KECE321 Communication Systems I

(Haykin Sec. 4.8 - Sec. 4.9)

Lecture #19, May 16, 2012

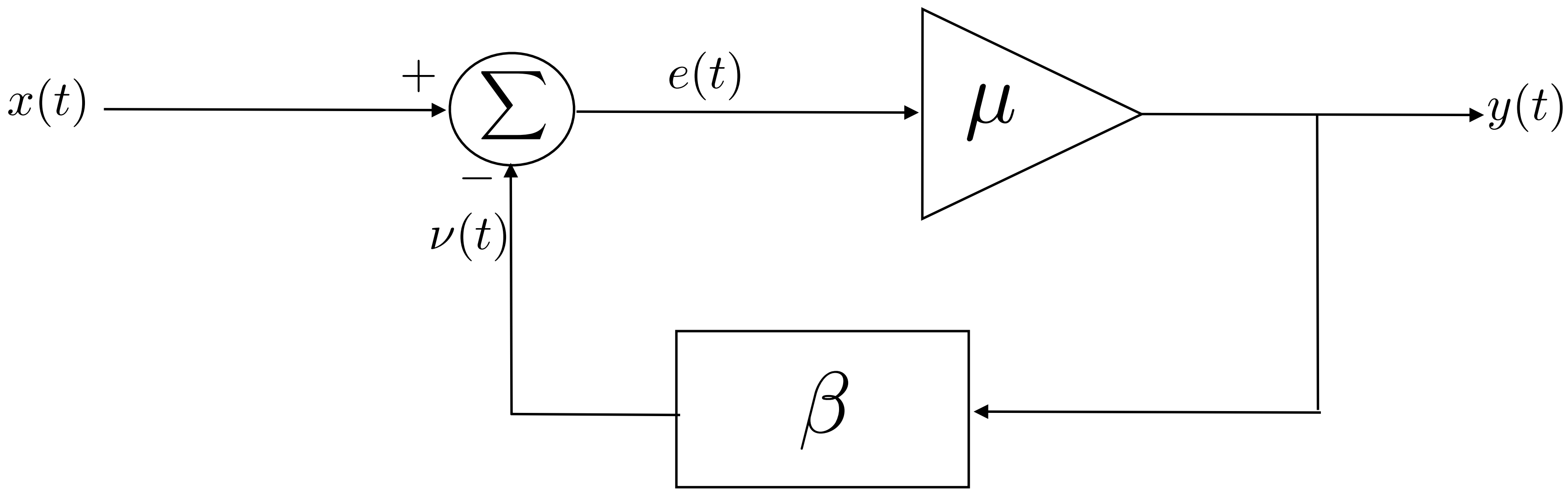
Prof. Young-Chai Ko

Summary

- Demodulation of FM signals
  - Phase locked loop (PLL)
- FM stereo multiplexing

Negative Feedback System

- Negative feedback system



$$[x(t) - \beta y(t)]\mu = y(t) \longleftrightarrow [X(f) - \beta Y(f)]\,\mu = Y(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{\mu}{1 + \mu\beta}$$

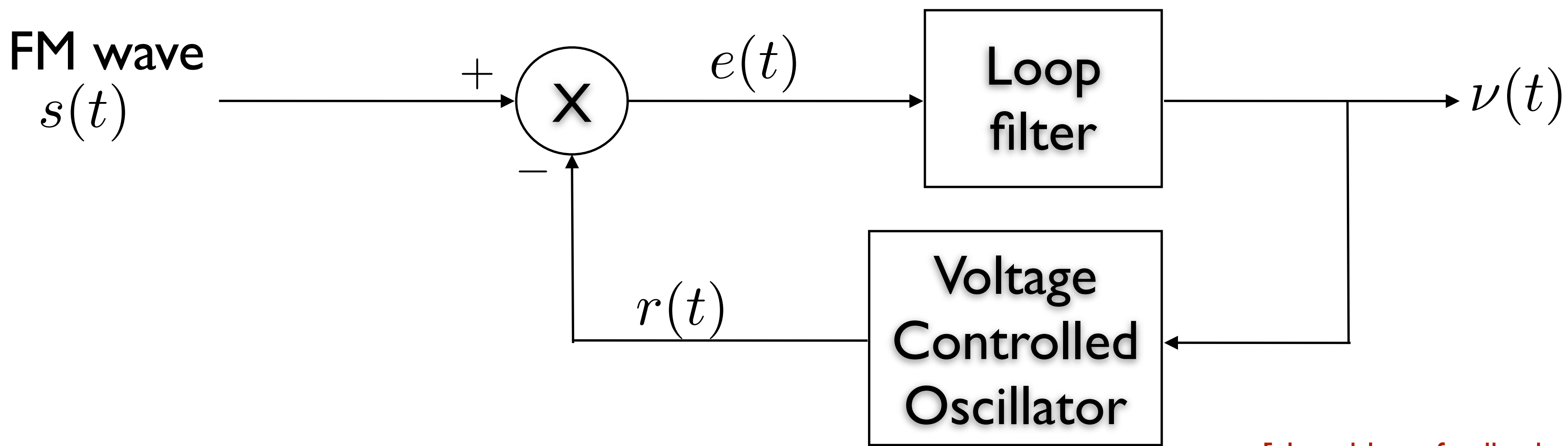
open-loop gain

For $\mu\beta >> 1$,

$$H(f) \approx \frac{1}{\beta}$$

Phase-Locked Loop (PLL)

- Block diagram of the PLL



[closed-loop feedback system]

- PLL consists of
  - VCO
  - Multiplier
  - Loop filter

- VCO
  - The frequency of the VCO is set precisely at the unmodulated carrier frequency f_c of the incoming FM wave $s(t)$.
  - The VCO output has a 90-degree phase-shift with respect to the unmodulated carrier wave.
- Suppose then that the incoming FM wave is defined by

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)]$$

where $\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$

  - FM wave produced by the VCO

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]$$

where $\phi_2(t) = 2\pi k_\nu \int_0^t \nu(\tau)\, d\tau$

- At the output of the multiplier, there are two components

1. double-frequency term

$$k_m A_c A_v \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)]$$

where k_m is the *multiplier gain.*

2. difference-frequency term

$$k_m A_c A_v \sin[\phi_1(t) - \phi_2(t)]$$

- With the loop filter designed to suppress the high-frequency components in the multiplier's output, we may henceforth discard the double-frequency term. Doing this, we may reduce the signal applied to the loop filter to

$$e(t) = k_m A_c A_v \sin[\phi_e(t)]$$

where $\phi_e(t)$ is the phase error defined by

$$\phi_e(t) = \phi_1(t) - \phi_2(t) = \phi_1(t) - 2\pi k_v \int_0^t \nu(\tau)\, d\tau$$

- Phase-lock

  - When the phase error $\phi_e(t)$ is zero, the phase-lock loop is said to be in *phase-lock*.

  - It is said to be near-phase-lock when the phase error $\phi_e(t)$ is small compared with one radian, under which condition we may use the approximation

$$\sin[\phi_e(t)] \approx \phi_e(t)$$

- Correspondingly we may approximate the error signal as

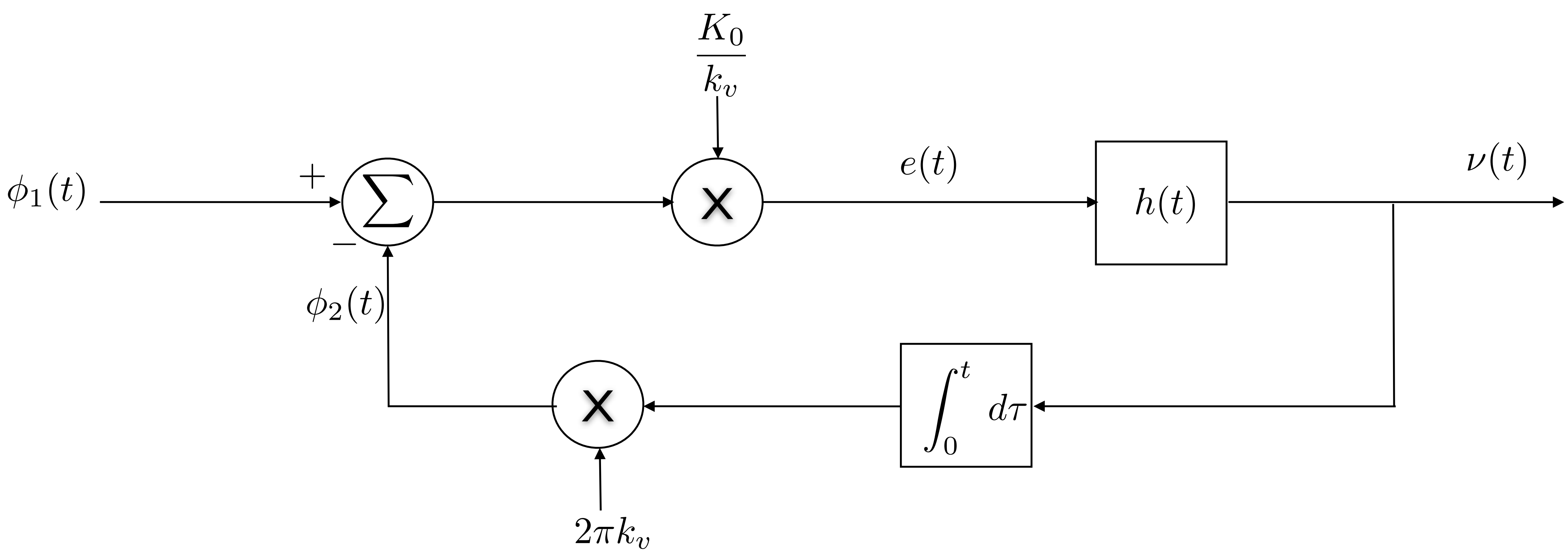
$$e(t) \approx k_m A_c A_v \phi_e(t) = \frac{K_0}{k_v}\phi_e(t)$$

where the new parameter $K_0 = k_m k_v A_c A_v$ is called the *loop-gain* parameter of the phase-lock loop.

- Let $h(t)$ be the impulse response of the loop filter. Then the output of the loop filter is

$$\nu(t) = \int_{-\infty}^{\infty} e(\tau)h(t-\tau)\,d\tau$$

- Linearized feedback model of the PLL



- Some observations from the linearized feedback model of the PLL

1. The feedback path is defined solely by the scaled integrator which is the VCO's contribution to the model. Correspondingly, the inverse of this feedback path is described in the time domain by the scaled differentiator:

$$\nu(t) = \frac{1}{2\pi k_v}\left(\frac{d\phi_2(t)}{dt}\right)$$

2. The closed-loop time-domain behavior of the phase-lock loop is described by the overall output $\nu(t)$ produced in response to the angle $\phi_1(t)$ in the incoming FM wave $s(t)$.

3. The magnitude of the open-loop transfer function of the phase-locked loop is controlled by the loop-gain parameter K_0.

- Assuming that the loop-gain parameter K_0 is large compared with unity, application of the feedback theorem to the model of the linearized feedback of the PLL teaches us that the closed-loop transfer function (i.e., closed-loop time-domain behavior) of the PLL is effectively determined by the inverse of the transfer function (i.e., time-domain behavior) of the feedback path.

- Accordingly, in light of the feedback theorem we may related the overall output $\nu(t)$ to the input angle

$$\nu(t) \approx \frac{1}{2\pi k_v} \left(\frac{d\phi_1(t)}{dt} \right)$$

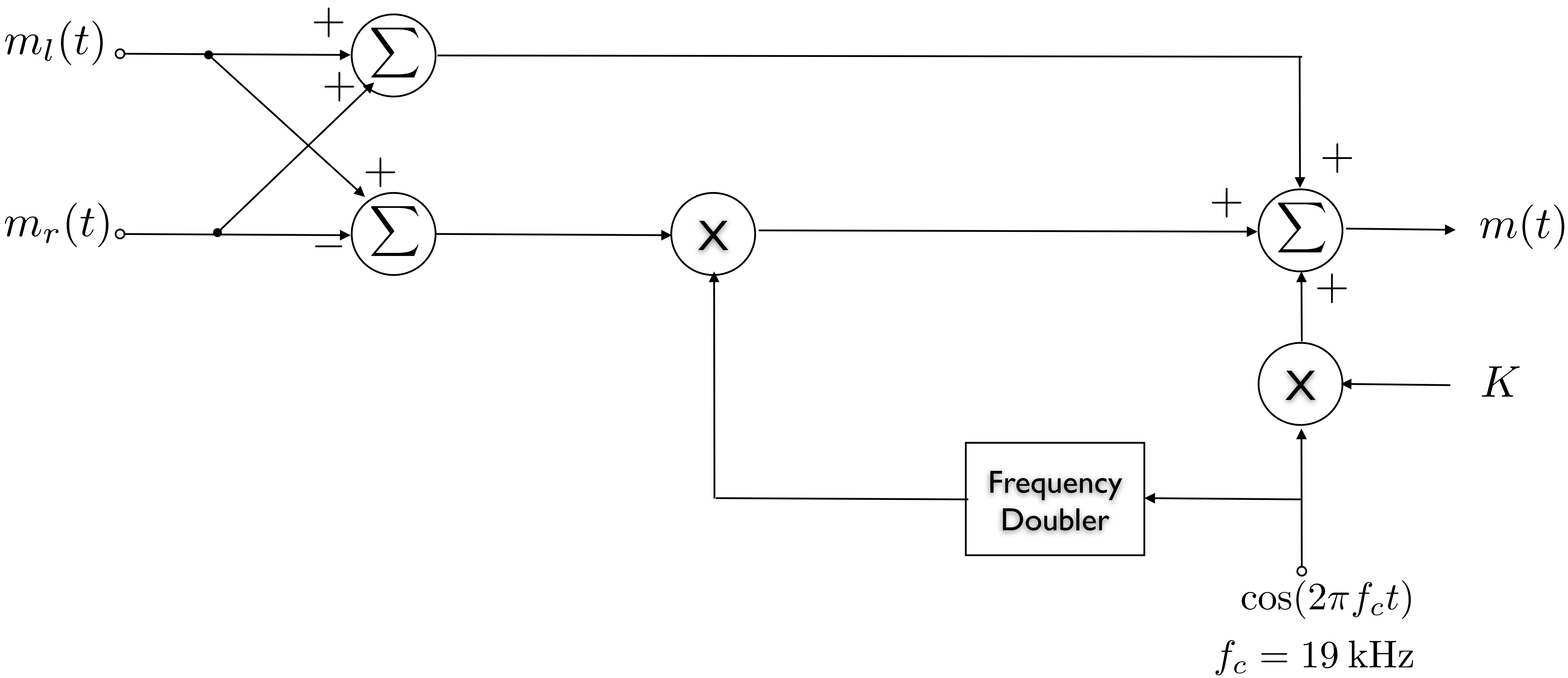
- Permitting K_0 to assume a large have has the effect of making the phase error $\phi_e(t)$ approaches zero. Under this condition, we have $\phi_1(t) \approx \phi_2(t)$.

- From the linearized feedback model of the PLL, we can obtain the following:

$$\begin{aligned} \nu(t) &= \frac{1}{2\pi k_v}\left(\frac{d\phi_2(t)}{dt}\right) = \frac{1}{2\pi k_v}\left(\frac{d\phi_1(t)}{dt} - \frac{d\phi_e(t)}{dt}\right) \\ &= \frac{1}{2\pi k_v}\left(\frac{d\phi_1(t)}{dt}\right) = \frac{1}{2\pi k_v}\frac{d}{dt}\left(2\pi k_f \int_0^t m(\tau)\, d\tau\right) \\ &= \frac{k_f}{k_v} m(\tau) \end{aligned}$$

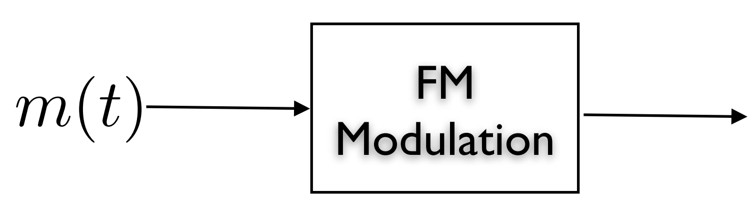
- When the system operates in the phase-lock mode or near phase lock and the loop-gain parameter K_0 is large compared with unity, frequency demodulation of the incoming FM wave $s(t)$ is accomplished.

FM Stereo Multiplexing

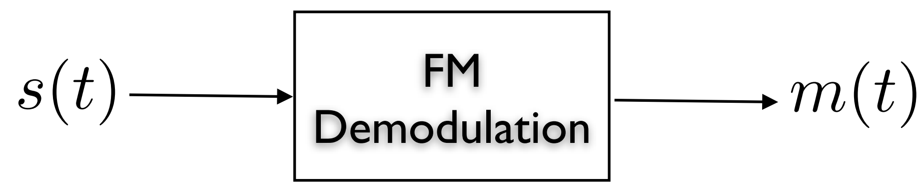


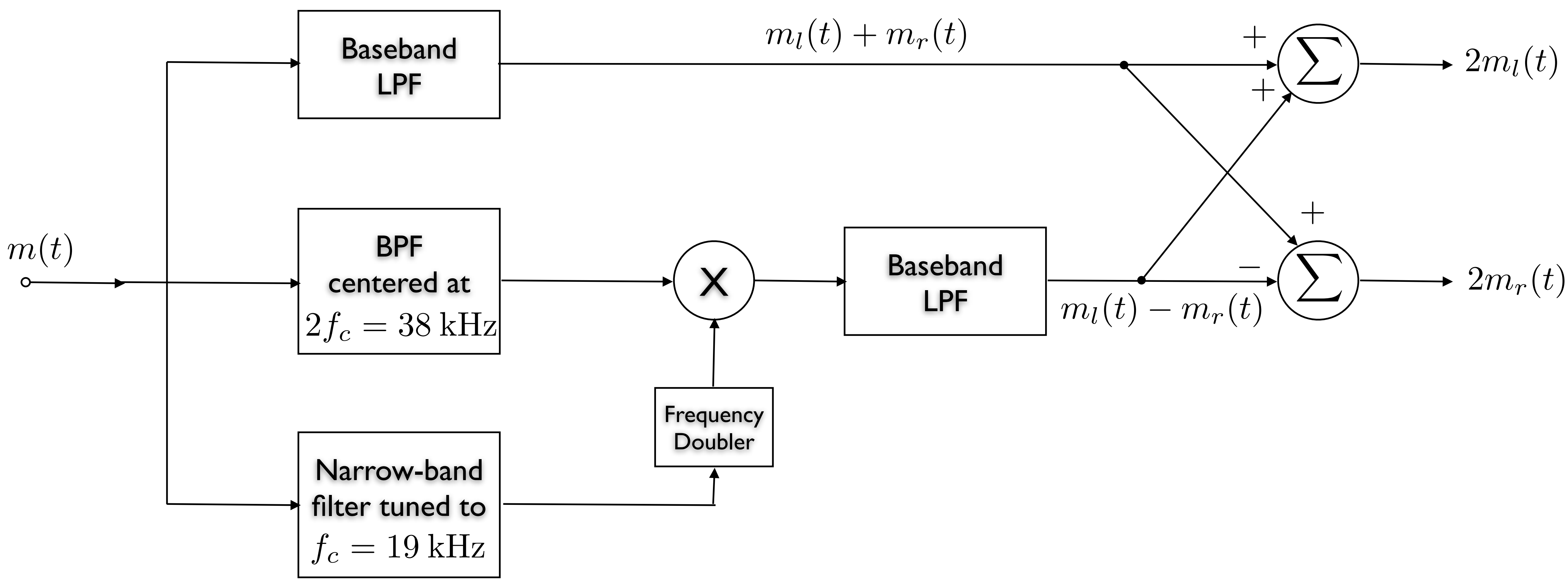
$$m(t) = [m_l(t) + m_r(t)] + [m_l(t) - m_r(t)]\cos(4\pi f_c t) + K\cos(2\pi f_c t)$$

Pilot symbol



$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

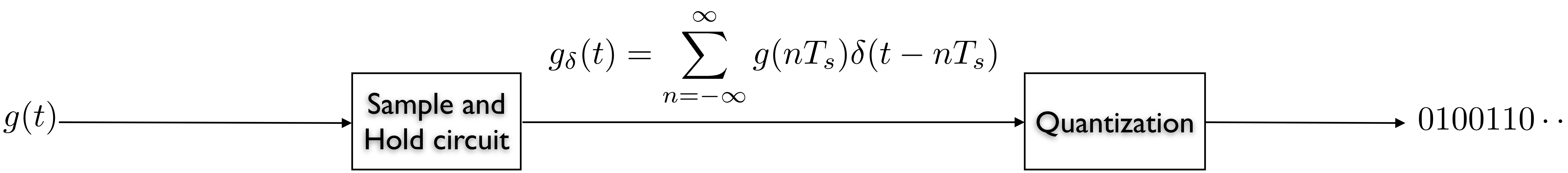

$s(t)$
FM Demodulation
$m(t)$


$m(t)$
Baseband LPF
BPF centered at $2f_c = 38\,\mathrm{kHz}$
Narrow-band filter tuned to $f_c = 19\,\mathrm{kHz}$
Frequency Doubler
X
Baseband LPF
$m_l(t) + m_r(t)$
$m_l(t) - m_r(t)$
+
+
+
−
$\sum$
$\sum$
$2m_l(t)$
$2m_r(t)$

Pulse Modulation: Transition from Analog to Digital Communications

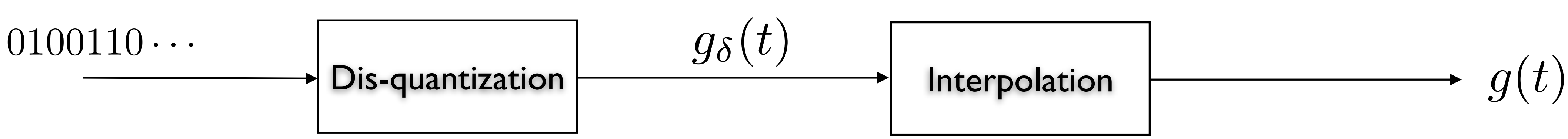
- Sampling process
- Pulse-amplitude modulation
- Pulse-position modulation
- Quantization
- Pulse code modulation
- Delta modulation

From Analog to Digital

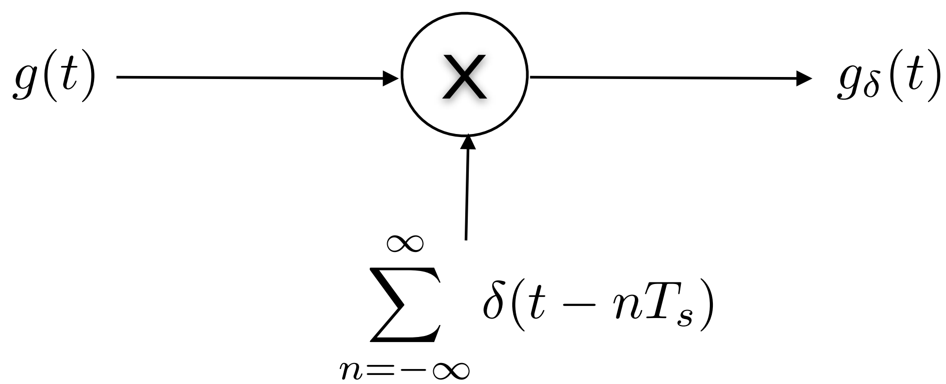


$$T_s \quad : \quad \text{sampling period}$$

$$f_s = \frac{1}{T_s} : \text{sampling rate}$$



Sampling Process



$$
\begin{aligned}
g_\delta(t) &= g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \\
&= \sum_{n=-\infty}^{\infty} g(t)\delta(t - nT_s) \\
&= \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)
\end{aligned}
$$

- Two questions
  - What are the restriction on $g(t)$ and T_s to allow perfect recovery of $g(t)$ from $g_\delta(t)$?
  - How is $g(t)$ recovered from $g_\delta(t)$?

- Fourier transform of $g_\delta(t)$

$$g_\delta(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad \longleftrightarrow \quad G_\delta(f) = G(f) * \mathcal{F}\left[\sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right]$$

  - Now let us find $\mathcal{F}\left[\sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right]$.

    - First of all, we note that the impulse train is periodic signal with a fundamental period T_s. Hence, it can be described in Fourier series form such as

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_s t}$$

where the Fourier coefficient c_n can be found as

$$c_n = \frac{1}{T_s} \int_{T_s} \delta(t) e^{-j2\pi n f_s t}\, dt = f_s$$

    - Hence, we have

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = f_s \sum_{n=-\infty}^{\infty} e^{j2\pi n f_s t}$$

- Now taking the Fourier transform of the impulse train in Fourier series form yields

$$
\begin{aligned}
\mathcal{F}\left[\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right] &= \int_{-\infty}^{\infty}\left(f_s\sum_{n=-\infty}^{\infty}e^{-j2\pi nf_st}\right)e^{-j2\pi ft}\,dt \\
&= f_s\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-j2\pi(f-nf_s)t}\,dt \\
&= f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)
\end{aligned}
$$

- where we make use of

$$\delta(t)\longleftrightarrow 1 \qquad \text{and} \qquad 1\longleftrightarrow\delta(f)$$

$$\Longrightarrow \int_{-\infty}^{\infty}e^{-j2\pi ft}\,dt=\delta(f)$$

- Hence, the Fourier transform of $g_\delta(t)$ can be written as

$$
\begin{aligned}
G_\delta(f) &= G(f) * \left[f_s \sum_{n=-\infty}^{\infty} \delta(f - n f_s) \right] \\
&= f_s \sum_{n=-\infty}^{\infty} G(f - n f_s)
\end{aligned}
$$

Sampling Process