Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

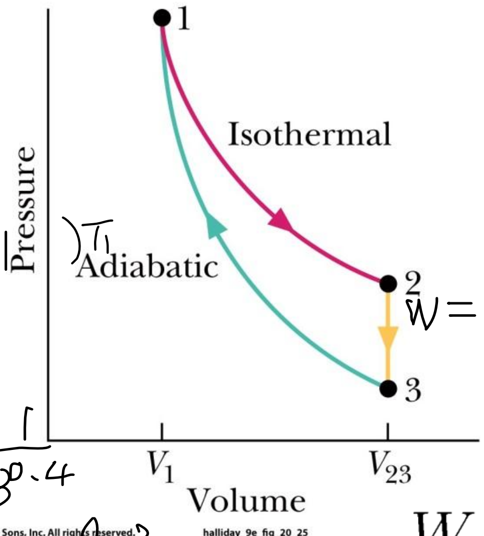
$dQ = dE + dW$

$\frac{dE}{T}$

Prob. 2

$\gamma = \frac{C_P}{C_V} = \frac{7}{5}$

Diatomic gas



$2 \to 3$

$\Delta E_{int} = \Delta Q$

$= nC_V \Delta T$

$= n\frac{5}{2}R\left(\frac{1}{3^{0.4}} - 1\right)T_1$

$\Delta S = nC_V \int \frac{dT}{T}$

$= n\frac{5}{2}R \ln \frac{1}{3^{0.4}}$

$= -\frac{5}{2}nR \cdot 0.4 \ln 3$

$= -nR\ln 3$

$W = 0$

$p_2/p_1,\ p_3/p_1,\ T_3/T_1$

$P_1V_1 = 3P_2V_1$

$\frac{P_2}{P_1} = \frac{1}{3}$

$1 \to 2$

PV^{γ}

$P_1V_1^{1.4} = P_3(3V_1)^{1.4}$

$\frac{P_3}{P_1} = \frac{1}{3^{1.4}}$

$\frac{T_3}{T_1} = \frac{1}{3^{0.4}}$

$TV^{\gamma-1}$

$T_1V_1^{0.4} = T_3(3V_1)^{0.4}$

$W/nRT_1,\ Q/nRT_1,\ \Delta E_{\text{int}}/nRT_1,\ \Delta S/nR$

$V_{23} = 3.0V_1$

$dW = pdV = \frac{nRT\,dV}{V}$

$W = \int_{V_1}^{V_{23}} nRT\frac{dV}{V} = nRT\ln 3$

$= Q$

$\Delta E_{int} = 0$

$\Delta S = \int \frac{dQ}{T} = \frac{1}{T}Q = \frac{W}{T}$

$= nR\ln 3$

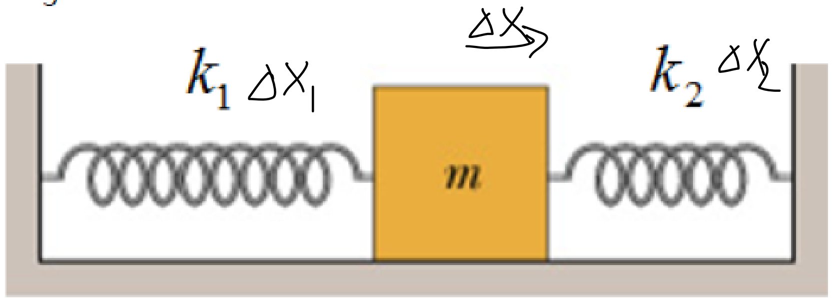
$-\Delta W = \Delta E_{int} = nC_V(T_1 - T_3)$

Problem 1. (25 points) Two springs each have unstressed (natural) lengths of 20 cm but different force constants: $k_1 = 40\ \mathrm{N/m}$ and $k_2 = 80\ \mathrm{N/m}$. They are fastened to a small mass, $m = 1.20\ \mathrm{kg}$, resting on a horizontal, frictionless surface [see Fig. 1]. The springs are stretched in opposite directions and fastened to the walls that are 70 cm apart. (Consider the mass m as a point particle.)

(a) How far from the left wall is the equilibrium position of the mass?

(b) What is the period τ_A of small harmonic oscillations of the mass along the direction of the springs?

(c) What is the period τ_{PE} of the potential energy when the mass undergoes a small harmonic oscillation?



<Fig. 1>

$$70 = 20 + \Delta x_1 + 20 + \Delta x_2$$

$$\Delta x_1 + \Delta x_2 = 30$$

$$\Delta x_1 = 2\Delta x_2$$

$$\frac{m\omega^2}{2} = kx^2$$

$$\Delta x_1 = 20,\quad \Delta x_2 = 10$$

$$F = -k_1\Delta x - k_2\Delta x$$
$$= -(k_1+k_2)\Delta x$$

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{\tau}$$

$$\omega\tau = 2\pi$$

$$\underline{\tau_A = 2\pi\sqrt{\frac{m}{k_1+k_2}}}$$

$$x = A\cos\omega t$$

$$V \propto x^2 \propto \cos^2\omega t$$

$$\tau_{PE} = \frac{1}{2}\tau_A = \pi\sqrt{\frac{m}{k_1+k_2}}$$

Problem 2-A. (25 points) A piano wire whose line density is $0.125\ \mathrm{g/cm}$ is under a tension of $500\ \mathrm{N}$.

(a) If its fundamental frequency of vibration is $220\ \mathrm{Hz}$ (musical note A3), find the length of the wire.

(b) If the amplitude of vibration at the center of the wire is $2\ \mathrm{mm}$, find the maximum transverse speed of the center.

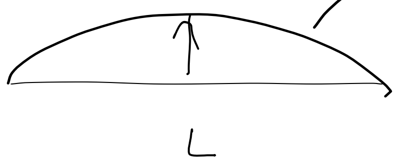
(c) How fast a person has to be approaching to the piano wire for the A3 note ($220\ \mathrm{Hz}$) to sound like the A#3 ($233\ \mathrm{Hz}$)? Take the speed of sound in air to be $3.30\times10^2\ \mathrm{m/s}$.

2mm

$$y = A\cos\omega t = A\cos 2\pi f t$$

$$\frac{dy}{dt} = -2\pi f A \sin 2\pi f t$$

$$v = 2\pi f A$$



$$\lambda = 2L$$

$$L = \frac{\lambda}{2}$$

$$v = \sqrt{\frac{T}{\mu}} = \lambda f = 2Lf$$

$$L = \frac{1}{2f}\sqrt{\frac{T}{\mu}} = \frac{1}{440}\sqrt{\frac{500}{1.25\times10^{-2}}} = \frac{1}{440}\sqrt{40000}$$

$$= \frac{200}{440}\,(\mathrm{m}) = \frac{5}{11}\ \mathrm{m}$$

$$f' = f\,\frac{v + v_D}{v}$$

$$\frac{233}{220} = 1 + \frac{v_D}{v}$$

$$\frac{v_D}{v} = \frac{13}{220}$$

$$v_D = 13\cdot\frac{3}{2} = \frac{39}{2}$$

Problem 2-B. (25 points) A nylon guitar string has a line density of $7.20\ \mathrm{g/m}$ and is under a tension of $162\ \mathrm{N}$. The fixed supports are distance $D = 90.0\ \mathrm{cm}$ apart. The string is oscillating in the standing wave pattern shown in Figure 2-B.

(a) Calculate the speed v, wavelength λ, and frequency f of the traveling waves whose superposition gives this standing wave.

(b) The string expanded a little as the room temperature has increased by $10\,^{\circ}\mathrm{C}$. In order to reproduce the same sound, the string tension needs to be adjusted. What should be the new tension? Assume that the coefficient of linear (thermal) expansion of the nylon string is $5 \times 10^{-5}/^{\circ}\mathrm{C}$. (Two end points are fixed.)

α

<Fig. 2-B>

$$v = \sqrt{\frac{\tau}{\mu}}$$

(a) $D = \frac{3}{2}\lambda \qquad \lambda = \frac{2}{3}D$

$$f = \frac{v}{\lambda} = \sqrt{\frac{\tau}{\mu}}\,\frac{3}{2D}$$

τ', μ'

$$\frac{\tau}{\mu} = \frac{\tau'}{\mu'}$$

$$\mu = \frac{m}{D}$$

$$\Delta L = \alpha D \Delta T \qquad \mu' = \frac{m}{D'}$$

$$\tau' = \tau\frac{\mu'}{\mu}$$

$$D' = (1+\alpha\Delta T)D \qquad \frac{\mu'}{\mu} = \frac{D}{D'}$$

$$= \tau\frac{1}{1+\alpha\Delta T}$$

Problem 3. (25 points) A bubble of n mol of helium is submerged in liquid water when the water (and thus the helium) undergoes a temperature increase from T_i to T_f at constant pressure P. As a result, the bubble expands from V_i to V_f. The helium is monatomic and ideal.

(a) How much energy is added to the helium as heat during the expansion?

(b) How much work W is done by the helium as it expands against the pressure of the surrounding water during the temperature increase.

(c) What will be ΔE_{int} (the change in internal energy) and ΔS (the change in entropy) of gas bubble caused by the thermal expansion?

$$\underline{\Delta Q = nC_p \Delta T} = \frac{5nR}{2}(T_f - T_i) = \frac{5}{2}P(V_f - V_i)$$

$$W = \int_{V_i}^{V_f} P\,dV = \underline{P}\,(V_f - V_i) = nR(T_f - T_i)$$

$$\Delta E = \Delta Q - W = \frac{3}{2}P(V_f - V_i)$$

$$dS = \frac{dQ}{T} = \frac{nC_p dT}{T} \qquad \Delta S = nC_p \ln\frac{T_f}{T_i}$$

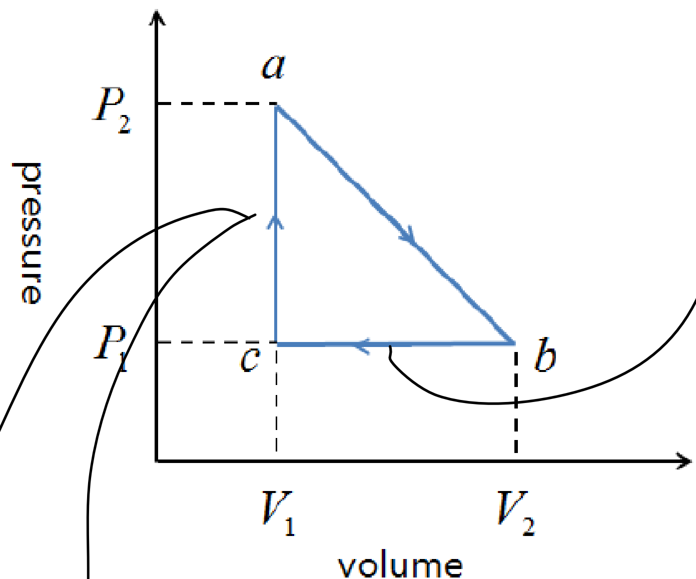
Problem 4. (25 points) A reversible engine operates in the cycle shown in Figure 4. Assume that the working substance is an ideal monatomic gas. ($T_b > T_a$)

(a) What will be the change in the internal energy ΔE_{int} of the gas during the process from a to b?

(b) What is the work W done by the engine during one complete cycle?

(c) What is the efficiency of this engine?

(d) What is ΔS_{ab} per mole. (ΔS_{ab} represents the change of entropy during the thermodynamic process from the 'a' to the 'b' state.)



<Fig. 4>

$$\frac{\Delta Q}{T} = nC_p\frac{\Delta T}{T}$$

$$\frac{\Delta S}{n} = C_V \ln\frac{T_c}{T_a} + C_p \ln\frac{T_b}{T_c}$$

$$= \frac{3}{2}R\ln\frac{P_1}{P_2} + \frac{5}{2}R\ln\frac{V_2}{V_1}$$

$$\Delta E = nC_V\frac{\Delta T}{T} = \frac{\Delta Q}{T}$$

$PV = nRT$

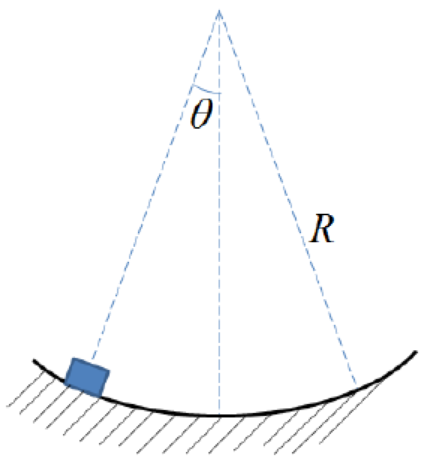
Problem 2. (25 points) A small block of mass m slides along the frictionless loop with radius R. The block is released from rest at the point that makes an angle θ_0 with the vertical. Use the approximation : $\sin\theta = \theta$ for small θ. The free fall acceleration is g.

(a) Find the net torque acting on the mass about the center of the loop when the mass makes a small angle θ with the vertical.

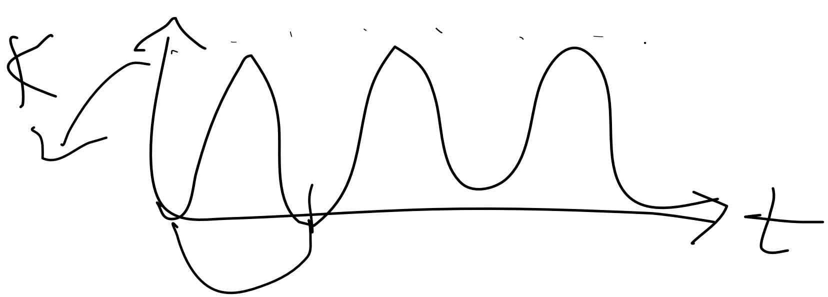
(b) Apply Newton's 2nd law to find the equation of motion for a small angle θ.

(c) Assume that $\theta(t)$ can be represented by $\theta(t) = A\sin\left(\frac{2\pi t}{T} + \phi\right)$. Find the constants A(amplitude), T(period), and ϕ(phase).

(d) Plot the graph of kinetic energy of the mass as a function of time.



<Fig. 2>



$$\theta(t) = \theta_0 \sin\left(\frac{2\pi t}{T} + \frac{\pi}{2}\right)$$

$$= \theta_0 \cos \frac{2\pi}{T} t$$

$T =$

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} m R^2 \omega^2$$

$$= \frac{mR^2}{2}\left(\frac{2\pi\theta_0}{T}\right)^2 \sin^2 \frac{2\pi t}{T}$$

$$= \frac{2\pi^2 m R^2 \theta_0^2}{T^2} \sin^2 \frac{2\pi t}{T}$$

$f_1 - f_2$

Problem 3. (25 points) Two pipes (length $0.30\ \mathrm{m}$ and $0.27\ \mathrm{m}$) open at both ends produce the sound waves that have the second lowest resonant frequencies for each pipes. The speed of sound is $300\ \mathrm{m/s}$.

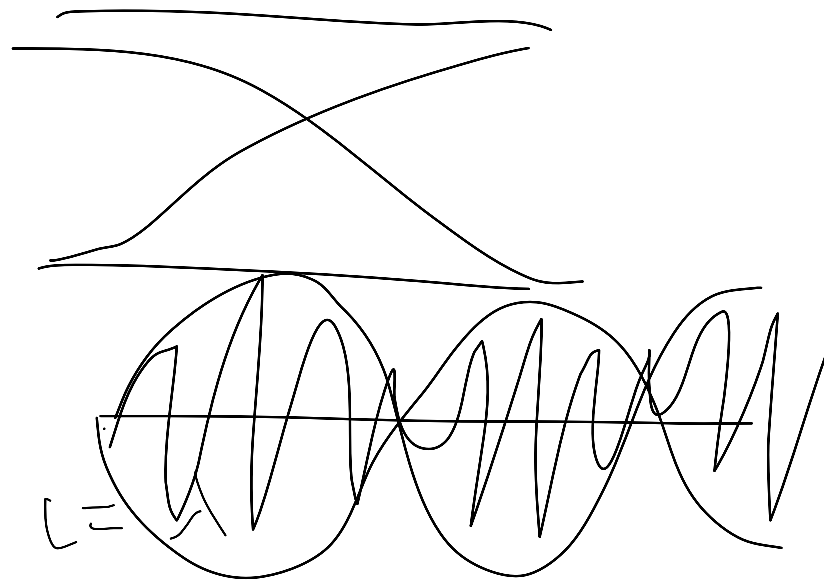
(a) Draw the wave pattern for the pipe of length $0.30\ \mathrm{m}$.

(b) Calculate the frequencies of the sound waves for the two pipes. Ignore the error smaller than $1\ \mathrm{Hz}$.

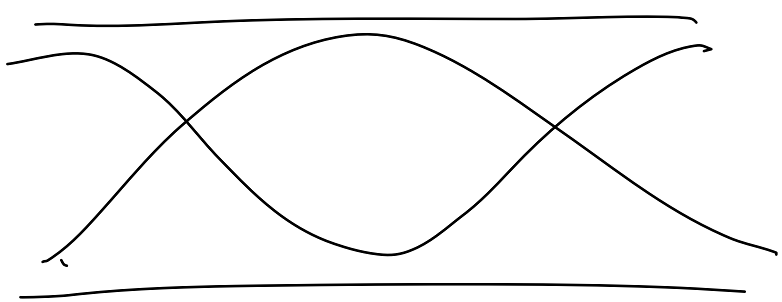
(c) Assume that the two waves are the sine waves of equal amplitude A and phase 0. Express the superposed wave in terms of time t, amplitude A, and the frequencies found in (b).

Use $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$.

(d) Plot the graph of the superposed wave as a function of time t at a fixed location.



L=30cm



$f = \dfrac{v}{\lambda} = \dfrac{v}{L}$

10000

$\dfrac{3000}{0.3}$ $\dfrac{\cancel{30000}}{0.279}$

1000 1111

$A\sin(2\pi f_1 t) + A\sin(2\pi f_2 t)$

$= 2A\sin(\pi(f_1+f_2)t)\cos(\pi(f_1-f_2)t)$

Problem 4. (25 points) An ideal gas of monatomic molecules undergoes a thermodynamic expansion (from volume V_0 to $8V_0$) determined by the relation $P = P_0(V_0/V)^{5/3}$, where P_0 is the pressure of the system when volume is V_0. The number of molecules in the gas is N, and the Boltzmann constant is k.

(a) Plot $P-V$ graph for the expansion.

(b) Using the definition of work, calculate the work done by the gas during the expansion.

(c) Using the fact that the internal energy of the gas is proportional to the temperature of the gas, calculate the change of the internal energy during the expansion.

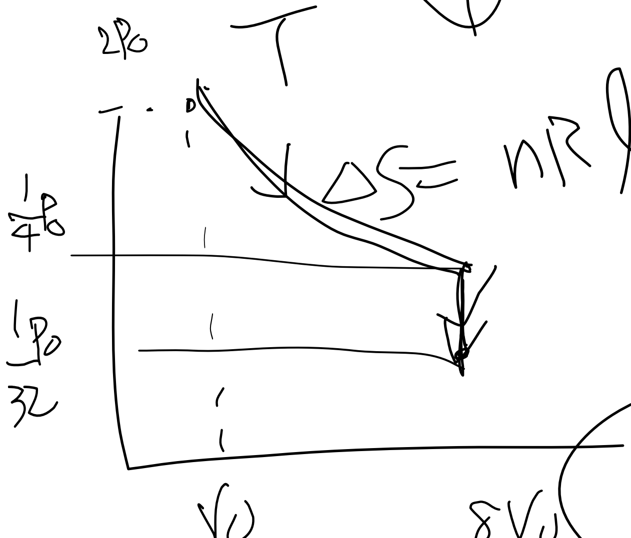
(d) If the gas takes an expansion (from volume V_0 to $8V_0$) determined by the relation $P = 2P_0(V_0/V)^2$, calculate the change of entropy for the process.

$\Delta S =$

PV

$nR\frac{1}{V}$

$$\frac{dQ}{T} = \frac{PdV}{T} + nC_V\frac{dT}{T}$$



$$\Delta S = nR\ln\frac{V_f}{V_i} + nC_V\ln\frac{T_f}{T_i}$$

$$nR\ln\frac{8V_0}{V_0} \Rightarrow$$

$$+\ nC_V\ln\frac{2}{1/4}$$

$$nR\ln 8 + \frac{3}{2}nR\ln 8$$

$$\Delta S = \cancel{nC_V}$$

$$= nR\frac{15}{2}\ln 2$$