \frac{A_c A_c'}{2}\cos(4\pi f_c t + \phi)m(t) + \frac{A_c A_c'}{2}\cos(\phi)m(t) \end{aligned}$$

- Signal at the output of the low-pass filter

$$v_0(t) = \frac{A_c A_c'}{2}\cos(\phi)m(t)$$

- The quadrature null effect
  - The zero demodulated signal occurs for $\phi = \pm\pi/2$
  - The phase error ϕ in the local oscillator causes the detector output to be attenuated by a factor equal to $\cos(\phi)$

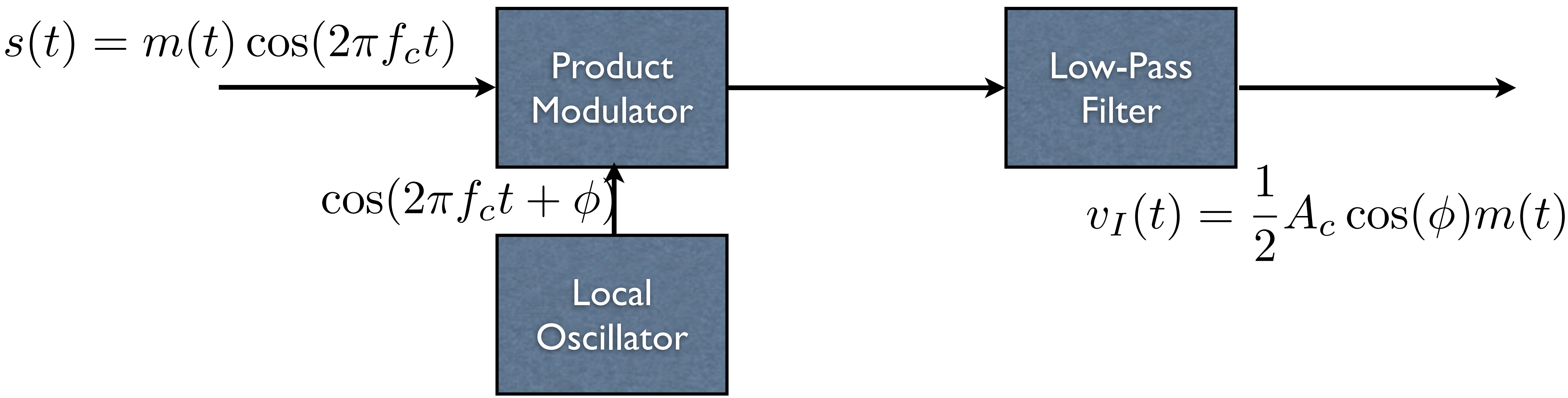


$$|V_0(0)| = \frac{A_c A_c'}{2} M(0)\cos(\phi)$$

Some Remarks on Coherent Detector

- Local oscillator may generate the incorrect frequency due to
  - temperature, aging, and so on.
- Coherent detection of a DSB-SC modulated wave requires
  - the locally generated carrier in the receiver needs to be synchronous in both frequency and phase with the carrier of the transmitter.
  - which is a rather demanding requirement.
- To overcome this problem it will be nice if we can estimate the phase error and the receiver with the estimation of phase error and correction based on the estimated phase error is called "Costas receiver".

In-Phase Coherent-Detector and Quadrature-Phase Coherent Detector



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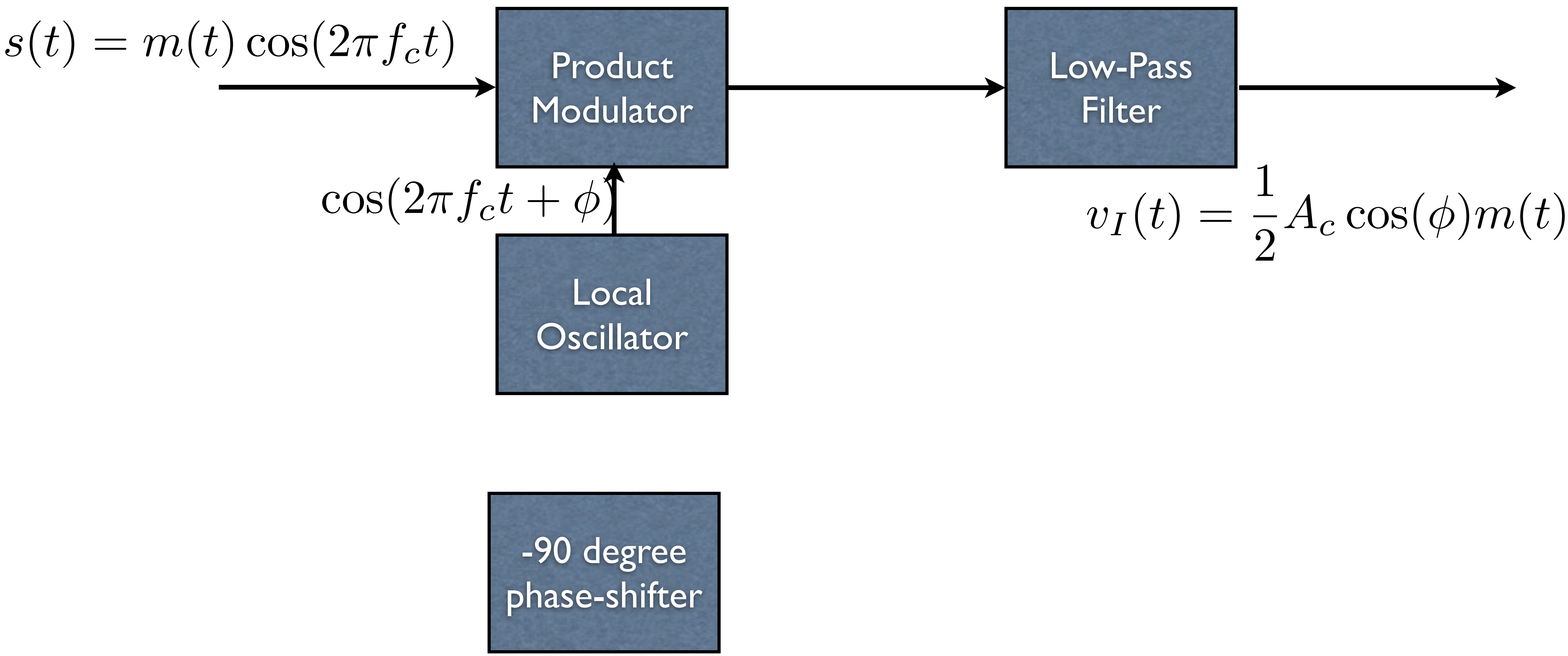

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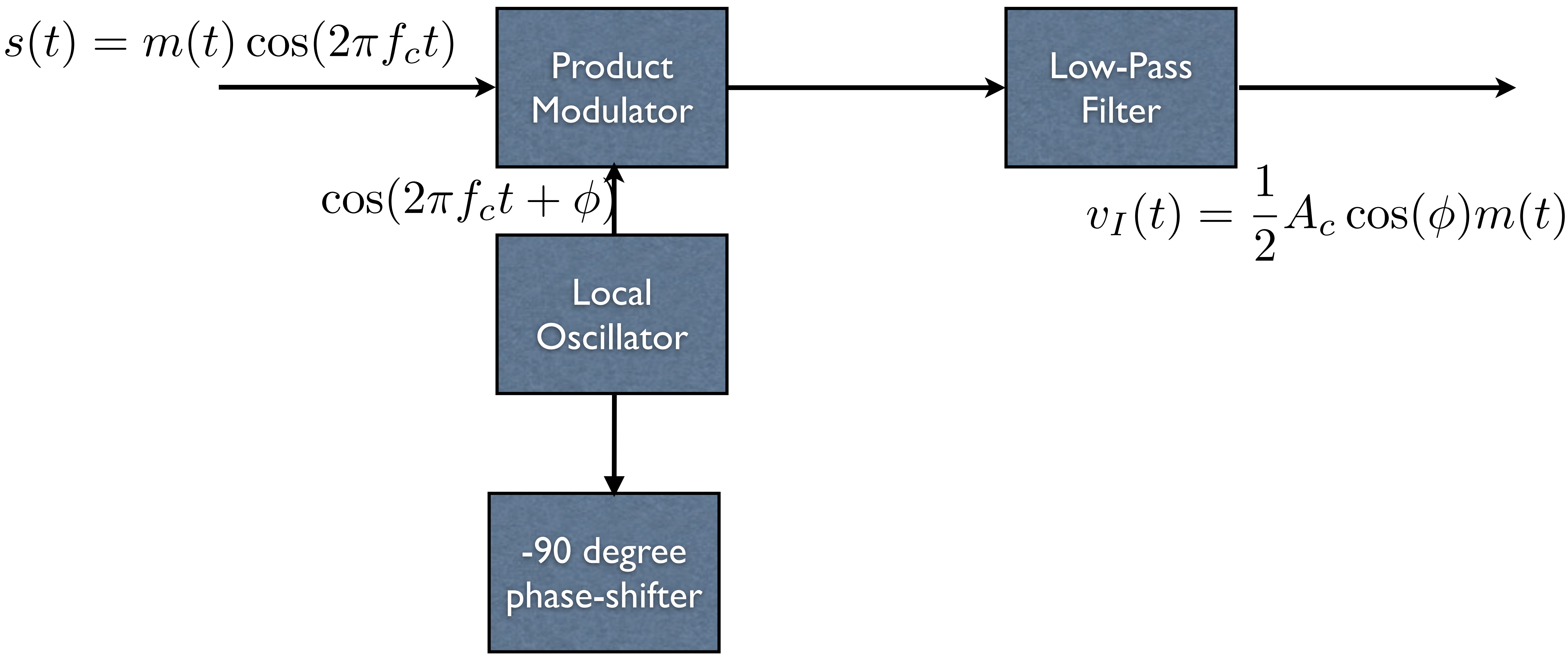

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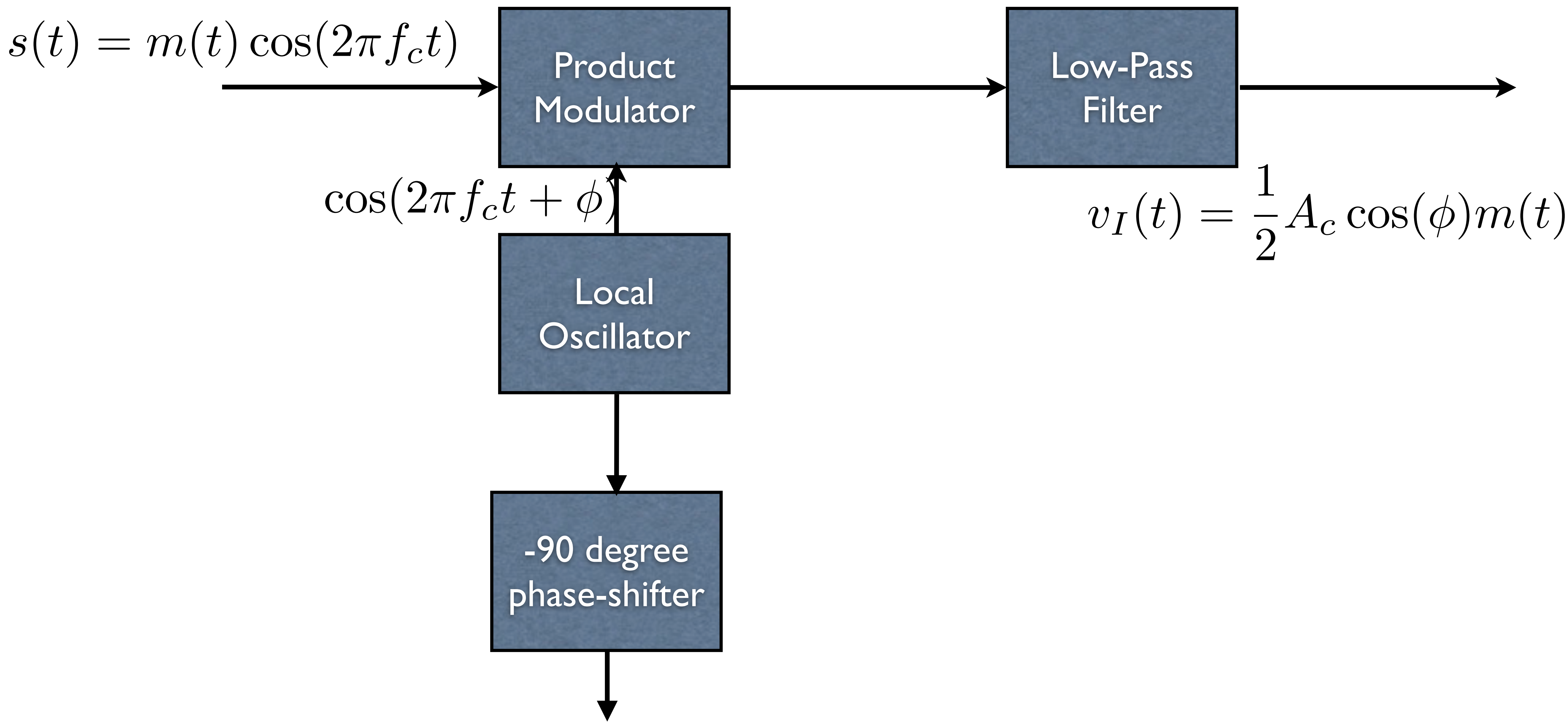

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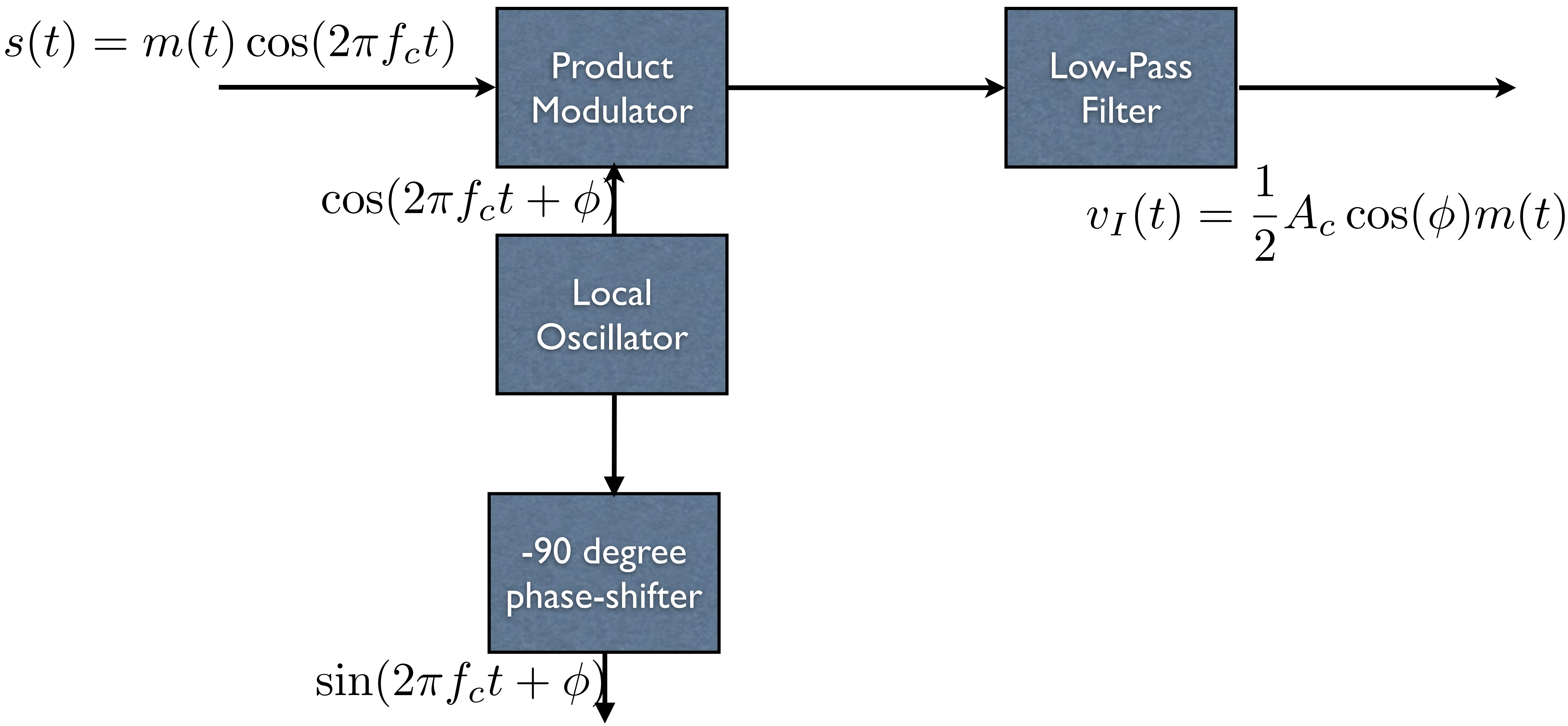

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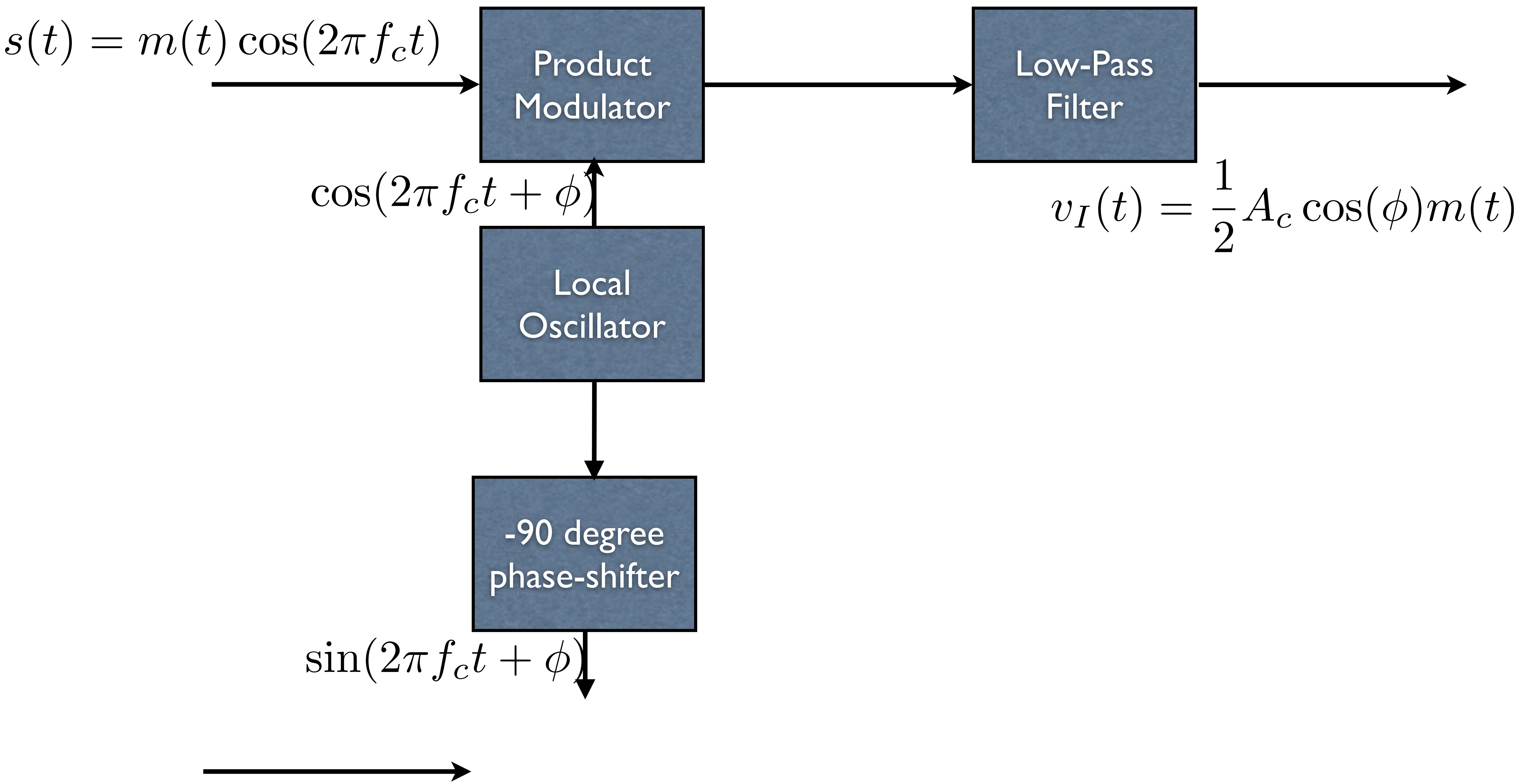

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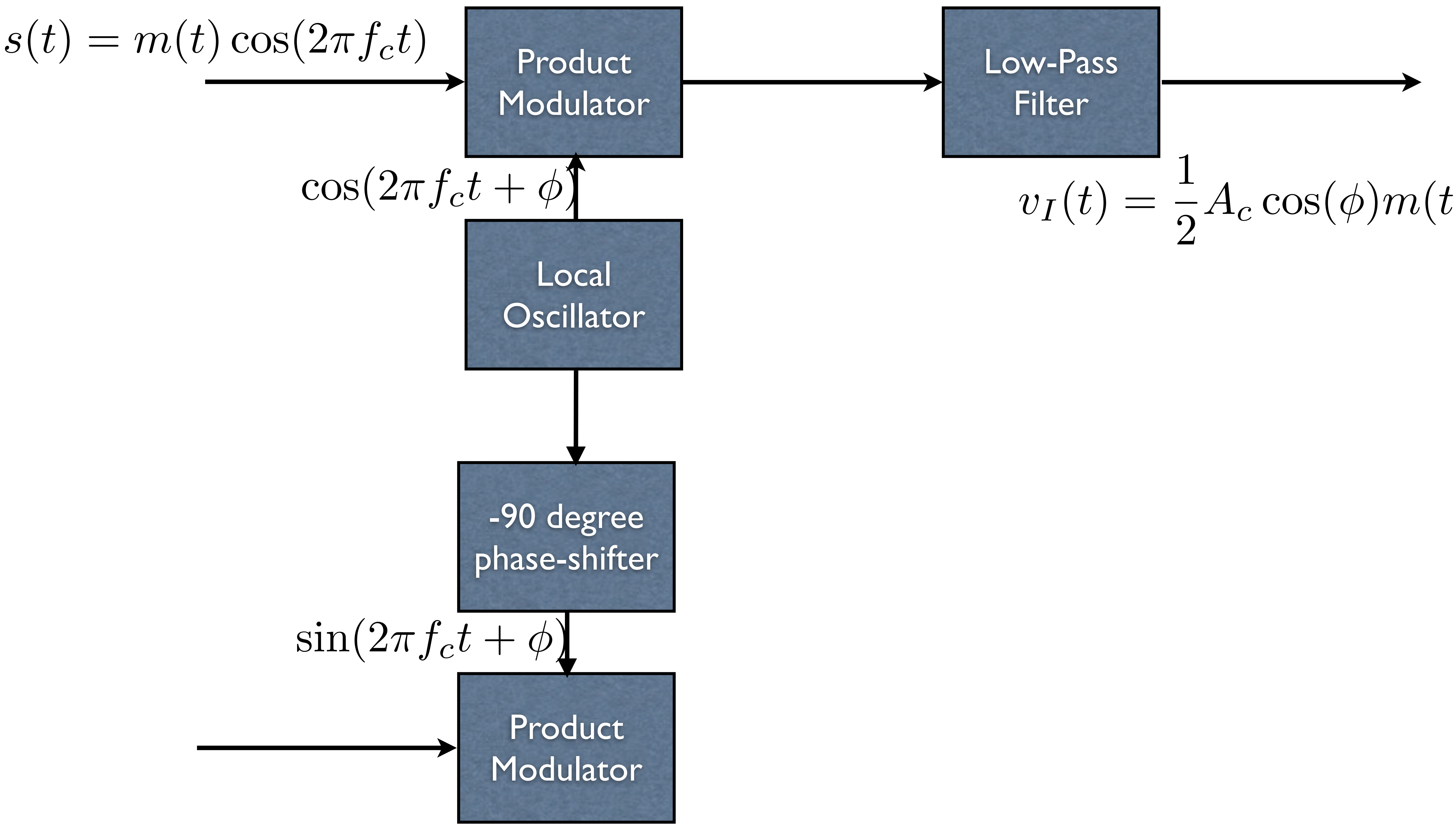

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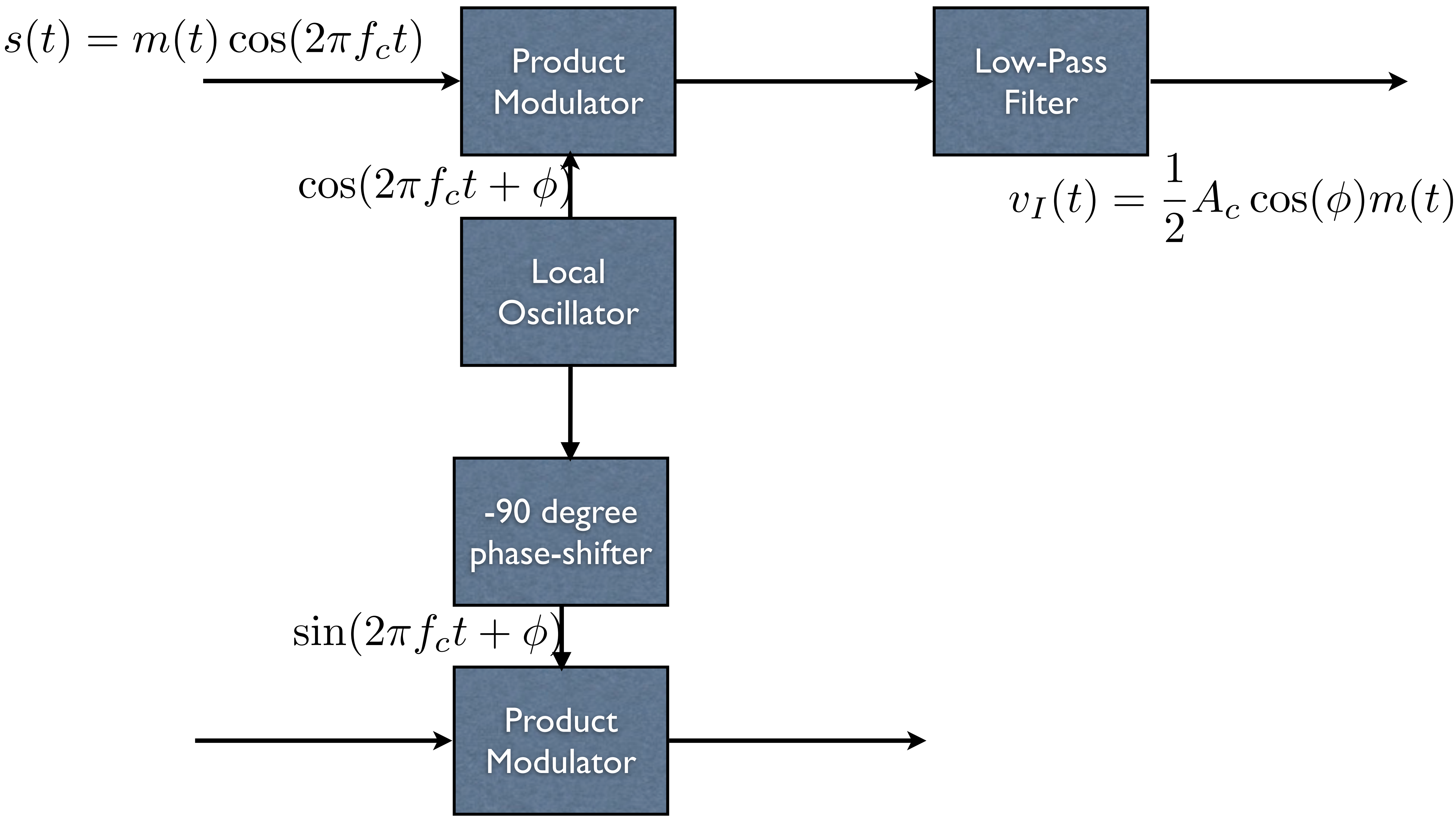

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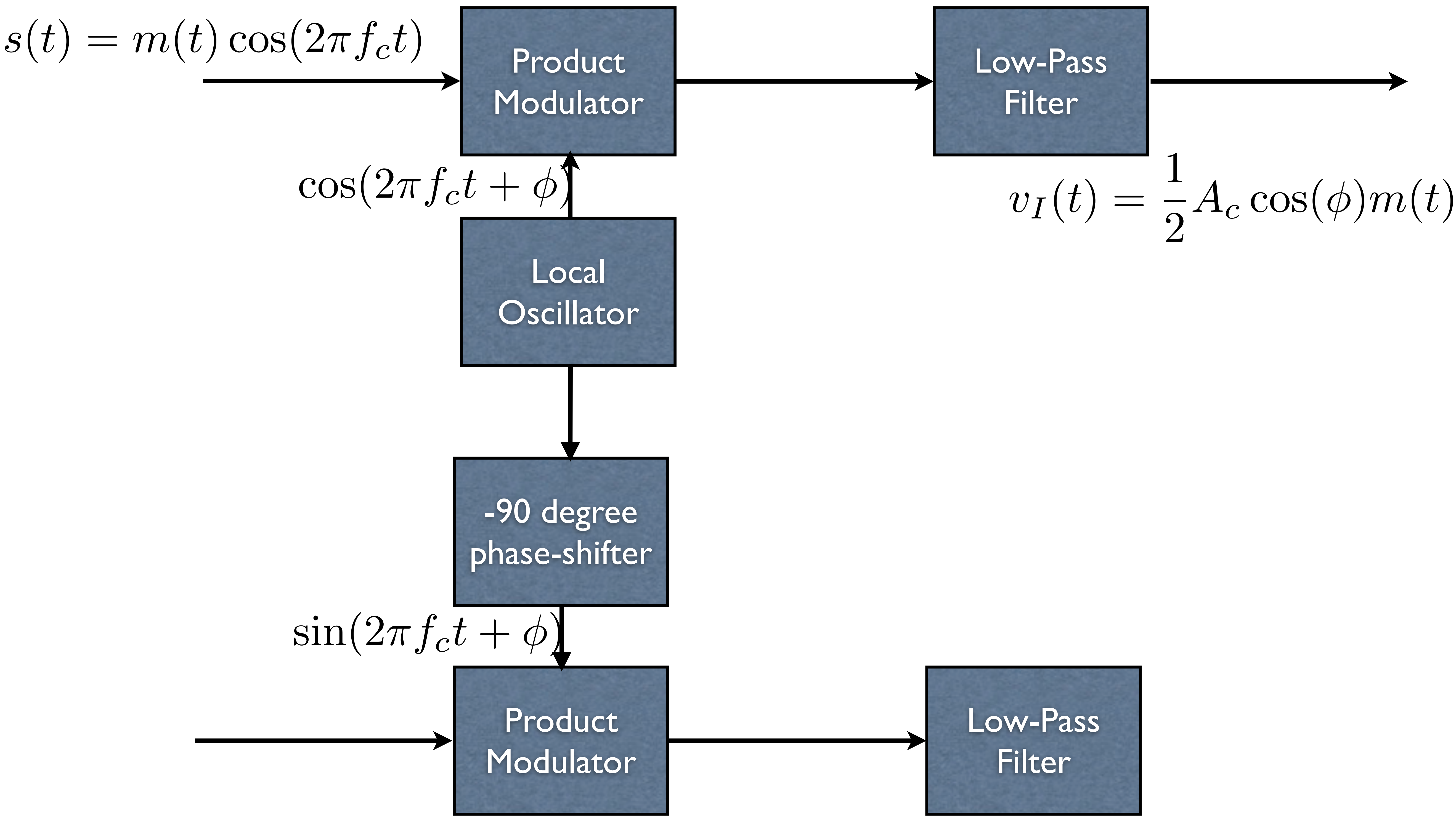

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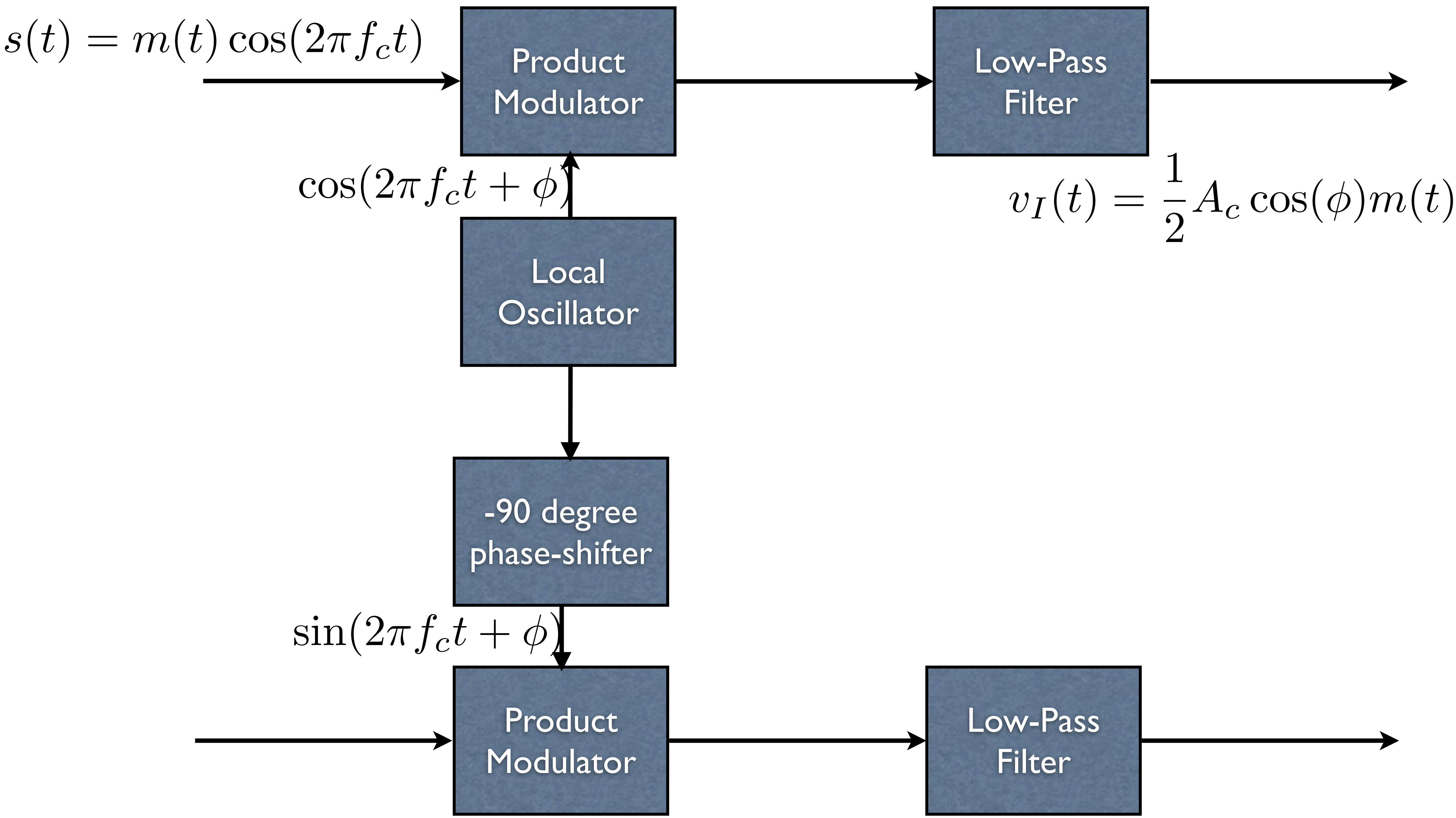

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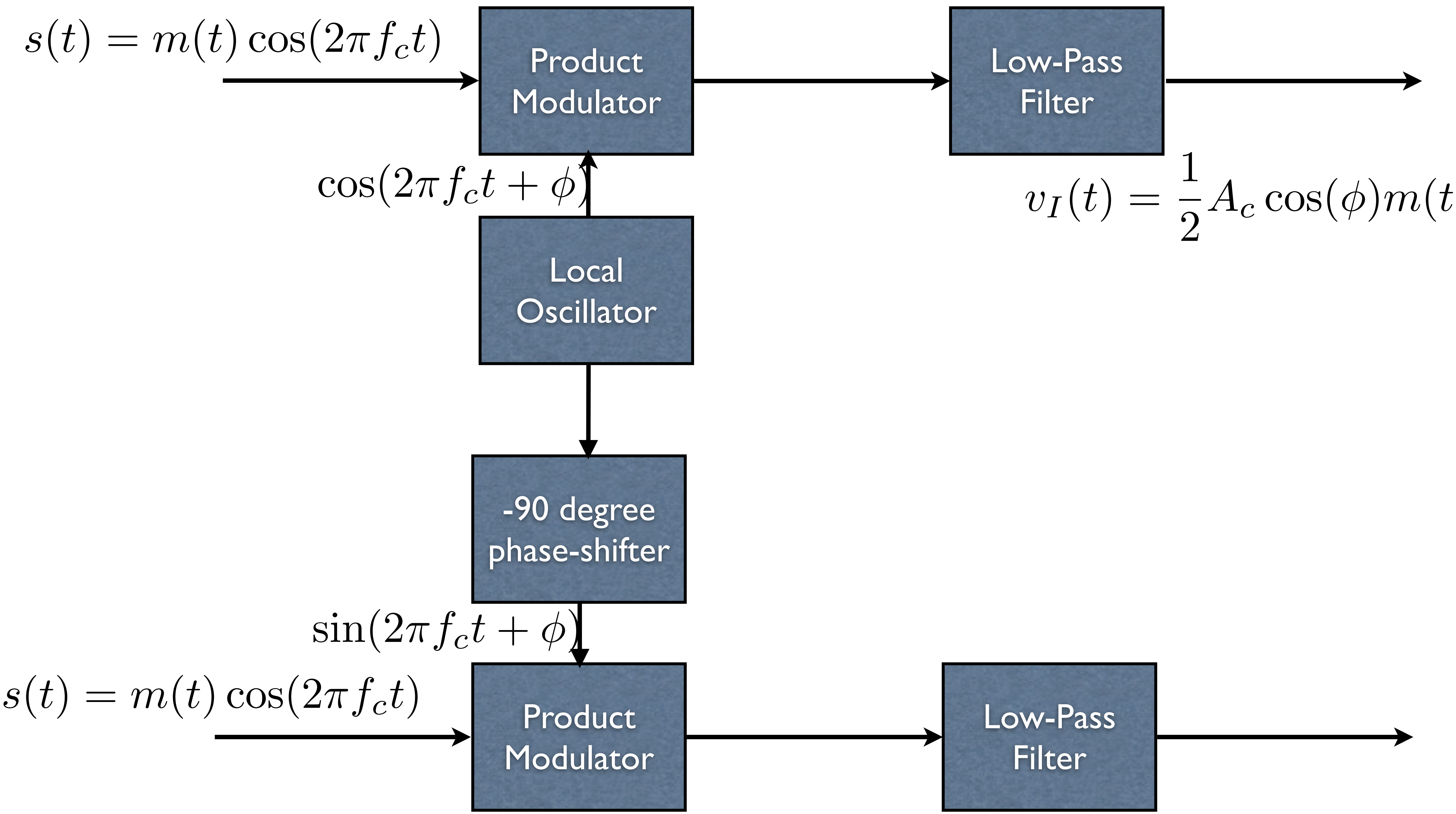

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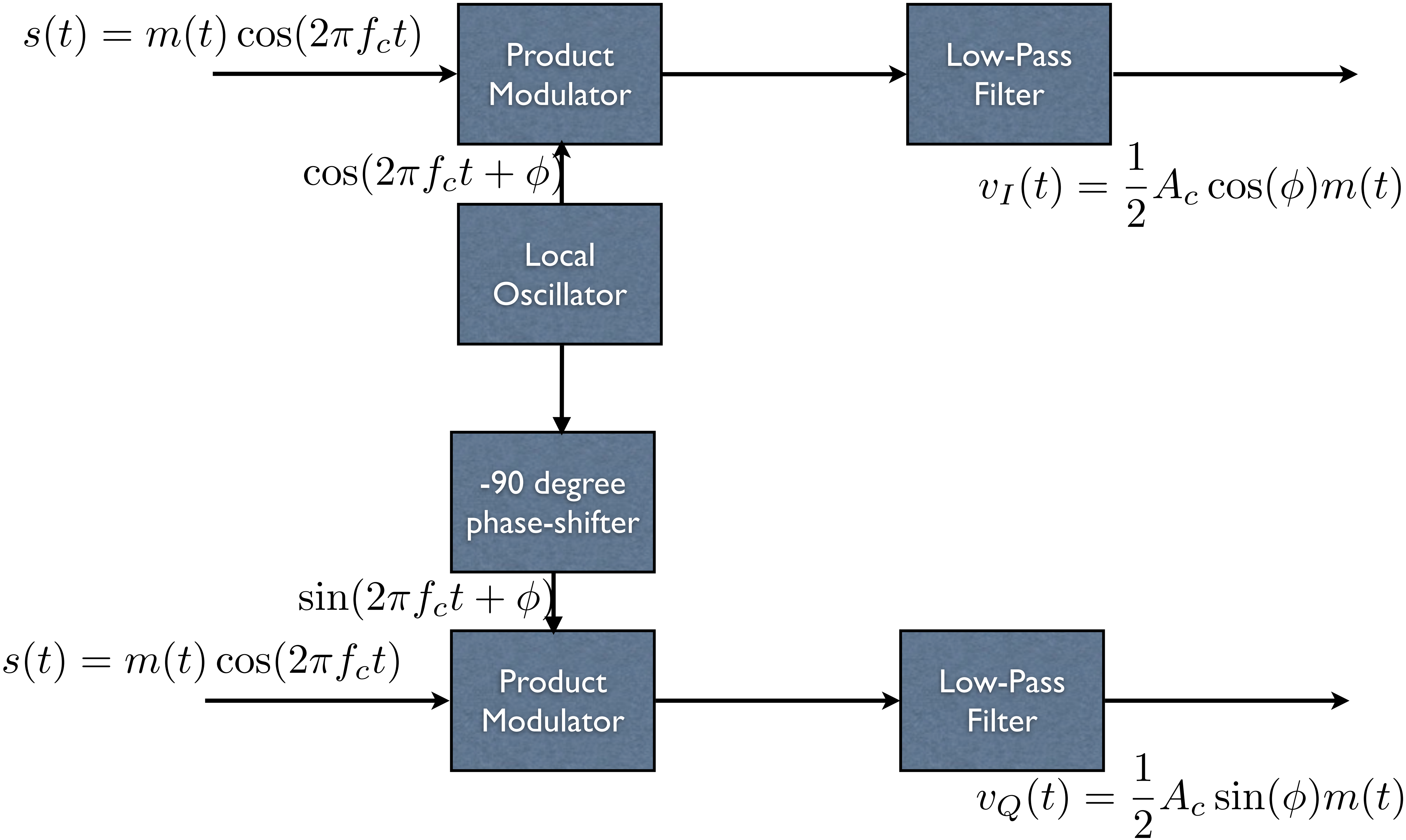

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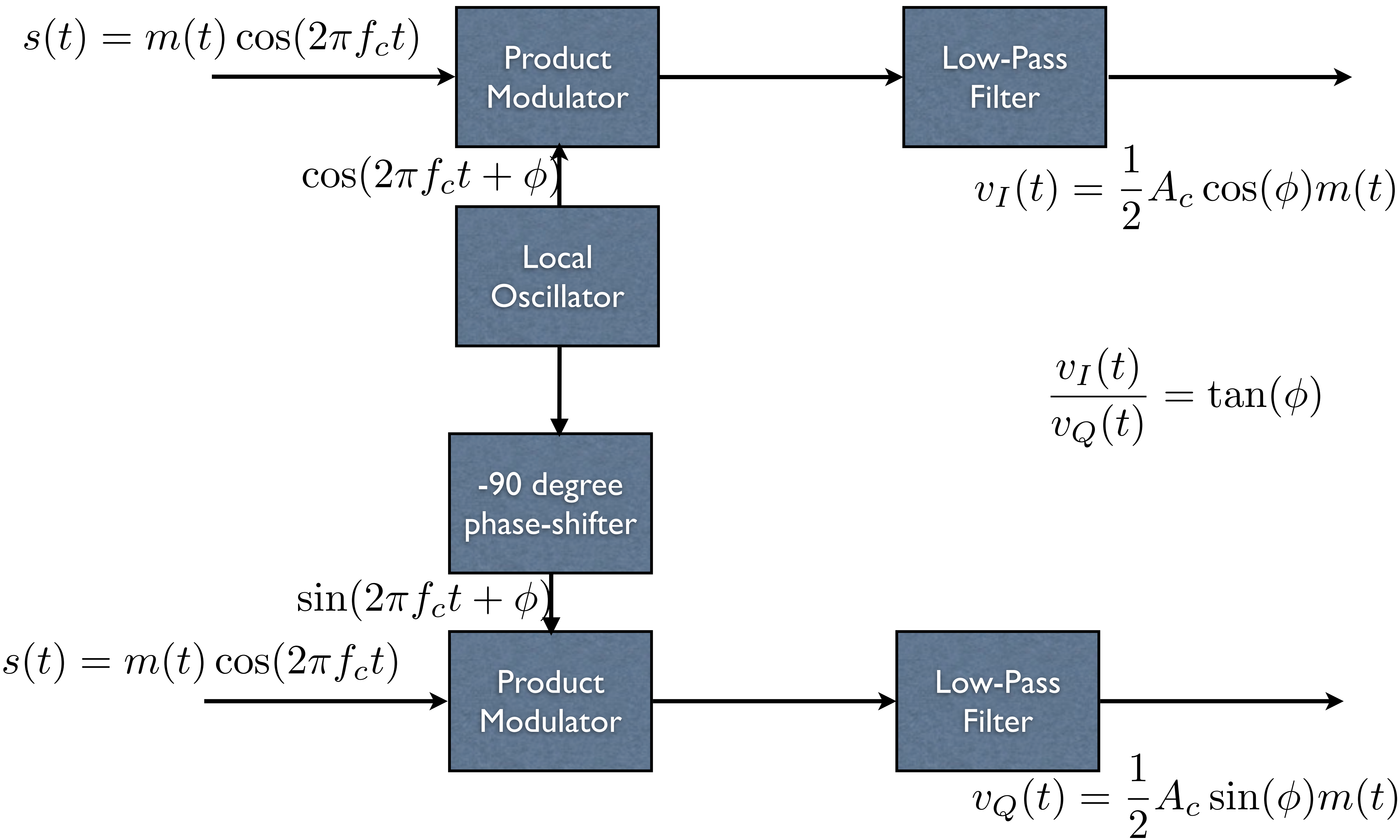


- Tagent function

$$\tan(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots, \qquad \text{for } |x| < \frac{\pi}{2}$$

- For small value of x

$$\tan(x) \approx x$$

Block Diagram of Costas Receiver

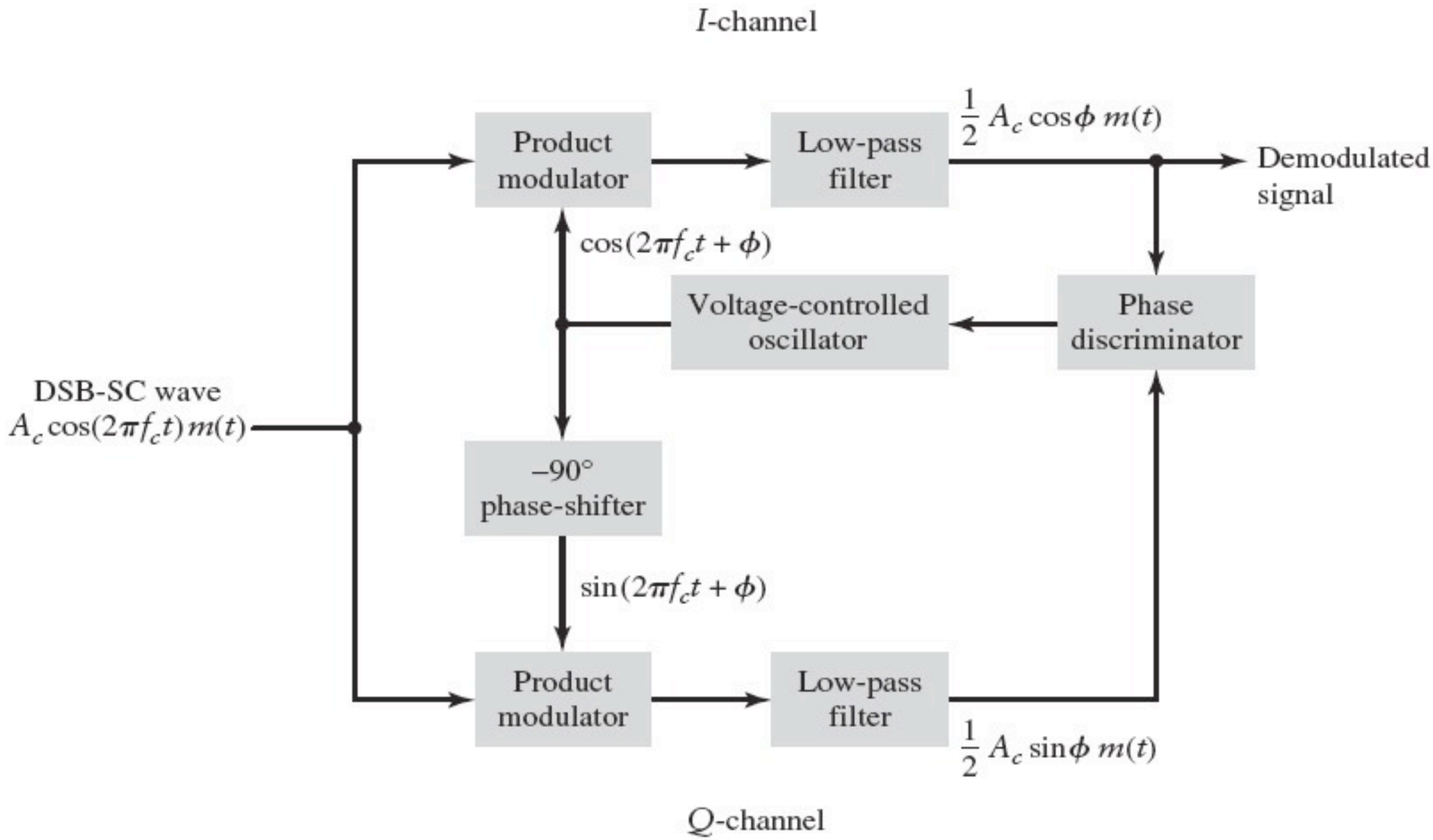


FIGURE 3.16 Costas receiver for the demodulation of a DSB-SC modulated wave.

[Ref: Haykin & Moher, Textbook]

Costas Receiver

- Consists of two coherent detectors supplied with the same input signal
    - Two local oscillators signals that are in phase quadrature with respect to each other.
        - I-Channel: In-phase coherent detector
        - Q-Channel: Quadrature-phase coherent detector
- Phase control in the Costas receiver ceases with modulation
    - Which means that phase-lock would have to be re-established with the reappearance of modulation.

Quadrature Amplitude Modulation (QAM)

- This scheme enables two DSB-SC modulated waves to occupy the same channel bandwidth.
- Bandwidth-conversion system
- This system send a pilot signal outside the passband of the modulated signal to maintain the synchronization

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

Block Diagram of QAM Modulator/Demodulator

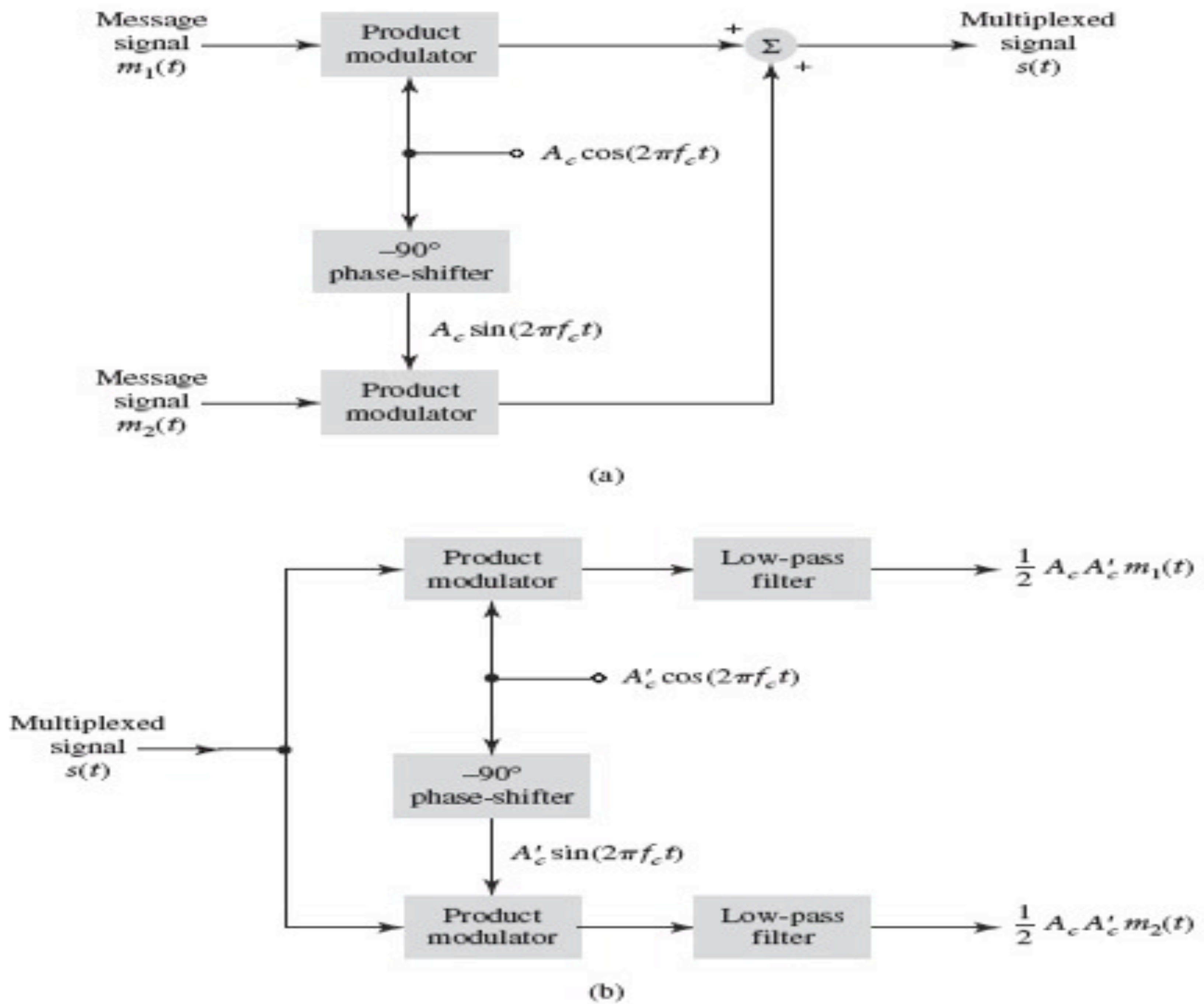


FIGURE 3.17 Quadrature-carrier multiplexing system: (*a*) Transmitter, (*b*) receiver.

[Ref: Haykin & Moher, Textbook]

Hilbert Transform

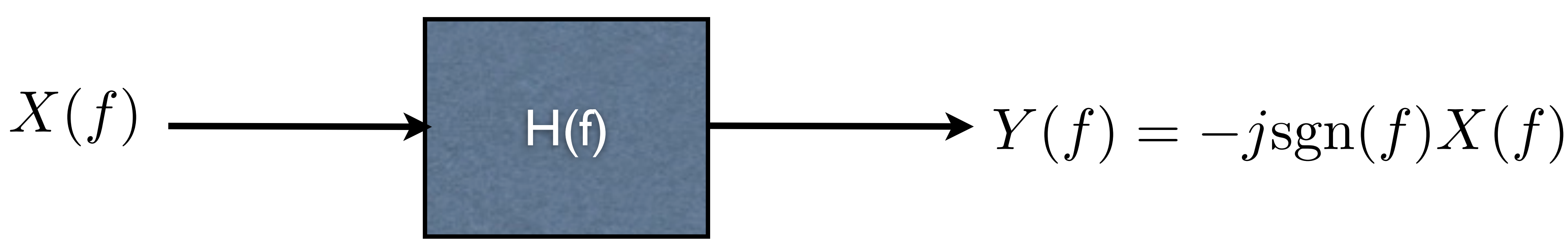
- Consider a filter that simply phase shifts all frequency components of its input by $-\pi/2$ radians, that is, its transfer function is

$$H(f) = -j\text{sgn} f$$

- Note that

$$|H(f)| = 1, \text{ and } \angle H(f) = \begin{cases} -\pi/2 & f > 0, \\ \pi/2 & f < 0 \end{cases}$$

- Input-Hilbert filter-Output signals



- Let us denote

$$\hat{x}(t) = \mathcal{F}^{-1}[Y(f)]$$

- Then

$$\hat{x}(t) = \mathcal{F}^{-1}[-j\mathrm{sgn}(f)X(f)] = h(t) * x(t)$$

- Now let us calculate the inverse transform of h(t).
  - Recall $\mathcal{F}[\mathrm{sgn}(t)] = \dfrac{1}{j\pi f}$, then using the duality property we have

$$\mathcal{F}^{-1}[\mathrm{sgn}(f)] = \frac{1}{j\pi(-t)} = \frac{j}{\pi t}$$

- We get the Fourier transform pair

$$\frac{j}{\pi t} \Longleftrightarrow \mathrm{sgn}(f) \qquad \text{or} \qquad \frac{1}{\pi t} \Longleftrightarrow -j\mathrm{sgn}(f)$$

- Now we obtain the output of the filter

$$\hat{x}(t) = x(t) * h(t) = \int_{-\infty}^{\infty} \frac{x(\lambda)}{\pi(t-\lambda)}\, d\lambda$$

- The function $\hat{x}(t)$ is defined as the Hilbert transform of $x(t)$.

- Remarks

  - The Hilbert transform corresponds to a phase shift of $-\pi/2$.

  - The Hilbert transform of $\hat{x}(t)$

$$\hat{\hat{x}}(t) = -x(t)$$

Properties of Hilbert Transform

1. Energies are equal

$$|\hat{X}(f)|^2 = |-j\text{sgn}(f)|^2|X(f)|^2 = |X(f)|^2$$

2. A signal and its Hilbert transform are orthogonal;

$$\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = 0 \qquad \int_{-\infty}^{\infty} X(f)\hat{X}(f)\,df = 0$$

Analytic Signals

- Definition of the analytic signal $x_p(t)$

$$x_p(t) = x(t) + j\hat{x}(t)$$

- Fourier transform of the analytic signal

$$X_p(f) = X(f) + j[-j\text{sgn}(f)X(f)] = X(f)[1 + \text{sgn}(f)]$$

or

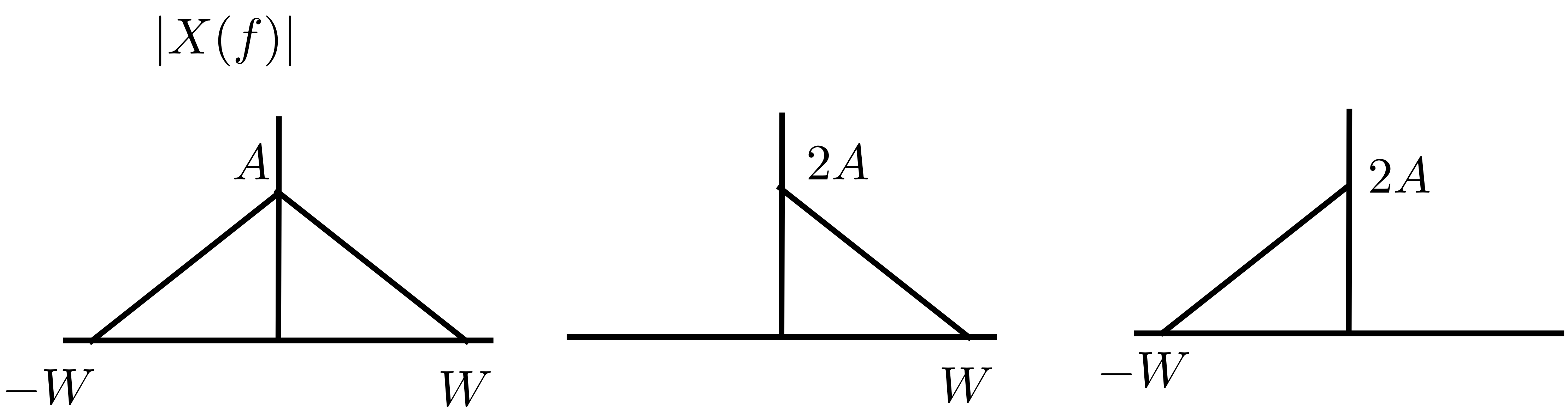
$$X_p(f) = \begin{cases} 2X(f), & f > 0 \\ 0, & f < 0 \end{cases}$$

- We can also show that

$$x_q(t) = x(t) - j\hat{x}(t)$$

and its Fourier transform

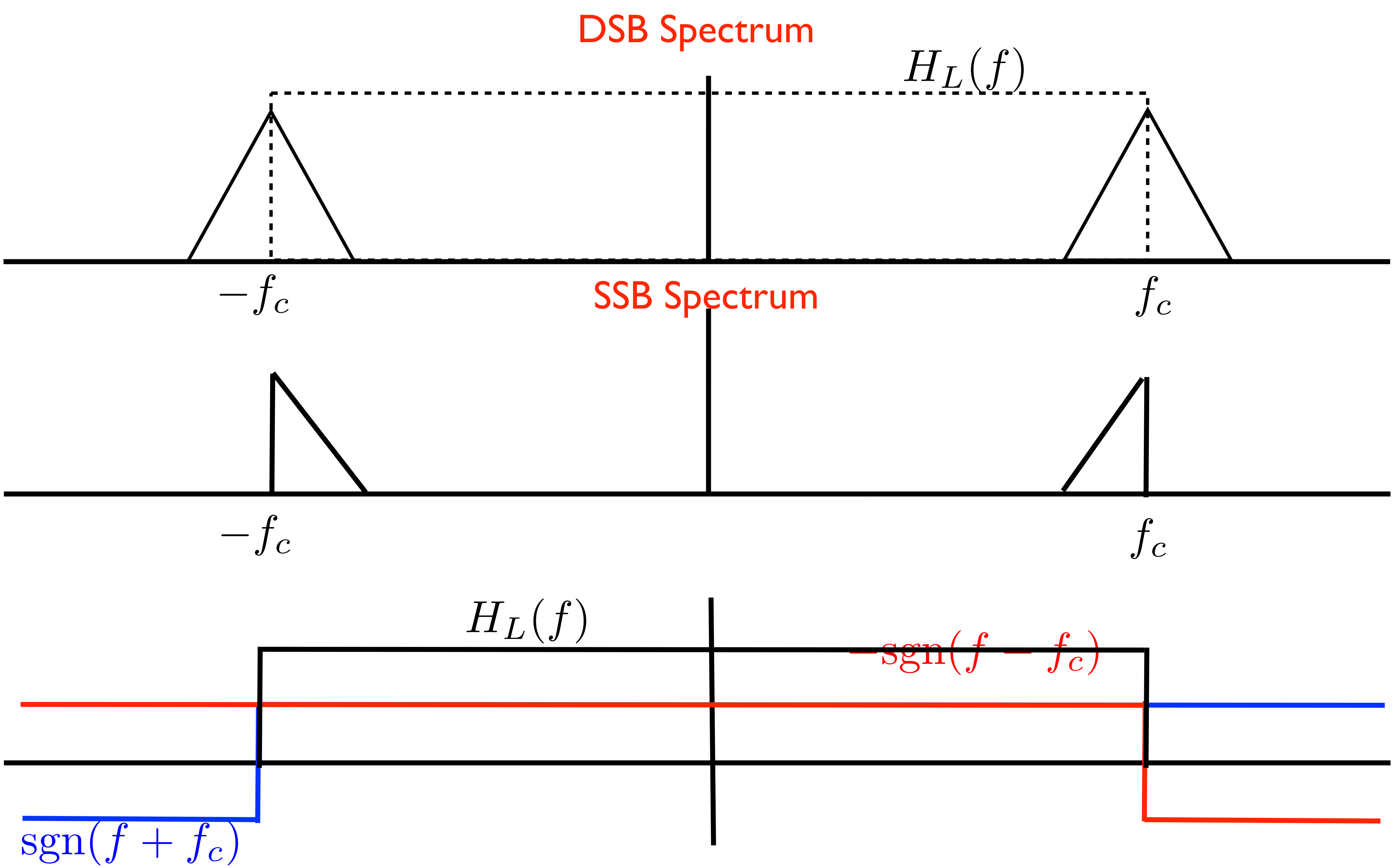
$$X_q(f) = X(f)\left[1 - \mathrm{sgn}(f)\right]$$
$$= \begin{cases} 0, & f > 0 \\ 2X(f), & f < 0 \end{cases}$$



Single-Sideband (SSB) Modulation

- SSB modulation
  - Suppress one of the two sidebands in the DSB-SC modulated wave prior to transmission
- Method of SSB Modulations
  - Time domain expression
    - SSB signal from DSB-SC is derived using the Hilbert transform.
  - Frequency domain expression
    - SSB signal from DSB-SC is generated Analytic signal is derived using the analytic signal.

Generation of LSB SSB



- Sideband filter

$$H_L(f) = \frac{1}{2}\left[\text{sgn}(f+f_c) - \text{sgn}(f+f_c)\right]$$

- Fourier transform of DSB-SC signal

$$S(f) = \frac{1}{2}A_c\left[M(f-f_c) + M(f+f_c)\right]$$

- Lower-Sideband SSB signal

$$= M(f-f_c)$$

$$S_{LSB}(f) = \frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f+f_c) + M(f-f_c)\text{sgn}(f+f_c)\right]$$

$$-\frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f-f_c) + M(f-f_c)\text{sgn}(f-f_c)\right]$$

or

$$= -M(f+f_c)$$

$$S_{LSB}(f) = \frac{1}{4}A_c\left[M(f+f_c) + M(f-f_c)\right] + \frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f+f_c) - M(f-f_c)\text{sgn}(f-f_c)\right]$$

- From our study of DSB

$$\frac{1}{2}A_c m(t)\cos(2\pi f_c t) \Longleftrightarrow \frac{1}{4}A_c\left[M(f+f_c)+M(f-f_c)\right]$$

- Also recall the Hilbert transform

$$\hat{m(t)} \Longleftrightarrow -j\text{sgn}(f)M(f),\;\; \hat{m}(t)e^{\pm j2\pi f_c t} \Longleftrightarrow -jM(f\mp f_c)\text{sgn}(f\mp f_c)$$

- Thus

$$\begin{aligned}\mathcal{F}^{-1}&\left\{\frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f+f_c)-M(f-f_c)\text{sgn}(f-f_c)\right]\right\}\\ &= -A_c\frac{1}{4j}\hat{m}(t)e^{-j2\pi f_c t}+A_c\frac{1}{4j}\hat{m}(t)e^{+j2\pi f_c t}\\ &= \frac{1}{2}A_c\hat{m}(t)\sin 2\pi f_c t\end{aligned}$$

- General form of a lower-sideband SSB signal

$$s_{LSB}(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) + \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

- Similarly, we can obtain the general form of a upper-sideband SSB signal

$$s_{USB}(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) - \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

- Block diagram for implementation

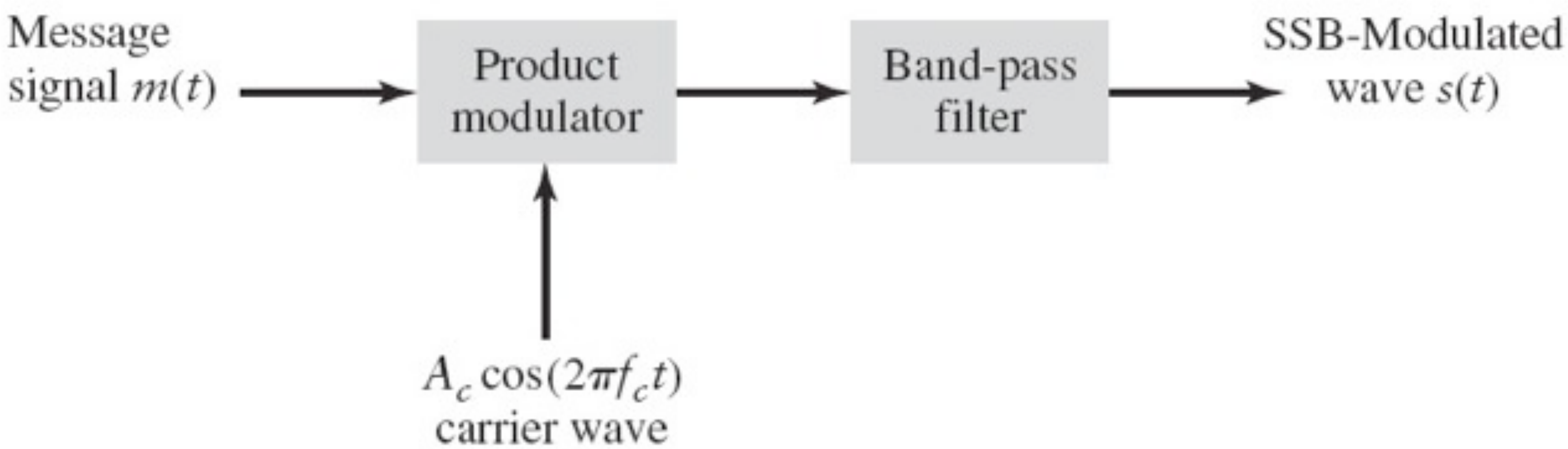


FIGURE 3.19 Frequency-discrimination scheme for the generation of a SSB modulated wave.

[Ref: Haykin & Moher, Textbook]

SSB Generation using Analytic Signal

- The positive-frequency portion of M(f)

$$M_p(f) = \frac{1}{2}\mathcal{F}\left[m(t) + j\hat{m}(t)\right]$$

- The negative-frequency portion of M(f)

$$M_n(f) = \frac{1}{2}\mathcal{F}\left[m(t) - j\hat{m}(t)\right]$$

- Upper-sideband SSB signal in the frequency domain

$$S_{USB}(f) = \frac{1}{2}A_c M_p(f - f_c) + \frac{1}{2}A_c M_n(f + f_c)$$

- Inverse Fourier transform

$$s_{USB}(t) = \frac{1}{4}A_c[m(t) + j\hat{m}(t)]e^{j2\pi f_c t} + \frac{1}{4}A_c[m(t) - j\hat{m}(t)]e^{-j2\pi f_c t}$$

or

$$s_{USB}(t) = \frac{1}{4}A_c m(t)[e^{j2\pi f_c t} + e^{-j2\pi f_c t}] - j\frac{1}{4}A_c\hat{m}(t)[e^{j2\pi f_c t} - e^{-j2\pi f_c t}]$$

Block Diagram for Implementation

- Wide-band phase-shifter is designed to produce the Hilbert transform in response to the incoming message signal.

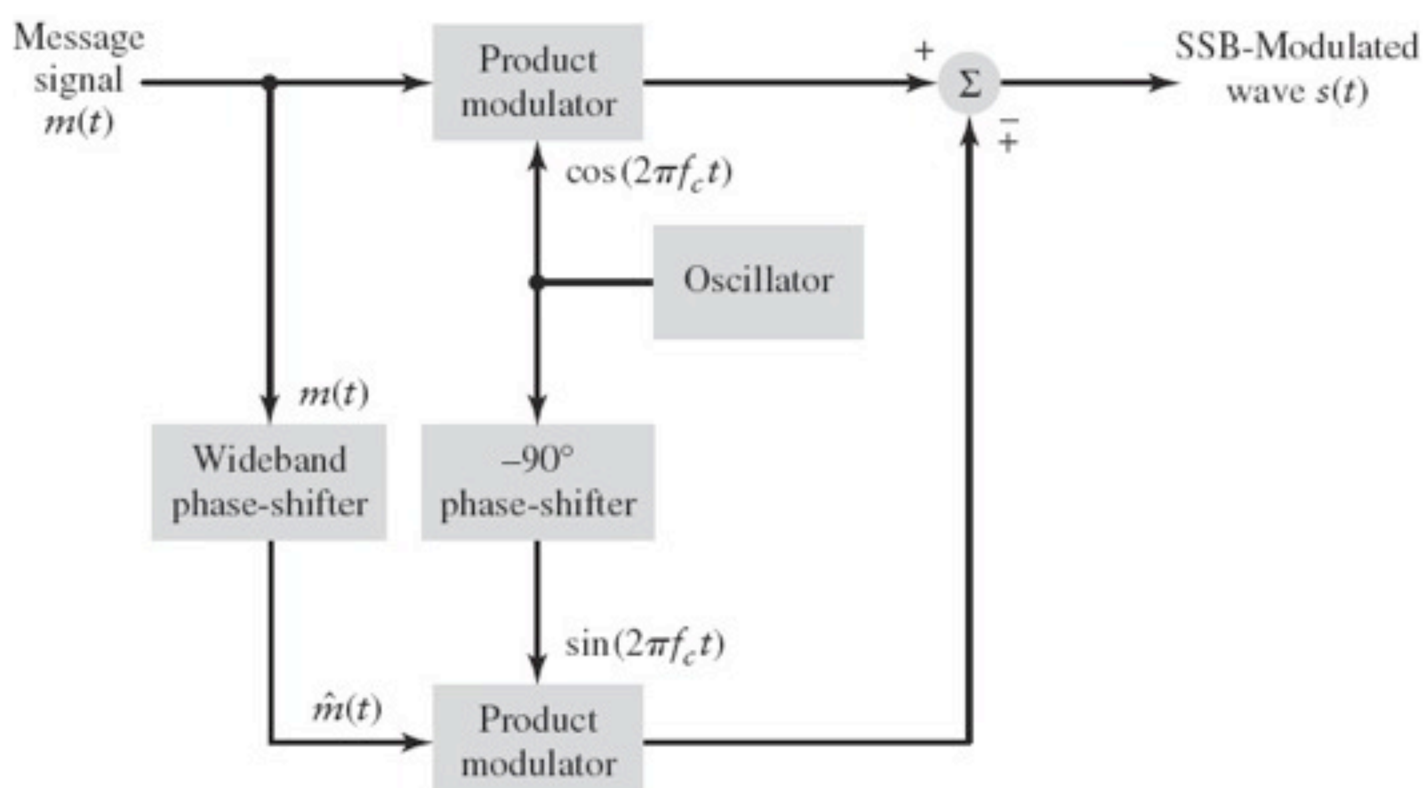


Figure 3.20 Phase discrimination method for generating a SSB-modulated wave. Note: The plus sign at the summing junction pertains to transmission of the lower sideband and the minus sign pertains to transmission of the upper sideband.

[Ref: Haykin & Moher, Textbook]

Coherent Detection of SSB

- Synchronization of a local oscillator in the receiver with the oscillator responsible for generating the carrier in the transmitter.
- Assume that the demodulation carrier has a phase error θ

$$
\begin{aligned}
d(t) &= \left[\frac{1}{2}m(t)\cos(2\pi f_c t) \pm \frac{1}{2}\hat{m}(t)\sin(2\pi f_c t)\right] \cdot \cos(2\pi f_c t + \theta) \\
&= \frac{1}{4}\left[m(t)\cos\theta + m(t)\cos(4\pi f_c t + \theta) \mp \hat{m}(t)\sin\theta \pm \hat{m}(t)\sin(4\pi f_c t + \theta)\right]
\end{aligned}
$$

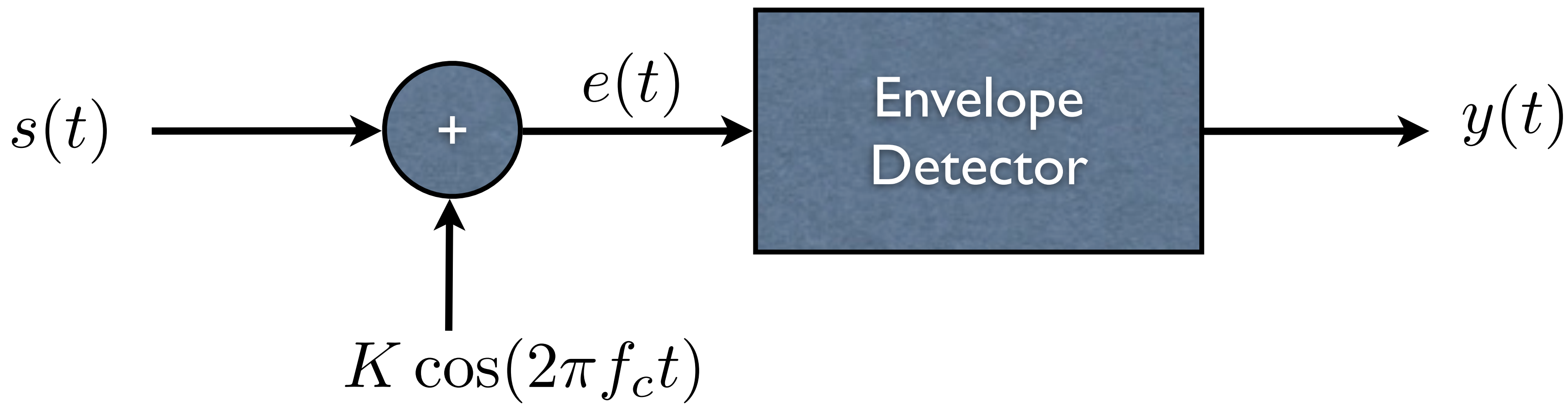
- Low-pass filtering and amplitude scaling yields

$$y(t) = m(t)\cos\theta \mp \hat{m}(t)\sin\theta$$

  - For θ equal to zero, the demodulator output is the desired message signal.

Envelope Detector of SSB Signal (Noncoherent)

- Consider the demodulator as follows:



$$e(t) = \frac{1}{2}[A_c m(t) + K]\cos(2\pi f_c t) \pm \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

$$y(t) = \sqrt{\left[\frac{1}{2}\left(A_c m(t) + K\right)\right]^2 + \left[\frac{1}{2}A_c \hat{m}(t)\right]^2}$$

- If we set K to be

$$\left[\frac{1}{2}A_c m(t) + K\right]^2 >> \left[\frac{1}{2}A_c \hat{m}(t)\right]$$

- Then,

$$y(t) \approx \frac{1}{2}A_c m(t) + K$$